The concept of Topological Spaces grew out of the study of real line and Euclidean spaces and the study of continuous functions on these spaces. The study of topological spaces, their continuous functions and general properties makes up one branch of topology known as General Topology. In 1965, L. A. Zadeh introduced the theory of fuzzy sets which proved to be an important mile stone in the development of various fields of science and engineering. Fuzzy Topology is a kind of topology defined on fuzzy sets. The theory of fuzzy topological spaces can be regarded as a generalization theory of topological spaces.

This thesis consists of four chapters. It is an elaborate study of a new type of stronger form of generalized closed sets in topological spaces called generalized minimal closed sets, their respective continuous maps, closed maps, homeomorphisms, regular spaces, normal spaces and their extensions to bitopological spaces and fuzzy topological spaces.

Chapter 1:

In 1970, one of the most important turning point that took place in topology was the introduction of the concept of generalized closed (briefly g-closed) sets in topological spaces by N. Levine in order to extend many of the important properties of closed sets to a larger family. Due to the introduction of this concept of generalized closed sets today, many authors have defined various forms of generalized closed sets and generalized continuous mappings in topological spaces, bitopological spaces and fuzzy topological spaces. Recently in the year 2001 and 2003, F. Nakaoka and
N. Oda introduced and investigated minimal open sets, minimal closed sets, maximal open sets and maximal closed sets in topological spaces. These new classes of sets can be applied to various aspects of general topology and fuzzy topology. Likewise these new classes of sets have been applied to generalized closed sets to introduce a new class of sets called generalized minimal closed sets in topological spaces, bitopological spaces and fuzzy topological spaces.

C. Balachandran, P. Sundaram and H. Maki introduced the class of generalized continuous maps that includes a class of gc-irresolute maps in topological spaces. Generalized closed maps, generalized open maps and generalized homeomorphisms in topological spaces were introduced and studied by S. R. Malghan, P. Sundaram and H. Maki respectively. T. Noiri and V. Popa studied generalized regular spaces and generalized normal spaces in topological spaces.

Section 1, of this chapter deals with the recent developments in general topology contributed by various authors and the definitions used by them are presented which are subsequently used in this chapter. Throughout this chapter (X, τ), (Y, σ) and (Z, η) denote the topological spaces on which no separation axioms are assumed unless otherwise explicitly mentioned. For any subset A of a topological space (X, τ), closure of A, interior of A and complement of A are denoted by cl (A), int (A) and A^c respectively.

Section 2, of this chapter deals with the definition and basic properties of a new class of sets called generalized minimal closed (briefly g-m_c closed) sets in topological spaces. A subset A of a topological space (X, τ) is said to be g-m_c closed set if cl (A) ⊆ U whenever A ⊆ U and U is a
minimal open set in $X$. This set is independent of minimal closed sets and closed sets but is stronger form of generalized closed sets. Every $g$-$m_i$ closed set in a topological space $(X, \tau)$ is a $\omega$-closed set and every $\omega$-closed set is a $g$-closed set. The behavior of subspaces and some other basic results have been obtained. The complement of a $g$-$m_i$ closed set in a topological space $(X, \tau)$ is called a generalized maximal open (briefly $g$-$m_i$ open) set. Also a set $A$ of a topological space $(X, \tau)$ is a $g$-$m_i$ open set iff $F \subseteq \text{int} A$ whenever $F \subseteq A$ and $F$ is a maximal closed set in $X$. Some of the important properties of this set have been proved. One of the important theorem is that, if $A$ is any $g$-$m_i$ open set in a topological space $(X, \tau)$, then $O = X$ whenever $O$ is an open set in $X$ and $\text{int} A \cup A^c \subseteq O$. Converse of this theorem need not be true.

In section 3, a new class of maps called generalized minimal continuous (briefly $g$-$m_i$ continuous) maps is studied that includes an analogy of $gc$-irresolute maps called generalized minimal irresolute (briefly $g$-$m_i$ irresolute) maps in topological spaces. A mapping $f: (X, \tau) \rightarrow (Y, \sigma)$, is said to be generalized minimal continuous (resp. $g$-$m_i$ irresolute) map if the inverse image of every minimal closed (resp. $g$-$m_i$ closed) set in $Y$ is $g$-$m_i$ closed (resp. $g$-$m_i$ closed) set in $X$. Every $g$-$m_i$ continuous map is $m_i$ $g$-continuous map. But the converse part, which is not true, is illustrated with the help of an example. Though every $g$-$m_i$ closed set is a $g$-closed set, $g$-$m_i$ continuous maps are independent of $g$-continuous maps. Some other basic results have been proved. Composition of $g$-$m_i$ continuous maps need not be a $g$-$m_i$ continuous map, but if $f: (X, \tau) \rightarrow (Y, \sigma)$ is a $g$-$m_i$ continuous map and $h: (Y, \sigma) \rightarrow (Z, \eta)$ is a maximal irresolute map, then $h \circ f: (X, \tau) \rightarrow (Z, \eta)$ is $g$-$m_i$ continuous map. The restricted map of a $g$-$m_i$ continuous map need not be a $g$-$m_i$ continuous map. An example has been given to show that
gc-irresolute maps and g-m_i irresolute maps are independent of each other. The restricted map of a g- m_i irresolute map need not be a g- m_i irresolute map. Composition of g- m_i irresolute map is a g- m_i irresolute map. Also if \( f: (X, \tau) \to (Y, \sigma) \) is a g- m_i irresolute map and \( h: (Y, \sigma) \to (Z, \eta) \) is a g- m_i continuous map, then \( h \circ f: (X, \tau) \to (Z, \eta) \) is a g- m_i continuous map.

In section 4, generalized minimal closed (briefly g-m_i closed) maps that include a class of generalized minimal* closed (briefly g- m_i* closed) maps, generalized minimal homeomorphisms (briefly g-m_i homeomorphisms) and generalized minimal* homeomorphisms (briefly g-m_i* homeomorphisms) are introduced and characterized in topological spaces. A map \( f: (X, \tau) \to (Y, \sigma) \) is said to be g-m_i closed (resp. g- m_i* closed) map if the image of every minimal closed (resp. g-m_i closed )set in \( X \) is g-m_i closed (resp. g-m_i closed )set in \( Y \). Every g-m_i closed map is m_i g-closed map. Some of the basic properties like restricted mappings, composition of mappings, etc have been investigated. Likewise a bijective map \( f: (X, \tau) \to (Y, \sigma) \), is said to be g-m_i homeomorphism (resp. g- m_i* homeomorphism) if \( f \) and \( f^{-1} \) are g-m_i continuous (resp. g-m_i irresolute) maps. Some of the important results of all these mappings have been studied.

The last section of this chapter deals with generalized minimal regular (briefly g-m_i regular) spaces and generalized minimal normal (briefly g-m_i normal) spaces. A topological space \((X, \tau)\) is said to be g-m_i regular space if for every g-m_i closed set \( F \) of \( X \) and each point \( x \in F^c \), there exist disjoint open sets \( U \) and \( V \) of \( X \) such that \( x \in U \) and \( F \subseteq V \). A topological space \((X, \tau)\) is said to be g-m_i normal space if for any pair of disjoint g-m_i
closed sets $A$ and $B$, there exists disjoint open sets $U$ and $V$ such that $A \subseteq U$ and $B \subseteq V$. Every $g$-regular (resp. $g$-normal) space is a $g$-$m_i$ regular (resp. $g$-$m_i$ normal) space. But the converses are not true, which is illustrated with the help of an example. Other basic results of both the spaces have been studied.

Chapter 2:

The triple $(X; \tau_1, \tau_2)$ where $X$ is a set and $\tau_1$ and $\tau_2$ are two topologies on $X$ is a bitopological space. J. C. Kelly initiated the systematic study of such spaces. After the work of J. C. Kelly, various authors turned their attention to generalization of various concepts of topology by considering bitopological spaces. T. Fukutake introduced and investigated the concept of generalized closed sets in bitopological spaces. H. Maki, P. Sundaram and K. Balachandran extended the notion of generalized continuous maps and some interesting results in topological spaces to bitopological spaces. In this chapter, generalized minimal closed sets and generalized minimal continuous mappings have been extended to bitopological spaces. Throughout this chapter $(X; \tau_1, \tau_2)$, $(Y; \sigma_1, \sigma_2)$ and $(Z; \eta_1, \eta_2)$ denote nonempty bitopological spaces on which no separation axioms are assumed unless otherwise mentioned and the fixed integers $i, j, k, e, m, n \in \{1, 2\}$.

Section 1, deals with the introduction and required definitions contributed by various authors. Section 2, of this chapter deals with generalized minimal closed sets in bitopological spaces. In a bitopological space $(X; \tau_1, \tau_2)$, a subset $A$ of $X$ is said to be a $(\tau_i, \tau_j)$-generalized minimal closed (briefly $(\tau_i, \tau_j)$-$g$-$m_i$ closed) set if $\tau_j$-$cl$ $(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is a $\tau_i$-minimal open set in $X$. The complement of a $(\tau_i, \tau_j)$-generalized
minimal closed in a bitopological space \((X; \tau_i, \tau_j)\) is called a \((\tau_i, \tau_j)\)-generalized maximal open (briefly \((\tau_i, \tau_j)\)-g-ma open) set. The behavior of subspaces and restricted spaces with respect to \((\tau_i, \tau_j)\)-g-ma closed sets and \((\tau_i, \tau_j)\)-g-ma open sets have been obtained in this section. Some other basic results have also been proved.

In section 3, generalized minimal continuous maps are introduced and investigated in bitopological spaces. A map \(f: (X; \tau_i, \tau_j) \to (Y; \sigma_i, \sigma_j)\) is said to be \((\tau_i, \tau_j)\)-\(\sigma_k\) generalized minimal continuous (briefly \((\tau_i, \tau_j)\)-g-ma continuous) map if the inverse image of every \(\sigma_k\)-ma closed set in \((Y; \sigma_i, \sigma_j)\) is \((\tau_i, \tau_j)\)-g-ma closed set in \((X; \tau_i, \tau_j)\). Every \((((\tau_i, \tau_j)\)-g-ma continuous map is \((\tau_i, \tau_j)\)-\(\sigma_k\)-ma-g-continuous map but the converse need not be true. The restricted map of \(((\tau_i, \tau_j)\)-g-ma continuous map need not be \(((\tau_i, \tau_j)\)-g-ma continuous map. Other related basic results have been proved. A map \(f: (X; \tau_i, \tau_j) \to (Y; \sigma_i, \sigma_j)\) is said to be \(((\tau_i, \tau_j)\)-\(\sigma_k\)\)-g-ma continuous (briefly \(((\tau_i, \tau_j)\)-g-ma continuous)) map if the inverse image of every \(\sigma_k\)-ma closed set in \((Y; \sigma_i, \sigma_j)\) is a \((\tau_i, \tau_j)\)-g-minimal closed set in \((X; \tau_i, \tau_j)\). Note that \(D((\tau_i, \tau_j) - D(\sigma_k, \sigma_e)\)-continuous maps and \((\tau_i, \tau_j)\)-\(\sigma_k\), \(\sigma_e\)-g-ma continuous maps are independent of each other. If \((\tau_i, \tau_j)\)-\(\sigma_k\), \(\sigma_e\) generalized minimal continuous maps are composed with \(((\sigma_k, \sigma_e)\)-\(\eta_m\)-generalized minimal continuous maps then we arrive at \(((\tau_i, \tau_j) - \eta_m\)-g-\(\eta_m\) continuous map. Besides these theorems, some of the other results have been investigated. Section 4 deals with generalized minimal closed maps and their characterizations in bitopological spaces. A mapping \(f: (X; \tau_i, \tau_j) \to (Y; \sigma_i, \sigma_j)\) is said to be \(((\tau_i)\)-\(\sigma_k\)\)-g-ma minimal closed (briefly \(\tau_i\)-\(\sigma_k\)-g-ma closed) map if the image of every \(\tau_i\) minimal closed set in \((X; \tau_i, \tau_j)\) is a \(\sigma_k\)-g-ma closed set in
(Y; σ₁, σ₂). A map \( f: (X; τ₁, τ₂) \rightarrow (Y; σ₁, σ₂) \) is said to be \(( (τᵢ, τⱼ, (σᵢ, σⱼ))-\text{generalized minimal closed (briefly } (τᵢ, τⱼ, (σᵢ, σⱼ))-\text{g-mi closed) map if the image of every } (τᵢ, τⱼ, g-mi \text{ closed set in } (X; τᵢ, τⱼ) \text{ is } (σᵢ, σⱼ)-\text{g-mi closed set in } (Y; σ₁, σ₂). \) Basic results have been obtained in bitopological spaces.

Chapter 3:

In this chapter, minimal open sets and minimal continuous mappings have been introduced in fuzzy topological spaces. Some of the important results of these classes of sets and classes of mappings have been obtained. Throughout this chapter (X, T), (Y, S) and (Z, P) denote fuzzy topological spaces (briefly fts) on which no separation axioms are assumed unless otherwise explicitly mentioned. For any fuzzy subset A of a fts (X, T), closure of A, interior of A and complement of A is denoted by cl (A), int (A) and \( A^{c} \) (or \( 1-A \)) respectively.

Section 1 of this chapter, is intended to provide a brief introduction to fuzzy subsets and fuzzy topology. The concept of a fuzzy subset, operations on fuzzy subsets, fuzzy subsets induced by mappings and fuzzy topological spaces are discussed in this section, which are subsequently used in this chapter.

Section 2 deals with fuzzy minimal open sets that include a class of fuzzy maximal open sets in fuzzy topological spaces. A nonempty fuzzy open set A of a fts (X, T) is said to be a fuzzy minimal open (briefly f- mi open) set if any fuzzy open set which is contained in A is either 0 or A. If A and B are any f- mi open sets then \( A \land B = 0 \) or \( A = B \). Basic results have been obtained. Recognition Principle for fuzzy minimal open sets is as
follows. Assume that $|\Lambda| \geq 2$. If $A_i$ is a $f$- open set for any element $i$ of $\Lambda$ and $A_i \neq A_j$ for any elements $i$ and $j$ of $\Lambda$ with $i \neq j$ then, $A_j = (\bigvee_{i \in \Lambda} A_i) \wedge (1 - \bigvee_{i \neq j \in \Lambda} A_i)$ for any element $j$ of $\Lambda$. Next assume that $|\Lambda| \geq 2$. If $A_i$ is a $f$- open set for any element $i$ of $\Lambda$ and $A_i \neq A_j$ for any elements $i$ and $j$ of $\Lambda$ with $i \neq j$. If $\bigvee_{i \in \Lambda} A_i = 1$, then $\{A_i / i \in \Lambda\}$ is the set of all $f$- open sets of a fuzzy topology $(X, T)$. A nonempty fuzzy open set $A$ of a fts $(X, T)$ is said to be a fuzzy maximal open ($f$- open) set if any fuzzy open set which contains $A$ is either $1$ or $A$. If $A$ and $B$ are $f$- open sets then $A \cup B = 1$ or $A = B$. Using this result we have proved some of the important results. One such result is that, if $A$, $B$ and $C$ are $f$- open sets such that $A \neq B$ and if $A \wedge B < C$, then $A = C$ or $B = C$. Decomposition theorem for $f$- open sets has been proved.

In section 3, a class of fuzzy minimal continuous maps that includes a class of fuzzy maximal continuous maps, fuzzy minimal irresolute maps and fuzzy maximal irresolute maps are studied in fuzzy topological spaces. A mapping $f: X \to Y$, from a fts $X$ into a fts $Y$ is said to be fuzzy-$m_i$ continuous (resp. fuzzy-$m_a$ continuous, fuzzy-$m_i$ irresolute and fuzzy-$m_a$ irresolute ) map if the inverse image of every $f$- open(resp. $f$- open, $f$-$m_i$ open and $f$-$m_a$ open) set in $Y$ is a fuzzy open (resp. fuzzy open, $f$-$m_i$ open and $f$-$m_a$ open) set in $X$. Relation between fuzzy continuous maps and $f$- continuous (resp. $f$-$m_a$ continuous) maps and $f$- irresolute (resp. $f$-$m_a$ irresolute) maps have been obtained. Other important results have been proved.
Section 4, deals with a class of fuzzy minimal open maps that includes a class of fuzzy maximal open maps, a class of fuzzy strongly minimal open maps and fuzzy strongly maximal open maps in fuzzy topological spaces. A map \( f : X \rightarrow Y \), from a fts \( X \) into a fts \( Y \) is said to be \( f\)-\( m \) open (resp. \( f\)-\( m_a \) open, \( f\)-\( s\)-\( m \) open and \( f\)-\( s\)-\( m_a \) open ) map if the image of every \( f\)-\( m \) open (resp. \( f\)-\( m_a \) open, \( f\)-\( m \) open and \( f\)-\( m_a \) open) set in \( X \) is a fuzzy open (resp. fuzzy open, \( f\)-\( m \) open and \( f\)-\( m_a \) open) set in \( Y \). Basic results of all these classes of maps have been characterized. Relation between fuzzy open maps and \( f\)-\( m \) open (resp. \( f\)-\( m_a \) open) maps and \( f\)-\( s\)-\( m \) open (resp. \( f\)-\( s\)-\( m_a \) open) maps have been obtained.

**Chapter 4:**

Generalized minimal closed sets and generalized minimal mappings in topological spaces have been extended to fuzzy topological spaces in this chapter by using fuzzy minimal open sets.

Section 1, of this chapter deals with the introduction and the required definitions that are used in this chapter. Throughout this chapter \((X, T), (Y, S)\) and \((Z, P)\) denote fuzzy topological spaces (briefly fts) on which no separation axioms are assumed unless otherwise explicitly mentioned. For any fuzzy subset \( \alpha \) of a fts \((X, T)\), closure of \( \alpha \), interior of \( \alpha \) and complement of \( \alpha \) are denoted by \( \text{cl}(\alpha) \), \( \text{int}(\alpha) \) and \( \alpha' \) (or (1-\( \alpha \))) respectively.

In section 2, a class of fuzzy generalized minimal closed sets have been introduced in fuzzy topological spaces. A fuzzy set \( \alpha \) of a fts \((X, T)\) is called a fuzzy generalized minimal closed (briefly f-g-\( m \) closed) set if \( \text{cl}(\alpha) \leq \sigma \) whenever \( \alpha \leq \sigma \) and \( \sigma \) is a fuzzy minimal open set in \((X, T)\). The behavior of subspaces of f-g-\( m \) closed sets have been obtained. The
complement of a \( f\)-\( g\)-\( m\) closed set in a fts \((X, T)\) is called a fuzzy generalized maximal open (briefly \( f\)-\( g\)-\( m\) open) set in \((X, T)\). Some of the important theorems on this set have been obtained in this chapter.

In section 3, a class of fuzzy generalized minimal continuous maps, that include a class of fuzzy generalized minimal irresolute maps are introduced and studied in fuzzy topological spaces. Some of the basic results have been proved in this section. A map \( f: X \to Y \), from fts \( X \) into fts \( Y \) is said to be fuzzy \( g\)-\( m\) continuous (resp. fuzzy \( g\)-\( m\) irresolute) map if the image of every fuzzy \( m\) closed (resp. fuzzy \( g\)-\( m\) closed) set in \( Y \) is a fuzzy \( g\)-\( m\) closed (resp. fuzzy \( g\)-\( m\) closed) set in \( X \). Fuzzy gc- irresolute maps and fuzzy \( g\)-\( m\) irresolute maps are independent of each other.

In section 4, fuzzy generalized minimal closed maps that includes a class of fuzzy generalized minimal\(^*\) closed maps, fuzzy generalized minimal homeomorphisms and fuzzy generalized minimal\(^*\) homeomorphisms are introduced and studied in fuzzy topological spaces. A mapping \( f: X \to Y \), from fts \( X \) into fts \( Y \) is said to be fuzzy \( g\)-\( m\) closed (resp. fuzzy \( g\)-\( m\)\(^*\) closed) map if the image of every fuzzy \( m\) closed (fuzzy \( g\)-\( m\) closed) set in \( X \) is a fuzzy \( g\)-\( m\) closed (fuzzy \( g\)-\( m\) closed) set in \( Y \). A bijective mapping \( f: X \to Y \), from fts \( X \) into fts \( Y \) is said to be fuzzy \( g\)-\( m\) (resp. fuzzy \( g\)-\( m\)\(^*\) ) homeomorphism if \( f \) and \( f^{-1} \) are fuzzy \( g\)-\( m\) continuous (resp. fuzzy \( g\)-\( m\) irresolute) maps. The equivalent properties of these classes of mappings have been proved.