4 Periodic energy switching of bright solitons in mixed coupled nonlinear Schrödinger equations with linear self and cross coupling terms

4.1 Introduction

It has been shown that pico-second pulse propagation in twisted birefringent fibres [52, 119] is governed by the following coupled CNLS type equations

\[ iq_{1z} + q_{1tt} + \rho q_1 + \chi q_2 + 2\mu (|q_1|^2 + |q_2|^2) q_1 = 0, \]  
\[ iq_{2z} + q_{2tt} - \rho q_2 + \chi q_1 + 2\mu (|q_1|^2 + |q_2|^2) q_2 = 0, \]

where \( \chi \) and \( \rho \) are the normalized linear coupling constants caused by the periodic twist of the birefringence axes and the phase velocity mismatch from resonance, respectively. It can be reduced to the standard Manakov equation by using the transformation [120]

\[ q_1 = \cos \theta e^{i\Gamma_z} q_{1M} - \sin \theta e^{-i\Gamma_z} q_{2M}, \]  
\[ q_2 = \sin \theta e^{i\Gamma_z} q_{1M} + \cos \theta e^{-i\Gamma_z} q_{2M}, \]
where the subscript $M$ refers to the Manakov model and $\Gamma = \sqrt{\rho^2 + \chi^2}$ and $\theta = \frac{1}{2} \tan^{-1} \left( \frac{\chi}{\rho} \right)$. In Ref. [120], the type-I SCC has been studied in the presence of linear self and cross coupling terms and several interesting features has been pointed out. As pointed out in Chapter 1, during a two soliton collision process, if a particular soliton in a given component experiences enhancement/suppression in its intensity due to its collision with the second soliton then the same soliton experiences suppression/enhancement in the other component. We refer such type of shape changing collision as type-I SCC. If there is a possibility of either enhancement or suppression of intensity in a given soliton in all the components then the corresponding collision scenario is referred as type-II SCC. Now it is of interest to investigate type-II SCC arising in the mixed CNLS system in the presence of linear self and cross coupling terms.

For this purpose, we consider the following coupled mixed CNLS system with specific type of linear couplings

\begin{align}
&iq_{1z} + q_{1tt} + \rho q_1 - \chi q_2 + 2\mu \left( |q_1|^2 - |q_2|^2 \right) q_1 = 0, \quad (4.3a) \\
&iq_{2z} + q_{2tt} - \rho q_2 + \chi q_1 + 2\mu \left( |q_1|^2 - |q_2|^2 \right) q_2 = 0, \quad (4.3b)
\end{align}

where $\rho$ and $\chi$ are the self and cross coupling coefficients, respectively. The physical significance of the above system has been discussed in the introductory chapter.

The main aim is to analyse type-II SCC behaviour of bright solitons in a possible integrable extension of mixed 2-CNLS equations involving the linear self and cross coupling terms which can be of physical interest. This study reveals the fact that in this system during the type-II SCC process there also occurs periodic energy switching due to the linear coupling terms which can be suppressed/enhanced by type-II SCC. The distinct feature is that the system exhibits large periodic energy switching between the components with very small linear self coupling strength.
Section 4.2 deals with the statement of the problem. Section 4.3 contains bright one and two soliton solutions of the mixed 2-CNLS equations with linear self and cross coupling terms. Shape changing collision of solitons with periodic energy switching is analyzed in section 4.4.

**4.2 Statement of the problem**

Lazarides and Tsironis [77] have obtained the following set of governing equations for electromagnetic pulse propagation in isotropic and homogeneous non-linear left handed materials,

\[
\begin{align*}
    iq_{1,z} + q_{1,t} + 2\mu \left( \sigma_1 |q_1|^2 + \sigma_2 |q_2|^2 \right) q_1 &= 0, \\
    iq_{2,z} + q_{2,t} + 2\mu \left( \sigma_1 |q_1|^2 + \sigma_2 |q_2|^2 \right) q_2 &= 0,
\end{align*}
\]  

by taking the effective permittivity and effective permeability to be intensity dependent and following a reductive perturbational approach. Here \(q_1\) and \(q_2\) are the electric and magnetic field components of the electromagnetic pulse, respectively, the subscripts \(z\) and \(t\) denote the partial derivatives with respect to normalized distance and retarded time respectively, while \(\mu\) is the measure of nonlinearity, \(\sigma_1\) and \(\sigma_2\) can be either +1 or -1. From a mathematical point of view the above equation reduces to the integrable mixed CNLS equation presented in Ref. [75,76] when \(\sigma_1 = 1\) and \(\sigma_2 = -1\). In the previous chapter, singular and nonsingular bright soliton solutions of Eq. (4.4) have been obtained. There it has been shown that even though system (4.4) admits SCC as in Manakov system, the intensity redistribution occurs in a completely different way which is not possible in the Manakov system [52]. As pointed out in the introductory chapter we denote such a collision picture as type-II SCC.

The next natural step is to study how the linear self coupling resulting from the same component and the cross coupling arising from the other compo-
nent influence the type-II SCC process. In this regard we consider the non-dimensional mixed CNLS equations with linear self and cross couplings (4.1).

4.3 Bright soliton solutions

We notice by a careful analysis of Eq. (4.1) that Eq. (4.1) can be reduced to the well known integrable mixed CNLS equations (4.4) discussed in previous section through the transformation.

\[
\begin{pmatrix}
q_1 \\
q_2
\end{pmatrix} =
\begin{pmatrix}
\cosh(\theta z) e^{i\Gamma z} & \sinh(\theta z) e^{-i\Gamma z} \\
\sinh(\theta z) e^{i\Gamma z} & \cosh(\theta z) e^{-i\Gamma z}
\end{pmatrix}
\begin{pmatrix}
q_{1m} \\
q_{2m}
\end{pmatrix},
\]

(4.5a)

where the real parameters \(\theta\) and \(\Gamma\) are expressed in terms of the coupling parameters \(\rho\) and \(\chi\) as

\[
\theta = \tanh^{-1}\left(\frac{\chi}{\rho}\right), \quad \Gamma = \sqrt{\rho^2 - \chi^2}, \quad \chi \leq \rho.
\]

(4.5b)

In Eq. (4.5a), \(q_{1m}\) and \(q_{2m}\) are the soliton solutions of Eq. (4.4) given in section 4.2. It is obvious that if the cross coupling term \(\chi\) becomes zero the above solution \((q_1, q_2)\) is the same as that of the mixed CNLS equations (4.4), with \(\sigma_1 = -\sigma_2 = 1\), except for a multiplicative phase factor \(e^{i\Gamma z}\) in the \(q_1\) component and \(e^{-i\Gamma z}\) in the \(q_2\) component. We now confine our analysis to the cases where \((q_{1m}, q_{2m})\) correspond to bright soliton solutions and analyze the nature of \((q_1, q_2)\) satisfying Eq. (4.1) through the relation (4.3).

4.3.1 Bright one-soliton solution

With the knowledge of the bright one soliton solution of the integrable mixed 2-CNLS equations for \((q_{1m}, q_{2m})\), given in the previous chapter, we write down
the one soliton solution of Eq. (4.1) by using the transformation (4.3) as

\[
\begin{pmatrix}
q_1 \\
q_2
\end{pmatrix} =
\begin{pmatrix}
\cosh(\theta) e^{i\Gamma z} & \sinh(\theta) e^{-i\Gamma z} \\
\sinh(\theta) e^{i\Gamma z} & \cosh(\theta) e^{-i\Gamma z}
\end{pmatrix}
\begin{pmatrix}
A_1 \\
A_2
\end{pmatrix} k_{1R} \text{sech} \left( \eta_{1R} + \frac{R}{2} \right) e^{i\eta_1}, \tag{4.6a}
\]

where

\[
\eta_1 = k_1(t + ik_1z) = k_{1R}(t - 2k_{1I}z) + i(k_{1I}t + (k_{1R}^2 - k_{1I}^2)z) \equiv \eta_{1R} + i\eta_{1I}, \tag{4.6b}
\]

\[
A_j = \frac{\alpha^{(j)}_1}{\left| \mu \left( \sigma_1 |\alpha_1^{(1)}|^2 + \sigma_2 |\alpha_1^{(2)}|^2 \right) \right|^{1/2}}, \quad j = 1, 2, \tag{4.6c}
\]

\[
e^{-R} = \frac{\mu \left( \sigma_1 |\alpha_1^{(1)}|^2 + \sigma_2 |\alpha_1^{(2)}|^2 \right)}{(k_1 + k_{1I})^2}, \quad \sigma_1 = -\sigma_2 = 1. \tag{4.6d}
\]

In the above expressions the suffices R and I denote the real and imaginary parts, respectively. The one soliton solution (4.6) for \((q_1, q_2)\) is characterized by three arbitrary complex parameters \(\alpha_1^{(1)}, \alpha_1^{(2)}, \text{ and } k_1\), in addition to the real coupling parameters \(\rho\) and \(\chi\). Also note that the value of \(\chi\) is restricted by Eq. (4.5b) as \(|\chi| \leq |\rho|\) since \(\tanh \theta = \chi/\rho\). As in the case of mixed CNLS equations, solution (4.6) can be both singular and nonsingular. The condition for non-singular solution is given by \(|\alpha_1^{(1)}| > |\alpha_1^{(2)}|\) [69]. In this work we deal with nonsingular solutions only as they are of specific physical interest.

### 4.3.1.A Analysis on bright one-soliton solution

Typical plot of non-singular bright one soliton solution (4.6) of Eq. (4.1) with the condition \(|\chi| \leq |\rho|\) is shown in Fig. 4.1. From the figure we observe that the role of the linear coupling terms in (4.1) is to induce spatially periodic intensity switching between the two components \(q_1\) and \(q_2\). The periodic oscillations Fig. 4.1a during the intensity switching depends particularly on the difference between the self and cross coupling terms \((\rho \text{ and } \chi)\) in addition to the soliton parameters \(k_1, \alpha_1^{(1)}\) and \(\alpha_1^{(2)}\). For comparison we have plotted the corresponding
one-soliton solution in the absence of coupling terms in Fig. 4.1b. It is interesting to note that this periodic intensity switching can be completely suppressed by suitably choosing $A_2$ or $a_1^{(2)}$. To see this, we compute the intensity of the soliton in the two components and write them as

\[
\begin{align*}
\frac{|q_1|^2}{P} &= k_{1R}^2 \left( |A_1|^2 \cosh^2 \left( \frac{\theta}{2} \right) + |A_2|^2 \sinh^2 \left( \frac{\theta}{2} \right) + 2 |A_1| |A_2| \cosh \left( \frac{\theta}{2} \right) \sinh \left( \frac{\theta}{2} \right) \cos(2\Gamma z + Q) \right), \\
\frac{|q_2|^2}{P} &= k_{1R}^2 \left( |A_1|^2 \sinh^2 \left( \frac{\theta}{2} \right) + |A_2|^2 \cosh^2 \left( \frac{\theta}{2} \right) + 2 |A_1| |A_2| \cosh \left( \frac{\theta}{2} \right) \sinh \left( \frac{\theta}{2} \right) \cos(2\Gamma z + Q) \right),
\end{align*}
\]
where

\[ P = \text{sech} \left( k_1 \left( t - 2k_1 z \right) + \frac{R}{2} \right), \tag{4.7c} \]

\[ Q = \tan^{-1} \left( \frac{A_{1j}}{A_{1R}} \right) - \tan^{-1} \left( \frac{A_{2j}}{A_{2R}} \right). \tag{4.7d} \]

It is clear from the above expressions that the oscillatory term \( \cos(2\Gamma z + Q) \) appearing in (4.7a) and (4.7b) leads to the periodic oscillations during energy switching. One can also verify that the spatial period of oscillation is \( Z = \frac{\pi}{\Gamma} \). Thus for larger \( \Gamma \) the width of spatial oscillations is smaller. Also the amplitude of oscillation \( 2|A_1||A_2|\cosh\left(\frac{\theta}{2}\right)\sinh\left(\frac{\theta}{2}\right) \) increases with decreasing \( \Gamma \) due to the dependence of \( \theta \) on \( \chi \) and \( \rho \) (see Eq. (4.5b)). We also note from Eqs. (4.7a) and (4.7b) that the oscillatory term (third term on the right hand side) vanishes when \( |A_2| = 0 \), that is \( \alpha_1^{(2)} = 0 \), or \( |A_1| = 0 \), that is \( \alpha_1^{(1)} = 0 \). At a first glance, it seems that the periodic energy switching scenario is similar to that of the Manakov system [52] with linear coupling terms arising in the context of twisted birefringent fibers [86]. But the way in which the switching occurs is different due to the hyperbolic terms in Eqs. (4.7a) and (4.7b), since \( 0 \leq \sinh^2 \theta/2 < \infty, 0 \leq \cosh^2 \theta/2 < \infty \). To be more precise, the amplitude of periodic oscillations and periodic switching of energy between the two components can vary exponentially but restricted by the condition (4.5b).

### 4.3.2 Two soliton solution

The bright two soliton solution of Eq. (4.1) can be obtained by applying the transformation (4.3) to the two soliton solution of the integrable mixed 2-CNLS equation given in Chapter 3. The explicit form of the solution is
4.3 Bright soliton solutions

Various quantities found in Eq. (4.8) are defined as below:

\[
\begin{align*}
q_1 &= \frac{1}{D} \left( \left( \alpha_1^{(1)} e^{i\Gamma z} \cosh(\frac{\theta}{2}) + \alpha_1^{(2)} e^{-i\Gamma z} \sinh(\frac{\theta}{2}) \right) e^{\eta_1} + \left( \alpha_2^{(1)} e^{i\Gamma z} \cosh(\frac{\theta}{2}) + \alpha_2^{(2)} e^{-i\Gamma z} \sinh(\frac{\theta}{2}) \right) e^{\eta_2} + \left( e^{\delta_{11}} \cosh(\frac{\theta}{2}) e^{i\Gamma z} + e^{\delta_{12}} \sinh(\frac{\theta}{2}) e^{-i\Gamma z} \right) e^{\eta_1 + \eta_1^* + \eta_2} + \left( e^{\delta_{21}} \cosh(\frac{\theta}{2}) e^{i\Gamma z} + e^{\delta_{22}} \sinh(\frac{\theta}{2}) e^{-i\Gamma z} \right) e^{\eta_2 + \eta_2^* + \eta_3} \right), \\
q_2 &= \frac{1}{D} \left( \left( \alpha_1^{(1)} e^{i\Gamma z} \sinh(\frac{\theta}{2}) + \alpha_1^{(2)} e^{-i\Gamma z} \cosh(\frac{\theta}{2}) \right) e^{\eta_1} + \left( \alpha_2^{(1)} e^{i\Gamma z} \sinh(\frac{\theta}{2}) + \alpha_2^{(2)} e^{-i\Gamma z} \cosh(\frac{\theta}{2}) \right) e^{\eta_2} + \left( e^{\delta_{11}} \sinh(\frac{\theta}{2}) e^{i\Gamma z} + e^{\delta_{12}} \cosh(\frac{\theta}{2}) e^{-i\Gamma z} \right) e^{\eta_1 + \eta_1^* + \eta_2} + \left( e^{\delta_{21}} \sinh(\frac{\theta}{2}) e^{i\Gamma z} + e^{\delta_{22}} \cosh(\frac{\theta}{2}) e^{-i\Gamma z} \right) e^{\eta_2 + \eta_2^* + \eta_3} \right),
\end{align*}
\]

(4.8a) (4.8b)

where \(\alpha_i^{(j)}\)'s are complex parameters and the denominator \(D\) is given by

\[
D = 1 + e^{\eta_1 + \eta_1^* + R_1} + e^{\eta_1 + \eta_2^* + \delta_0} + e^{\eta_1^* + \eta_2 + \delta_0^*} + e^{\eta_2 + \eta_2^* + R_2} + e^{\eta_1 + \eta_1^* + \eta_2 + \eta_2^* + R_3}.
\]

(4.8c)

Various quantities found in Eq. (4.8) are defined as below:

\[
\begin{align*}
\eta_i &= k_i(t + ik_i z), \quad e^{\theta_0} = \frac{\kappa_{12}}{k_1 + k_2}, \quad e^{R_1} = \frac{\kappa_{11}}{k_1 + k_1^*}, \quad e^{R_2} = \frac{\kappa_{22}}{k_2 + k_2^*}, \\
e^{\delta_{ij}} &= \frac{(k_1 - k_2)(\alpha_i^{(j)} \kappa_{21} - \alpha_j^{(j)} \kappa_{11})}{(k_1 + k_1^*)(k_1 + k_2)}, \quad e^{\delta_{ij}} = \frac{(k_2 - k_1)(\alpha_2^{(j)} \kappa_{12} - \alpha_1^{(j)} \kappa_{22})}{(k_2 + k_2^*)(k_1 + k_2^*)}, \\
e^{R_3} &= \frac{|k_1 - k_2|^2}{(k_1 + k_1^*)(k_2 + k_2^*)[k_1 + k_2]^2(\kappa_{11} \kappa_{22} - \kappa_{12} \kappa_{21})},
\end{align*}
\]

(4.8d)

and

\[
\kappa_{ij} = \mu \left( \sigma_1 \alpha_i^{(1)} \alpha_j^{(1)*} + \sigma_2 \alpha_i^{(2)} \alpha_j^{(2)*} \right), \quad i, j = 1, 2,
\]

(4.8e)
where \( \sigma_1 = 1 \) and \( \sigma_2 = -1 \). This solution represents the interaction of two bright solitons in the presence of self and cross coupling terms. Although the above solution features both singular and nonsingular solutions in the following we consider only the nonsingular soliton solution which results for the choice \([69]\)

\[
\kappa_{11} \geq 0, \quad \kappa_{22} \geq 0, \quad \kappa_{11} \kappa_{22} - |\kappa_{12}|^2 > 0, \tag{4.9a}
\]

and

\[
\frac{1}{2} \sqrt{\frac{\kappa_{11} \kappa_{22}}{k_1 R k_2 R} + \frac{|k_1 - k_2|}{k_1 + k_2^2} \sqrt{\frac{\kappa_{11} \kappa_{22} - |\kappa_{12}|^2}{k_1 R k_2 R}}} > \frac{|\kappa_{12}|}{|k_1 + k_2^2|}, \tag{4.9b}
\]

and analyze their collision behaviour. In a similar way the multi-soliton solution of Eq. (4.1) can be obtained by applying the transformation to the multi-soliton solution of the mixed 2-CNLS equations which is given in Chapter 3.

### 4.4 Shape changing collision of solitons with periodic energy switching

We have already explained the nature of type-II SCC of bright solitons in section 4.3. In this section, we analyze the influence of linear cross coupling terms on the above mentioned type-II SCC. We perform an asymptotic analysis \([69]\) for the choice \( k_{1R}, k_{2R} > 0 \) and \( k_{1I} > k_{2I} \). To facilitate the understanding of the collision dynamics we consider the intensities of the two colliding solitons in the asymptotic limits at \( z \to -\infty \) (before collision) and \( z \to \infty \) (after collision). The asymptotic forms the intensities of solitons in the limits \( z \to \pm \infty \) read as

\[
\left| \frac{q_j^{n\pm}}{P_n} \right|^2 = k_n R \left( |A_1^{n\pm}|^2 \cosh^2 \left( \frac{\theta}{2} \right) + |A_2^{n\pm}|^2 \sinh^2 \left( \frac{\theta}{2} \right) \right.
\]

\[
+ 2 |A_1^{n\pm}| |A_2^{n\pm}| \cosh \left( \frac{\theta}{2} \right) \sinh \left( \frac{\theta}{2} \right) \cos(2\Gamma z + Q_n) \right), \tag{4.10a}
\]
where

\[ P_n = \text{sech}(k_n R(t - 2k_n z + R_n)), \quad (4.10b) \]

\[ Q_n = \tan^{-1}\left( \frac{A_{1j}^{n\pm}}{A_{1j}^{n\mp}} \right) - \tan^{-1}\left( \frac{A_{2j}^{n\pm}}{A_{2j}^{n\mp}} \right), \quad n = 1, 2, j = 1, 2. \quad (4.10c) \]

Here the quantities \( A_j^{n^-}k_{nR} \) and \( A_j^{n^+}k_{nR}, \quad j, n = 1, 2, \) are the amplitudes of the \( n \)th soliton in \( j \)th component before and after interaction respectively in the absence of linear coupling \((\chi = \rho = 0)\), where \( A_j^n \)'s take the following forms before and after interaction \([69]\):

\[
\begin{pmatrix}
A_1^{1-} \\
A_2^{1-}
\end{pmatrix} = \begin{pmatrix}
\alpha_1^{(1)} \\
\alpha_1^{(2)}
\end{pmatrix} \frac{e^{-R_1/2}}{(k_1 + k_1^*)},
\]

\[
\begin{pmatrix}
A_1^{2-} \\
A_2^{2-}
\end{pmatrix} = \begin{pmatrix}
e^{\delta_{11}} \\
e^{\delta_{12}}
\end{pmatrix} \frac{e^{-(R_1+R_3)/2}}{(k_2 + k_2^*)},
\]

\[
\begin{pmatrix}
A_1^{1+} \\
A_2^{1+}
\end{pmatrix} = \begin{pmatrix}
e^{\delta_{21}} \\
e^{\delta_{22}}
\end{pmatrix} \frac{e^{-(R_2+R_3)/2}}{(k_1 + k_1^*)},
\]

\[
\begin{pmatrix}
A_1^{2+} \\
A_2^{2+}
\end{pmatrix} = \begin{pmatrix}
\alpha_2^{(1)} \\
\alpha_2^{(2)}
\end{pmatrix} \frac{e^{-R_2/2}}{(k_2 + k_2^*)}.
\]

Various quantities occurring in Eqs. (4.11) are defined in Eq. (4.8). From Eqs. (4.10) it can also be verified that

\[
\left| \frac{q_1^{n\pm}}{P_n} \right|^2 - \left| \frac{q_2^{n\pm}}{P_n} \right|^2 = k_n^2 R \left( |A_1^{n\pm}|^2 - |A_2^{n\pm}|^2 \right) = \frac{k_n^2 R}{\mu}, \quad n = 1, 2, \quad (4.12)
\]

which is a consequence of the conservation law of intensities in the mixed CNLS system. The role of linear coupling parameters on type-II SCC and vice-versa can be well understood by analysing the asymptotic expressions (4.10) which clearly shows that these terms induce periodic switching of intensity between the two colliding solitons in both the components \( q_1 \) and \( q_2 \). At a first sight it seems that the periodic intensity switching in a given soliton (say soliton \( S_1 \))
is influenced only by the same soliton present in the other component. But a careful analysis shows that the presence of the second soliton (say soliton $S_2$) plays an indirect but predominant role in controlling the switching process through type-II SCC and vice-versa. We discuss below the various possibilities of periodic energy switching in system (4.1):

1. The coupling results in periodic oscillations in the energy switching process throughout the collision process due to the oscillatory term $\cos(2\Gamma z + Q_n)$ appearing in Eq. (4.10a). As in the case of one soliton solution here also the amplitude and width of the periodic oscillations increase with decreasing $\Gamma$. Thus the important feature of such collision process is that the amplitude of periodic energy switching can be large depending upon the relative signs of linear coupling terms $\rho$ and $\chi$. This periodic energy
switching behaviour, in the presence of coupling, depends on the $\alpha$ and $k$-parameters and also on the linear coupling coefficients. Thus the oscillating energy switching process co-exists with type-II SCC for $\frac{\alpha_1^{(1)}}{\alpha_1^{(2)}} \neq \frac{\alpha_2^{(1)}}{\alpha_2^{(2)}}$. Such a two soliton collision process with periodic intensity switching is shown in Fig. 4.2 for $\alpha_1^{(1)} = 0.7226 + 1.1254i$, $\alpha_1^{(2)} = 0.8484 + 0.2625i$, $\alpha_2^{(1)} = 0.5511 + 0.8584i$, $\alpha_2^{(2)} = 0.1923 + 0.0595i$, $\rho = 1$, $\chi = 0.5$, $k_1 = 1 + i$, $k_2 = 1.1 - i$. Eq. (4.10) also shows that the coupling enhances the amplitude of the soliton in a given component before and after interaction due to the contribution from the other component as compared with the bright soliton collision case in the absence of coupling.
2. The distinct feature of this collision process is that the intensity redistribution can be used to control the switching dynamics. One interesting possibility is complete suppression of oscillation either before or after collision in a particular soliton say "$S_n$" by making any one of $|A_j^+|$ or $|A_j^-|$, $j, n = 1, 2$, to be zero, respectively, with commensurate changes in the other soliton. As the nonsingular condition (4.9a) of the solution rules out the possibility of making $|A_i^{n\pm}|$ to be zero, the complete suppression of periodic oscillation of intensities in both the components of soliton $S_n$ before (after) collision can be obtained by choosing $|A_2^-|$ ($|A_2^+|$) = 0. This suppression (enhancement) of intensities of a particular soliton in
a given component during the type-II SCC results in the enhancement (suppression) of amplitude of periodic oscillations in the other colliding soliton as inferred from Eq. (4.10). Fig. 4.3 shows the type-II SCC scenario in which the oscillations in the \(q_1\) and \(q_2\) components of \(S_2\) after interaction are completely suppressed, for the choice \(\alpha_1^{(1)} = 0.6093 + 0.9489i, \alpha_1^{(2)} = 0.4978 + 0.1540i, \alpha_2^{(1)} = 0.5403 + 0.8415i, \alpha_2^{(2)} = 0, \rho = 1, \chi = 0.5, k_1 = 1 + i, k_2 = 1.1 - i\). The reason for this is that in the absence of coupling terms soliton \(S_2\) undergoes type-II SCC with \(S_1\) and its intensity in \(q_2\) component after interaction is exactly zero for the given parametric choice. Similarly Fig. 4.4 shows the suppression of oscillations in \(q_1\) and \(q_2\) components of \(S_1\) before interaction for the parametric choice \(\alpha_1^{(1)} = 1, \alpha_1^{(2)} = 0, \alpha_2^{(1)} = 1.0201,\)
\( \alpha_2^{(2)} = 0.2013, \rho = 1, \chi = 0.5, k_1 = 1 + i, k_2 = 1.1 - i. \) This kind of switching process arises from the fact that in the absence of coupling the intensity of \( S_1 \) in \( q_2 \) component (that is, \( |A_2^1|^2 k_2^2 R \)) is zero before it collides with \( S_2 \).

3. The standard elastic collision with periodic energy switching only arises for the choice \( \frac{\alpha_1^{(1)}}{\alpha_2^{(1)}} = \frac{\alpha_1^{(2)}}{\alpha_2^{(2)}}. \) This is shown in Fig. 4.5 for the parametric choice \( \alpha_1^{(1)} = 0.6782 + 1.0562i, \alpha_1^{(2)} = 0.6782 + 1.0562i, \alpha_2^{(1)} = 0.7247 + 0.2242i, \alpha_2^{(2)} = 0.7247 + 0.2242i, \rho = 1, \chi = 0.5, k_1 = 1 + i, k_2 = 1.1 - i. \)

In order to appreciate the significance of the present system, we compare the soliton collision behaviour with that of twisted birefringent fibers [86] which involve focusing type nonlinearities. The crucial difference follows from Eq. (4.11), which says that the energy exchange between the two components \((q_1, q_2)\) before and after collision is constant and as a result a given soliton experiences the same effect (either suppression or enhancement of intensity) in both the components during its collision with other soliton contrary to the twisted birefringent system. Thus the amplitude of oscillation due to coupling can be simultaneously enhanced/suppressed after collision in both the components as a consequence of type II-SCC, a situation which is not possible in twisted birefringent fibers. Another important advantage is the efficiency of switching due to linear couplings. Here the coupling terms influence the energy switching exponentially due to the hyperbolic terms (see Eq. (4.11)). This suggests large switching of energy with small self coupling strengths, as compared with Manakov system with linear couplings, a desirable property in fiber couplers.