Chapter 1

Introduction
A signal is a physical quantity conveying information that varies with time, space or any other single or multiple independent variables. The number of independent variables decides the dimension of the signal. The independent variable itself decides the domain to which the signal belongs. If the independent variable is time, the signal is referred to as time domain signal. Speech, electrocardiogram are the examples of information bearing signals that are functions of single independent variable, time. Image is an example of two-dimensional signal and it is a function of two independent spatial variables. A signal, which is continuous valued with respect to independent variable is referred to as analog signal and a signal, which is defined only at certain specific values of independent variable is referred to as discrete signal. A signal discretized both with respect to independent variable and function value is considered as digital signal.

Signal processing refers to the manipulation of original signal so as to obtain more useful form of the signal than the original signal. A physical device that performs signal processing is referred to as a system. A broad definition of the system also includes the associated software realizations of the signal processing operation. Removal of interference, extraction of feature parameters, signal compression, modulation of signal for transmission purpose and filtering are some of the examples of signal processing. The output of the signal processing system may be another signal in the same domain as the input signal. Signal denoising, modulation
are examples for this type of signal processing. The system output may represent features or attributes of the input signal such as number of zero crossings in a signal, formant frequency of speech signal, identification of individual objects in an image, etc. Some of the system output may be entirely in different domain. Given signal or function can be equally represented in other domain as function of variables of that domain. For certain signal processing applications such a representation may be more useful to bring out the hidden information, which may not be explicitly available in the original domain. In such signal processing applications, mathematical transformations need to be applied on a signal available in its original form thereby representing the signal in another domain.

Rapid growth in the field of digital signal processing is attributed to significant advances in digital computer technology and integrated circuit fabrication. Powerful, smaller, faster and inexpensive digital computers and special purpose digital hardware have made it possible to construct highly sophisticated digital systems capable of performing complex digital signal processing functions and tasks which are too difficult or expensive to be performed by analog signal processing systems. In addition, digital signal processing hardware allows programmable operations and achieves higher order precision. Due to these attractive features of the digital signal processing, offline processing applications are being replaced by real time signal processing applications.
In time domain, the signal is represented in time-amplitude form. This representation of the signal is not always the best representation of the signal for most of the signal processing applications. Changing the representation of the signal from one form to another form by applying mathematical transformations is referred to as transform. A signal function and its transform are two faces of the same information. A transform expresses the signal in terms of set of orthogonal basis functions, basis vectors or kernel /1/. Weightage given to each of the basis function in expressing a signal in transform domain is called transform coefficient. Transform can also be viewed as projection of the signal on a set of basis functions. Each coefficient is then the inner product of basis function with the given signal. Unitary transforms preserve the signal energy even after transformation. This implies the information carried by the signal is preserved under unitary transformation. Transforms have tendency to pack a large fraction of energy of the signal into a relatively few components of the transform coefficients. Due to decomposition of the signal in terms of orthogonal basis functions, the transform coefficients become highly decorrelated /1,2/. Apart from satisfying the above-mentioned properties, transform invertibility is sometimes important because it allows performing signal processing in transform domain, such as noise suppression and then return to original domain without loosing any useful information.
There are number of transforms frequently used by engineers and mathematicians in signal processing. They differ from each other with respect to the basis functions employed. Fourier transform (FT) is the most widely used transform. Basis functions employed in FT are complex sinusoids. By computing the FT of a signal, it is decomposed into its individual frequency components. FT coefficients depict the relative significance or contribution of each frequency component represented as complex sinusoid. For this reason, FT domain is sometimes referred to as frequency domain. Other frequently used transforms are /1,2/ Cosine transform, sine transform, Karhunen and Loeve transform, Haar transform, and Hadamard transform etc. Every transformation technique has unique area of applications, with advantages and disadvantages. Karhunen and Loeve transform is termed as optimal transform because of high decorrelation among its coefficients /1/. However, its kernel is signal dependent and it is computationally intensive. Cosine transform has well defined signal independent kernel. It represents the signal in compact form with its performance close to Karhunen and Loeve transform.

The transform domain provides an excellent compromise between computational complexity and performance, because transform domain manipulations are simple and sometimes more advantageous than time domain operations. For example, in signal denoising applications, noise component can easily be separated from signal as the signal and noise
occupy different regions in transform domain /3/. Noise suppression is achieved by applying a mask, which is equivalent to multiplication of the transform of noisy signal with the mask. The process is viewed in time domain as convolution of input signal with the transfer function of the system. Convolution operation involves fold, shift and cumulative multiply-add operations. In FT domain, it gets simplified to the multiplication of FT of the two signals to be convolved. The magnitude square of the FT of the signal is known as power spectrum. Bandwidth of speech signal for various applications, such as cellular phones and radio transmitter, has been standardized based on power spectrum of speech signal. Signal compression is possible in transform domain, as large fraction of signal energy requires only relatively few transform coefficients. Due to high decorrelation among these coefficients, simple coding becomes more effective in transform domain than in the original signal domain /4/.

Although FT is the widely used transform in signal processing, it does not suit all the requirements of signal processing. As the computation of FT involves inner product of signal and the complex sinusoid over the entire time duration, there is no way to distinguish the local characteristics of the signal under analysis from its global characteristics during the computation of FT. Local characteristics of the signal are treated as though they are global characteristics /5/, due to which FT fails to retain the information about the time duration of occurrence of different frequency components in
the original signal. FT assumes the signal as stationary signal, i.e., a signal with no change in its frequency spectrum with respect to time. There exist natural and man-made nonstationary signals, which have time varying spectra. While sunlight is an example of signal with slowly varying spectral contents, speech signal is an example of signal with rapidly varying spectral contents. FT does not suit for the analysis of nonstationary signals, as the time resolution is zero in its domain. For nonstationary signal analysis, there is a need for a transform, which is a joint function of time and frequency that describes the energy density or signal intensity simultaneously in the time and frequency plane. This is referred to as time frequency representation (TFR) /5/. Such a TFR helps in finding fraction of energy in certain time and frequency range. It also helps in calculating the distribution of frequency at a particular time, the global and local moments of the distribution such as the mean frequency and its local spread. Also, it would be a powerful tool for the construction or synthesis of signals with desirable time-frequency characteristics /5/. In other words, TFR carry information of a signal in a convenient and precise way. Fractional Fourier transform, short time Fourier transform and wavelet transform are examples of linear TFRs /2,3,5,6/, whereas Active Unterberger distribution (AUD) /6/, Wigner distribution (WD) /3,5,6/, Born-Jordan distribution (BJD) /5,6/, Page distribution (PD) /5,6/ are examples of quadratic TFRs and signal adaptive Radially Gaussian
kernel distribution (RGD) /6/, Cohen's nonnegative distribution (CND) /6/ are nonlinear and non quadratic TFRs.

The principle of TFR can be understood with the help of short time Fourier transform (STFT) /5,6/. In STFT, the entire time duration of the signal is divided into windows of short duration and FT is computed for each window. The window width is so chosen that the signal is stationary within the window so that FT is applicable. The window width conveys the duration of FT computation and this localizes the frequency analysis and hence the name for the technique as short time Fourier transform. Such representation is generally called spectrogram. For the analysis of a slowly varying signal, wider window width is preferred and for a fast varying signal, narrow window width is preferred in order to capture the signal variation effectively. With the fixing of the analysis window, time and frequency resolutions are fixed throughout the analysis. Varying time and frequency resolution is possible in wavelet transform (WT) /2,7/. The analysis is carried out with scaled and shifted versions of basis function called wavelet. Scaling and shifting of wavelet during signal analysis results in variable time and frequency resolution. The wavelet function can be chosen based on the characteristics of signal under analysis. As the time width of the window reduces, frequency resolution becomes poor and vice versa. It implies that WT cannot discriminate signals with too close high frequencies. This considerably complicates wavelet based singularity
extraction and signal modeling. Use of nonideal low pass and high pass filters in computation of WT coefficients upsets the forward and inverse transforms leading to artifacts in the reconstructed signal /7/. The dual-tree complex wavelet transform overcomes the drawback of real WT but at the cost of simplicity.

In nonstationary signal analysis, it is customary to use the time-frequency plane, with two orthogonal time and frequency axes. This enables representation of signal parameters such as signal energy, signal intensity etc. simultaneously with respect to time and frequency. FT can be interpreted as a rotation of time domain signal by an angle $\pi/2$ in the time-frequency plane and represented as an orthogonal signal representation for sinusoidal signal. Two successive forward FT operations will result in the reflected version of the original signal. A rotation by integer multiple of $\pi/2$ can thus be defined through repeated application of FT. A rotation angle $\alpha = a\pi/2$, with $a$ being a real number, leads to a domain that represents the signal both with respect to time and frequency. Signal representation along this intermediate axis making an angle $\alpha$ with time axis, in time-frequency plane has nonzero time and frequency resolutions. As the angle of rotation is fraction $a$ of $\pi/2$, the transform is known as fractional Fourier transform (FRFT) with an order parameter $a$. Thus, FRFT is a generalization of the FT with an order parameter $a/3$. The first order FRFT, with $a = 1$, is the ordinary FT and the zeroth order FRFT, with $a = 0$, is
the signal itself. The FRFT is also called rotational Fourier transform or angular Fourier transform. It performs a rotation of signal in the continuous time-frequency plane by an arbitrary angle and serves as an orthonormal signal representation for the chirp signal, a signal whose frequency varies linearly with time. FRFT has been proved to relate to other time varying signal analysis tools, such as Wigner distribution, wavelet transform etc. Computational complexity of FRFT is same as that of FT. An added advantage of FRFT is the freedom in the choice of orientation of transform axis with respect to time axis. This makes FRFT a powerful tool in solving differential equations /8/, optical signal processing /3/, time variant filtering and multiplexing /3/, swept frequency filters, pattern recognition /3/, digital watermarking /9/, time-frequency signal analysis /3/, Fourier optics and optical information processing /3/.

Similarly, attempts have been made to represent other transforms as fractional transforms. Fractional cosine transform /10,11/, Fractional Hilbert transform /12/, Fractional Hartley transform /13/ and Fractional WT /14/ are some examples. Fractional cosine transform (FRCT) is a generalization of continuous cosine transform. While cosine transform gives the spectral characteristics of the signal, FRCT interpolates between the signal in original domain and cosine-modulated form of the signal. It has similar relationship with FRFT as the continuous cosine transform has with FT /10,11/. FRCT can be computed as the FRFT of the evenly extended two-
sided function. Many of the properties of the FRFT are also satisfied by FRCT. Computational complexity of FRCT is same as that of FRFT. It has same application area as that of FRFT /10,11/. Fractional sine transform (FRST) is a generalization of continuous sine transform. It is interpreted as an interpolation between the signal and sine-modulated form of the signal. It has similar relationship with FRFT as the continuous sine transform has with FT /10,11/. It has same application area as that of FRFT /10,11/. Computational complexity of FRST is same as that of FRCT.

Signal or data compression, one of the signal processing applications, is an ancient activity; abbreviation and other devices for shortening the length of transmitted messages have no doubt been used in human society /15/. Claude Shannon created a formal intellectual discipline for both lossless and lossy compression. According to Claude Shannon /15/, with a source as a Gaussian zero-mean stochastic process, compression scheme amounts extracting the low frequency coefficients and retaining as few low frequency coefficients as required based on distortion level. In this definition, frequency components are extracted by applying FT that suits only for stationary signals. The important point to observe is that the processing is in transform domain. For a digital signal, the signal compression refers to the process of starting with a source of data in digital form and creating a representation for it using fewer bits than the original signal /2/. Audio coding or audio compression is an important application area of signal
processing. Attempts are being continuously made to improve audio coding techniques to increase the efficiency in transmission and storage while maintaining the audio signal quality. Lossless audio coding of CD quality stereo digital audio signals is very much essential for digital music distribution over the Internet. Lossy compression techniques such as MPEG or MP3 may not be acceptable for this application /16/. Some of the lossless compression algorithms /16/ are AudioPak, DVD, LTAC, MUSICompress, OggSquish, Philips, Shorten, Sonarc and WA. Amongst these techniques, lossless transform audio compression (LTAC) is the only algorithm based on transform coding and it employs discrete cosine transform (DCT). However, in this algorithm error signal is also transmitted along with the coded coefficients. Moreover, DCT can perform well only when the signal is stationary, and the energy is exclusively concentrated in certain frequency bands.

In real time applications, samples of input signal arrive in a sequential manner. While processing these samples, a block of $N$ input samples is considered at a time. If the processing time is less than the time for arrival of the required number of samples, the processing system has to remain idle until the arrival of all the samples. To make the output available at a faster rate, newly arrived sample can be processed along with the past $N-1$ samples. This technique is referred to as sliding technique. Discrete Fourier transform (DFT) for real time applications is implemented following this
method and is known as sliding DFT (SDFT) /17,18/. The processing system need not wait for the accumulation of samples.

Applications of digital image processing stems from two principal areas—improvement of pictorial information for human interpretations and processing of image data for storage, transmission and representation for autonomous machine perception. Low-level image processing such as denoising, image enhancement are considered as preprocessing step for further processing of the image. For example, segmentation and recognition of objects are referred to as mid level image processing. Image attributes are extracted in mid level image processing. Edges, one of the attributes, characterize object boundaries and are useful in several applications of image processing such as segmentation, object identification and registration. There are several methods to determine the edges in the image in spatial domain /1,19/. All are based on finding the n\(^{th}\) order derivative of the image. These methods ideally yield list of pixels lying on the edges. Very recently a scheme /20/ for block edge detection in DCT compressed domain has been proposed employing theory of correlation based pattern classification. In Ref. 19, DCT coefficients resulting from ideal edge patterns were examined to determine the edge parameters in terms of DCT coefficients. In the methods using DCT, a few low frequency DCT coefficients are considered. However, presence of edges results in abrupt changes in the gray level leading to higher frequency components. Hence,
The use of low frequency DCT coefficients is not justified. WT based edge detection suffers from poor frequency resolution at high frequency region where edge information is strongly present. In an optical system involving many lenses separated by arbitrary distances, the amplitude distribution of light at intermediate planes can be expressed in terms of FRFT. In optical and optical digital image analysis applications, the data can be acquired directly in the Fourier domain for real time feature extraction in the focal plane. It would be ideal if the edge detection were carried out in transform domain itself.

Thus, signal processing in the transform domain provides an attractive compromise between computational complexity and performance. In the present work, we have attempted to use the discrete FRFT (DFRFT) and discrete FRCT (DFRCT) for signal compression. The attempt meets dual purpose of perceptually lossless quality signal compression retaining time-frequency information. We have developed the method for edge detection in TFR domain using DFRFT and DFRCT. It enables us to find the edges in the image when the source data originates directly in the transform domain. Results of edge detection using DFRFT and DFRCT are presented for test images.

In chapter 2, various definitions of FRFT, FRCT and FRST are described. Different definitions lead to different physical interpretations and make these transforms suitable for variety of applications. There are many
approaches to discretize fractional transforms /22-26/. Each approach has unique advantages and disadvantages in terms of properties satisfied, cost of computation, ease of implementation and closeness to continuous fractional transforms. These are discussed in detail in this chapter. A generalized expression for the three discrete fractional transforms is developed in this chapter.

In chapter 3, single dimensional signal compression employing DFRFT as well as DFRCT is presented. An iterative method of determination of compact domain for both DFRFT and DFRCT is developed. The same is employed for the purpose of single dimensional signal compression. Certain elementary signals are considered for demonstrating the possibility of signal compression using DFRFT and DFRCT. The performance measures are compared with those obtained with DCT. The signal compression technique is tested using sound quality assessment material (SQAM) signals /27/ and results are presented.

In chapter 4, a generalized implementation scheme for the three sliding discrete fractional transforms: sliding discrete fractional Fourier transform (SDFRFT), sliding discrete fractional cosine transform (SDFRCT) and sliding discrete fractional sine transform (SDFRST) is presented. They are compared in terms of certain performance measures.

In chapter 5, the applications of DFRFT, DFRCT and DFRST for two-dimensional signal processing are presented. After a brief introduction to the
schemes presently available for edge detection, a scheme of edge detection in images using DFRFT, DFRCT and DFRST is presented. Results are presented for test images and compared with those obtained with spatial domain approach of edge detection.

Summary and conclusions of the present work are given in chapter 6.
REFERENCES


/27/ http://www.tnt.unihannover.de/project/mpeg/audio"