

9 Summary and future outlook

In this chapter we summarize the work presented in this thesis and indicate some of the potential further studies.

9.1 Summary of the thesis

In the present thesis, we have made a detailed and systematic study on the soliton dynamics of the (1+1)D multicomponent CNLS system and (2+1)D multicomponent LSRI system. The study starts with the derivation of multicomponent bright and dark higher order soliton solutions by using Hirota’s bilinearization method and identification of the shape changing collision of bright solitons. Also the effect of the coupling terms in the mixed CNLS system is studied which leads to a new type of shape changing collision of bright solitons. And the collision scenario of degenerate and non-degenerate solitons in the coherently coupled NLS system is presented. We list below the important observations made in our investigation.

1. We have given bright multisoliton solution of multicomponent focusing and mixed CNLS system in Gram determinant form. Also we explicitly prove that the determinant form indeed satisfies the corresponding bilinear equations.

2. The bright soliton solutions of the mixed CNLS equations with two components (2-CNLS) with linear self- and cross-coupling terms have been
obtained by identifying a transformation that transforms the corresponding equation to the integrable mixed 2-CNLS equations. The study on the collision dynamics of bright solitons shows that there exists periodic energy switching, due to the coupling terms. This periodic energy switching can be controlled by a new type of shape changing collisions of bright solitons arising in a mixed 2-CNLS system, characterized by intensity redistribution, amplitude dependent phase shift, and relative separation distance. We also pointed out that this system exhibits large periodic intensity switching even with very small linear self-coupling strengths.

3. Mixed-type (bright-dark) soliton solutions of the integrable N-coupled nonlinear Schrödinger CNLS equations with mixed signs of focusing- and defocusing-type nonlinearity coefficients have been obtained by using Hirota’s bilinearization method. Generally, for the mixed N-CNLS equations the bright and dark solitons can be split up in N-1 ways. By analyzing the collision dynamics of these coupled bright and dark solitons systematically we pointed out that for \( N > 2 \), if the bright solitons appear in at least two components, nontrivial effects, such as onset of intensity redistribution, amplitude-dependent phase shift, and change in relative separation distance take place in the bright solitons during collision. However their counterparts, the dark solitons, undergo elastic collision but experience the same amplitude-dependent phase shift as that of bright solitons. Also we have presented bright-dark soliton solutions of CNLS system with defocusing nonlinearity.

4. Explicit forms of one- and two-soliton solutions of the coherently coupled NLS equations have been obtained using a non-standard type of Hirota’s bilinearization method. Analyzing the nature of the bright one-soliton solution we have reported degenerate solitons and non-degenerate solitons. Particularly, for non-degenerate solitons the density profile can vary from
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single hump to double hump profile including flat-top solitons. Our analysis on the collision dynamics revealed the fact that separate collisions among degenerate solitons alone or among non-degenerate solitons alone are elastic. On the other hand, collision of a degenerate soliton with a non-degenerate soliton exhibits nontrivial behaviour resulting in an intensity switching of the non-degenerate soliton spread up in the two components leaving the other soliton unaltered. This property will have immediate applications in soliton collision based computing. Apart from the switching, we have also observed that this collision transforms the soliton profile from single hump to double hump including flat-top profile or vice versa. We expect that this property can find application in pulse shaping in the context of nonlinear optics.

5. We have obtained explicitly the multi bright plane soliton solutions of physically interesting integrable (2+1) dimensional $(n+1)$-wave system by applying Hirota’s bilinearization procedure. We have also presented the results in a Gram determinant form for the multisoliton solutions of the multicomponent LSRI system along with the necessary proof. We have observed that the solitons in the short wave components can be amplified by merely reducing the pulse width of the long wave component. The study on collision dynamics shows that the solitons appearing in the short wave components undergo shape changing collisions with intensity redistribution and amplitude-dependent phase shift. This gives the exciting possibility of soliton collision based computing in higher dimensional integrable systems also. However, the solitons in the long wave component always undergo elastic collision.
9.2 Future outlook

1. In the present thesis we explicitly proved the bright-bright multisoliton solution given in Gram determinant form of (1+1)D CNLS and (2+1)D LSRI system indeed satisfies the corresponding bilinear equations. It will be of interest to give the proof for the existence of bright-dark multisoliton solution of CNLS and LSRI system.

2. We have given a detailed study of (1+1)D focusing, defocusing, mixed CNLS systems in the present thesis. In future, we wish to discretize the above mentioned systems and find the soliton solutions.

3. In the present thesis, we have studied the shape changing collision of bright solitons in multicomponent LSRI system. It will be of interest to induce breathers in LSRI system by suitably choosing the parameters and to study their dynamics.

4. We have obtained bright-dark soliton solutions of LSRI system. In future, we wish to induce breathers in bright and dark solitons and controlling them by suitably choosing the parameters.

5. We have not considered exact (2+1) dimensional exponentially localized solutions in CNLS/LSRI systems. This is a challenging problem which we hope to pursue in future.

6. Also we have not considered any (3+1) dimensional CNLS system in our study. This is again a challenging future task.

7. Also, we have not considered nonintegrable perturbations of the systems considered in this thesis. They are again very fascinating systems to investigate.

We hope to explore these and other related problems in the near future.