Chapter 5

Edge Detection using Discrete Fractional Transforms
5.1 Introduction

Digital image processing is implemented for improving the pictorial information for human interpretations and processing of image data for storage, transmission and representation for autonomous machine perception. Image processing can be considered to have three types of computerized processes: low-level image processing, mid-level image processing and high-level image processing. A low-level processing is characterized by both the input and output of the process being images. Low-level image processing such as denoising, image enhancement are considered as preprocessing step for further processing of the image. Determination of edges and boundaries of objects, segmentation, classification and recognition of objects are some of the mid-level image processing tasks, leading to extraction of image attributes. High-level processing involves assigning meaning to an ensemble of recognized objects or performing the cognitive functions associated with vision. In mid-level processing and high-level image processing the output is not another image. Certain image processing tasks may involve all the three levels of processing.

Edges, one of the attributes of the image, characterize object boundaries and are useful in several applications of image processing such as segmentation, object identification and registration. Spatial distribution of edges in an image is a useful texture descriptor for similarity search and
retrieval. The different gray shades of a monochrome image are generally
classified into three types of regions /1/, a region with constant gray level, a
region with onset and end of discontinuities, and a region with gray level
ramps. These discontinuities can be used to model lines and edges in an
image. An edge is defined as set of points where the image gray level or
intensity has sharp transitions. An edge is a set of connected pixels that lie
on the boundary between the two regions. It is a local concept based on a
measure of gray level discontinuity at a point. In practice, factors such as
limitation on the sampling rate, illumination conditions during image
acquisition and the quality of the acquisition system result in edges that are
blurred. Thus, an edge may not be sharp. Ramp like signal is generally used
to model an edge.

There are several methods for determining the edges in an image in
spatial domain /1,2/. All the methods are based on finding the n\textsuperscript{th}
derivative of the image. The magnitude of the first derivative is used to
detect the presence of an edge and the sign of the second derivative is used
to determine whether an edge pixel lies on the dark or bright side of an edge.
Roberts, Prewitt and Sobel /1,2/ masks used to compute gradient at every
pixel location. Laplacian masks /1,2/ are based on finding second order
derivative. However, the second order derivative around an edge has an
undesirable property that it gives two values for every edge in an image.
Laplacian of Gaussian masks are used to smoothen the image and to provide
zero crossings to find the location of edges. It reduces the effect of noise while detecting the edges. These methods ideally yield list of pixels lying on the edges. Hence, linking procedures to assemble edge pixels into meaningful edges typically has to follow the edge detection algorithms. As eye is not sensitive to derivatives of the order more than two, order of derivative is selected as two in these methods. Edges are formed from pixels with derivative values that exceed a preset threshold.

In recent years, due to the rapid growth in multimedia communication, there has been significant development in multimedia standards. These standards are used to facilitate the search and retrieval of multimedia contents such as images, video, text and audio. The standards /3/ include H.261, H.263, H.263+, H.26L, H.323, MPEG-1, MPEG-2, MPEG-4, and MPEG-7. The major difference between the earlier video coding standards and those of recent are in the representation and compression of the information. The most early video coding standards, including H.261, MPEG-1, and MPEG-2, use discrete cosine transform (DCT) and the recent standards use wavelet transform (WT) /3/. In order to avoid the unnecessary decompression and recompression operations in indexing and / or searching processes, it is desired to extract image and video information directly from the compressed form.

Very recently a scheme /4/ for block edge detection in DCT-compressed images has been proposed based on theory of correlation dependent pattern
classification. The image is divided into blocks of $8 \times 8$ and each block is further divided into four $4 \times 4$ subregions. A set of measures were developed to arrive at the orientation of edge in spatial domain in terms of the average intensity value of each subregion, which was the sum of the pixel values in that subregion. In order to detect the edges in DCT-compressed domain, the intensity value of each subregion is expressed in terms of DCT coefficients. In Ref. 5, pattern of the two dimensional DCT kernel is analyzed. DCT coefficients resulting from ideal edge patterns were examined to arrive at the measures of edge orientations in terms of DCT coefficients. Ten DCT coefficients have been considered in each block of $8 \times 8$ in the measures of edge orientations.

In real WT based edge detection /6/, peaks in detail sub bands of WT are identified across several levels of decomposition, at concordant locations. The process involves decomposing the image to several levels, manipulating the coefficients and then taking the inverse transform. WT based edge detection suffers from poor frequency resolution of WT in high frequency region /6,7/ where edge information is strongly present. The Dual-Tree Complex Wavelet Transform /7/ overcomes the drawback of real WT, but, at the cost of simplicity. Dyadic sampling of the image, inherent in wavelet transform, may fail to identify all edges /7/.

In an optical system involving many lenses separated by arbitrary distances, the amplitude distribution of light at intermediate planes can be
expressed in terms of fractional Fourier transform (FRFT) /8/. In optical and optical digital image analysis applications, the data can be acquired directly in the Fourier domain for real time feature extraction in the focal plane /8/. If the spatial domain approach for finding edges is to be employed, the source data need to be converted into to spatial domain, in these situations. DCT and WT based edge detection require the image in the respective domains. If the image is obtained in FRFT domain, these methods for edge detection are not suitable. There is thus, a need for development of method of edge detection in FRFT domain. A method of block edge detection when the source data originates in FRFT, fractional cosine transform (FRCT) or fractional sine transform (FRST) domain is described in this chapter. Results of edge detection are presented for some test images. The performance of each fractional transform is analyzed. The results are also compared with spatial domain method.

5.2 Block edge detection in DFRFT domain

The ideal binary edges with five orientations, no edge, 0°, 90°, 45° and –45°, are considered for a block within which any ideal edge is defined. Each block size is chosen as \((2N+1)\times(2N+1)\) with maximum value of \(N\) being 4. This is to meet the requirement of odd number of input samples for the closed form discrete FRFT (DFRFT) and discrete FRCT (DFRCT)
computation as explained in chapter 2. A block with an ideal binary edge has only ones and zeros. A logical zero is assigned for a pixel in dark region and logical one for that in bright region. For each of the five edge orientations, two possibilities of ideal binary edges are considered. The existence of an edge is ascertained with zeros on one side of the edge and ones on the other side, and vice versa. This is to ensure that only one edge orientation measure is arrived at for each edge irrespective of which side of the edge is dark or bright. Two-dimensional DFRFT is applied to each of these blocks. Equation 2.59 is followed in computing two-dimensional DFRFT. For further computation, each element in the block is considered as the absolute values of individual DFRFT coefficients. Each of these blocks is partitioned further into 4 sub blocks, $A$ of size $5 \times 5$, $B$ of size $5 \times 4$, $C$ of size $4 \times 5$ and $D$ of size $4 \times 4$ as shown in Fig. 5.1 (a). The sum of the elements in the corresponding sub blocks is assigned as the value for the sub block $A$, $B$, $C$ or $D$ as shown in Fig. 5.1 (b). The ideal binary edges and the corresponding pattern of distribution of the coefficients for the same are shown, respectively, in Fig. 5.2 (a) and 5.2 (b). The pattern of the sums of coefficients in blocks $A$, $B$, $C$ and $D$ for each ideal edge is shown in Fig. 5.2 (c). For each ideal binary edge, the coefficient distribution and the sub block value distribution have unique pattern. By referring to these patterns, the empirical formulae for edge orientations in any block have been arrived at
and are listed in table 5.1. These measures remain the same even when the block sizes of 7×7 and 5×5 are considered.

Figure 5.1: (a) Partitioning of block of 9×9 DFRFT coefficients into four subblocks.
(b) Sums of coefficients in 4 subblocks of 9×9 block.

Figure 5.2: (a) Ideal binary edges.
(b) Distribution of absolute values of DFRFT coefficients.
(c) A, B, C and D for each ideal edge.
Table 5.1: Measures for five orientations of edge based on DFRFT coefficients.

<table>
<thead>
<tr>
<th>Edge orientation (in degrees)</th>
<th>Measure</th>
</tr>
</thead>
<tbody>
<tr>
<td>No edge</td>
<td>$\theta_{\text{no_edge}} =</td>
</tr>
<tr>
<td>0</td>
<td>$\theta_0 = \frac{(A + C) - (B + D)}{2}$</td>
</tr>
<tr>
<td>90</td>
<td>$\theta_{90} = \frac{(A + B) - (C + D)}{2}$</td>
</tr>
<tr>
<td>45</td>
<td>$\theta_{45} = \frac{1.5(</td>
</tr>
<tr>
<td>-45</td>
<td>$\theta_{-45} = \frac{A + B + C + D}{4}$</td>
</tr>
</tbody>
</table>

For example, for $90^\circ$ edge orientation, one of the two blocks for ideal binary edges considered has $9 \times 5$ ones on the left side of the edge and $9 \times 4$ zeros on the right side of the edge. The other block has $9 \times 5$ zeros on the left side of the edge and $9 \times 4$ ones on the right of the edge. The pattern of the edges, coefficients and sums of coefficients shown in Fig. 5.2, is repeated in Fig. 5.3 for only $90^\circ$ edge orientation. It has $A$ and $B$ values displayed as white region and $C$ and $D$ values displayed as black region in Fig. 5.3 (c). Thus, the average of $A$ and $B$ will be more than the average of $C$ and $D$ for edge orientation of $90^\circ$. The empirical formula $\theta_{90} = \frac{(A + B)/2 - (C + D)/2}{|A - (B + C + D)|}$ will result in larger value in comparison with the other measures of edge orientation.
orientation whenever the edge orientation is $90^0$. The other measures of edge orientation are arrived at in a similar way.

Figure 5.3: (a) Ideal binary edges for $90^0$ orientation. (b) Distribution of absolute values of DFRFT coefficients. (c) $A$, $B$, $C$ and $D$ in sub blocks.

In determining the edges in an image, it is divided into several blocks. DFRFT is found for each block. $A$, $B$, $C$ and $D$ values are computed for each block as explained above. Measures of orientations $\theta_i$ are computed with $i =$ no edge, $0^0$, $90^0$, $45^0$ and $-45^0$. The value of $i$ with the maximum measure of orientation, $\theta_i$, is considered as the orientation of that block. Depending on the value of $i$, one of the patterns shown in Fig. 5.4 is stored as edge label for each of the block in the original image, forming another image showing only the edges. Edge labels are blocks having zeros along the direction of the edge in a block with the rest of the elements as ones. The size of the
edge label block is chosen to be same as the size of the block into which image is divided.

![Edge labels](image)

**Figure 5.4: Edge labels for**
(a) No edge (b) 0° (c) 90° (d) 45° (e) -45° edges.

In DFRFT based block edge detection, the presence of an edge and its orientation are determined in terms of DFRFT coefficients. The measures of five different orientations are independent of $a$. Thus if the image is already represented in terms of DFRFT coefficients, the value of $a$ need not be known to find the edges in such an image. There is no need to optimize the value of $a$ as demanded by the other applications of DFRFT, such as signal compression /8/. Complexity is less, as there is no need to compute inverse transform. In this work, five different orientations have been considered. For each ideal edge, two possibilities are considered and only one measure is arrived at. The empirical formula, derived for any edge orientation, identifies the edge irrespective of which side of the edge is dark or bright.

**Results**

Three different block sizes, 9×9, 7×7 and 5×5 are considered. Figure 5.5 shows the original image of cameraman. Figure 5.6 shows its edges as
determined by the method discussed, for block size of $9 \times 9$, $7 \times 7$ and $5 \times 5$. Similarly, Fig. 5.7 shows the original coins image and Fig. 5.8 shows the results for coins image. The results bring out a point clear that the measures of orientations as listed in table 5.1 are robust and stable even for nonideal edges present in images. As observed, when the block size is smaller, the better is the edge detection due to better localization. However, the block size should not be smaller than $5 \times 5$ as details in the block will be too less to decide the edge. Similarly, the block size should not exceed $9 \times 9$ as there is every possibility of more than one edge falling in such block. The merit of the proposed method is that there is no need to employ threshold to decide the presence of an edge or its orientation. Preprocessing such as image blurring to widen the edge is also not required.

Fig. 5.5: Original image of cameraman
Edges as detected in DFRFT domain

(a)

Edges as detected in DFRFT domain

(b)
Edges as detected in DFRFT domain

Figure 5.6: Edge detection based on DFRFT for
(a) Block size of 9 x 9.
(b) Block size of 7 x 7.
(c) Block size of 5 x 5.

Fig. 5.7: Original image of coins
Edges as detected in DFRFT domain

(a)

Edges as detected in DFRFT domain

(b)
Number of edges as determined in DFRFT domain with different block sizes is compared with that obtained in spatial domain. The measures as in ref [4] are used in determining the edges in spatial domain. Table 5.2 lists the number of edges as a percentage of total edges in cameraman image. Table 5.3 lists the number of edges as a percentage of total edges in coins image. There is no significant difference in the number of edges detected in DFRFT domain in comparison with the edges in spatial domain, maximum difference being only 1.6%.
Table 5.2: Number of edges detected in DFRFT and spatial domain as a % of total blocks for cameraman image.

<table>
<thead>
<tr>
<th>% of edges</th>
<th>9×9</th>
<th>7×7</th>
<th>5×5</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>DFRFT</td>
<td>Spatial</td>
<td>DFRFT</td>
</tr>
<tr>
<td>No edge</td>
<td>69.1</td>
<td>70.7</td>
<td>76.7</td>
</tr>
<tr>
<td>0°</td>
<td>6.5</td>
<td>6.0</td>
<td>2.9</td>
</tr>
<tr>
<td>90°</td>
<td>5.2</td>
<td>4.8</td>
<td>6.25</td>
</tr>
<tr>
<td>45°</td>
<td>8.9</td>
<td>8.7</td>
<td>9.3</td>
</tr>
<tr>
<td>-45°</td>
<td>10.2</td>
<td>9.8</td>
<td>4.9</td>
</tr>
</tbody>
</table>

Table 5.3: Number of edges detected in DFRFT and spatial domain as a % of total blocks for coins image.

<table>
<thead>
<tr>
<th>% of edges</th>
<th>9×9</th>
<th>7×7</th>
<th>5×5</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>DFRFT</td>
<td>Spatial</td>
<td>DFRFT</td>
</tr>
<tr>
<td>No edge</td>
<td>81.8</td>
<td>82.0</td>
<td>87.0</td>
</tr>
<tr>
<td>0°</td>
<td>3.4</td>
<td>3.4</td>
<td>3.1</td>
</tr>
<tr>
<td>90°</td>
<td>3.0</td>
<td>2.9</td>
<td>1.5</td>
</tr>
<tr>
<td>45°</td>
<td>5.9</td>
<td>6.3</td>
<td>4.6</td>
</tr>
<tr>
<td>-45°</td>
<td>5.8</td>
<td>5.4</td>
<td>3.7</td>
</tr>
</tbody>
</table>

142
5.3 Block edge detection in DFRCT domain

Five ideal binary edge orientations are considered, with two possibilities for each ideal binary edge. The block size of \((2N+1)\times(2N+1)\) is chosen within which any ideal edge is defined, with the maximum value of \(N\) being 4. Two-dimensional DFRCT as defined in Eqn. (2.62) is applied to each block. For further computation, the terms \(A1\) to \(A3\) are defined as matrices, respectively, of absolute values, real values and imaginary values of DFRCT coefficients. The terms \(B\) to \(G\) are defined as the sum of row, column or certain diagonal elements of \(A1\), \(A2\) and \(A3\). The terms used in the computation of measures of edge orientation are listed in table 5.4. The ideal binary edges and the corresponding pattern of distribution of absolute values, real values and imaginary values of DFRCT coefficients for the same are shown in Fig. 5.9. By referring to the patterns of distribution of coefficients shown in Fig. 5.9, the empirical formulae for edge orientation are obtained and listed in table 5.5.
Figure 5.9: (a) Ideal binary edges.
(b) Distribution of absolute values of DFRCT coefficients.
(c) Distribution of real values of DFRCT coefficients.
(d) Distribution of imaginary values of DFRCT coefficients.

Table 5.4: Definition of the terms \( A1 \) to \( A3, B \) to \( G \).

<table>
<thead>
<tr>
<th>Term</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A1 )</td>
<td>Matrix of abs value of DFRCT coefficients</td>
</tr>
<tr>
<td>( A2 )</td>
<td>Matrix of real value of DFRCT coefficients</td>
</tr>
<tr>
<td>( A3 )</td>
<td>Matrix of imaginary value of DFRCT coefficients</td>
</tr>
<tr>
<td>( B )</td>
<td>( \sum ) first row of ( A1 )</td>
</tr>
<tr>
<td>( C )</td>
<td>( \sum ) first column of ( A1 )</td>
</tr>
<tr>
<td>( D )</td>
<td>( \sum ) Principal diagonal of ( A1 )</td>
</tr>
<tr>
<td>( E )</td>
<td>( \sum ) subdiagonal of ( A1 ) above Principal diagonal</td>
</tr>
<tr>
<td>( F )</td>
<td>( \sum ) subdiagonal of ( A1 ) below Principal diagonal</td>
</tr>
</tbody>
</table>
| \( G \) | \( \sum \) ((subdiagonal of \( A2 \) above Principal diagonal-

subdiagonal of \( A2 \) below Principal diagonal)+
(subdiagonal of \( A3 \) above Principal diagonal-
subdiagonal of \( A3 \) below Principal diagonal)) |
Table 5.5: Measures for five orientations of edge based on DFRCT coefficients.

<table>
<thead>
<tr>
<th>Edge orientation (in degrees)</th>
<th>Measure</th>
</tr>
</thead>
<tbody>
<tr>
<td>No edge</td>
<td>$\theta_{\text{no edge}} =</td>
</tr>
<tr>
<td>0</td>
<td>$\theta_0 =</td>
</tr>
<tr>
<td>90</td>
<td>$\theta_{90} =</td>
</tr>
<tr>
<td>45</td>
<td>$\theta_{45} = 3D + 3.5 \min(E, F)$</td>
</tr>
<tr>
<td>-45</td>
<td>$\theta_{-45} = 3D + 4G$</td>
</tr>
</tbody>
</table>

For $0^\circ$ edge orientation, one of the two blocks for ideal binary edges considered has $9 \times 5$ ones on the top of the edge and $9 \times 4$ zeros below the edge. The other block has $9 \times 5$ zeros on the top of the edge and $9 \times 4$ ones below the edge. The pattern of the DFRCT coefficients is as shown in Fig. 5.9 (b). The first column elements are of larger values than any other element in the block. The first row elements are of larger values for $90^\circ$ edge orientation. Thus, the empirical formula $|3.5C - B|$ will result in larger value in comparison with the other measures of orientation whenever the edge orientation is $0^\circ$. The other measures are formulated in a similar way. They are listed in table 5.5. Due to the identical pattern of absolute value of coefficient distribution for $45^\circ$ and $-45^\circ$ orientations, real and imaginary value distribution are taken into account to arrive at the empirical formula.
for these orientations. These measures remain the same even when the block sizes of 7×7 and 5×5 are considered.

To determine the edges in an image, it is divided into several blocks and DFRCT is found for each block. A1 to A3 and B to G are computed for each block as explained above. Measures of orientations $\theta_i$ are computed with $i$ = no edge, $0^\circ$, $90^\circ$, $45^\circ$ and $-45^\circ$ using the relations in table 5.5. The value of $i$ with the maximum measure of orientation, $\theta_i$, is considered as the orientation of that block. The corresponding edge label shown in Fig. 5.4 is stored for each of the block in the original image, forming another image showing only the edges.

In DFRCT based block edge detection, the presence of an edge and its orientation are determined in terms of DFRCT coefficients. The measures of five different orientations listed in Table 5.5 are independent of $a$. There is no need to compute inverse transform to determine the edges. Since signs of DFRCT coefficients are not considered, the edge detection is irrespective of which side of the edge is dark. There is no need to optimize the value of $a$ as demanded by the other applications of DFRCT [9].

Results

Three different block sizes, 9×9, 7×7 and 5×5 are considered. Figure 5.10 shows the edges of cameraman image as determined by the method discussed, for block size of 9×9, 7×7 and 5×5. Similarly, Fig. 5.11 shows
the results for coins image. The results indicate that the measures of orientations are robust and stable even for nonideal edges present in images. There is no need to employ threshold to decide the presence of the edge. Preprocessing such as image blurring to widen the edge is not required.

Edges as detected in DFRCT domain

(a)

Edges as detected in DFRCT domain

(b)
Figure 5.10: Edge detection based on DFRCT for
(a) Block size of 9×9.
(b) Block size of 7×7.
(c) Block size of 5×5.
Figure 5.11: Edge detection based on DFRCT for
(a) Block size of 9×9.
(b) Block size of 7×7.
(c) Block size of 5×5.

Number of edges determined in the image in DFRCT domain with
different block sizes is compared with that obtained in spatial domain. Table
5.6 and table 5.7 lists the number of edges as a percentage of total edges respectively in cameraman and coins image. There is no noticeable difference in the number of edges detected in DFRCT domain in comparison with the edges in spatial domain, maximum difference being only 1.1%.

Table 5.6: Number of edges detected in DFRCT and spatial domain as a % of total blocks for cameraman image.

<table>
<thead>
<tr>
<th>% of edges</th>
<th>Edge orientation</th>
<th>9×9</th>
<th>7×7</th>
<th>5×5</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>DFRCT</td>
<td>Spatial</td>
<td>DFRCT</td>
<td>Spatial</td>
</tr>
<tr>
<td>No edge</td>
<td>71.7</td>
<td>70.7</td>
<td>76.0</td>
<td>77.1</td>
</tr>
<tr>
<td>0°</td>
<td>5.9</td>
<td>6.0</td>
<td>3.7</td>
<td>3.1</td>
</tr>
<tr>
<td>90°</td>
<td>5.1</td>
<td>4.8</td>
<td>5.7</td>
<td>5.7</td>
</tr>
<tr>
<td>45°</td>
<td>8.4</td>
<td>8.7</td>
<td>8.7</td>
<td>8.7</td>
</tr>
<tr>
<td>-45°</td>
<td>8.9</td>
<td>9.8</td>
<td>5.9</td>
<td>5.4</td>
</tr>
</tbody>
</table>

Table 5.7: Number of edges detected in DFRCT and spatial domain as a % of total blocks for coins image.

<table>
<thead>
<tr>
<th>% of edges</th>
<th>Edge orientation</th>
<th>9×9</th>
<th>7×7</th>
<th>5×5</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>DFRCT</td>
<td>Spatial</td>
<td>DFRCT</td>
<td>Spatial</td>
</tr>
<tr>
<td>No edge</td>
<td>82.8</td>
<td>82.0</td>
<td>86.6</td>
<td>86.9</td>
</tr>
<tr>
<td>0°</td>
<td>4.0</td>
<td>3.4</td>
<td>2.9</td>
<td>2.9</td>
</tr>
<tr>
<td>90°</td>
<td>2.7</td>
<td>2.9</td>
<td>1.8</td>
<td>1.7</td>
</tr>
<tr>
<td>45°</td>
<td>6.2</td>
<td>6.3</td>
<td>4.2</td>
<td>4.2</td>
</tr>
<tr>
<td>-45°</td>
<td>4.3</td>
<td>5.4</td>
<td>4.4</td>
<td>4.2</td>
</tr>
</tbody>
</table>
5.4 Block edge detection in DFRST domain

Five ideal binary edge orientations are considered, with two possibilities for each ideal binary edge. The block sizes of $5 \times 5$, $7 \times 7$ and $9 \times 9$ are chosen within which any ideal edge is defined. Two-dimensional DFRST as defined in Eqn. (2.79) is applied to each block. $A1$ and $A2$ are the matrices, respectively, of real values and imaginary values of DFRST coefficients. The terms $B$ to $G$ are defined as the sum of row, column or certain diagonal elements of $A1$ and $A2$. The terms used in the computation of measures of edge orientation are listed in Table 5.8. The ideal binary edges and the corresponding pattern of real values and imaginary values of DFRST coefficients for the same are shown in Fig. 5.12. By referring to the patterns of distribution of coefficients shown in Fig. 5.12, the empirical formulae for edge orientation are obtained and listed in Table 5.9.

![Figure 5.12: (a) Ideal binary edges. (b) Distribution of real values of DFRST coefficients. (c) Distribution of imaginary values of DFRST coefficients.](image-url)
Table 5.8: Definition of the terms $A_1$, $A_2$ and $B$ to $G$.

<table>
<thead>
<tr>
<th>Term</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_1$</td>
<td>Matrix of real value of DFRST coefficients</td>
</tr>
<tr>
<td>$A_2$</td>
<td>Matrix of imaginary value of DFRST coefficients</td>
</tr>
<tr>
<td>$B$</td>
<td>$\text{Abs} \left( \sum \text{first row of } A_1 \right)$</td>
</tr>
<tr>
<td>$C$</td>
<td>$\text{Abs} \left( \sum \text{subdiagonal of } A_1 \text{ above Principal diagonal} \right) + \text{Abs} \left( \sum \text{subdiagonal of } A_1 \text{ below Principal diagonal} \right)$</td>
</tr>
<tr>
<td>$D$</td>
<td>$\sum \text{first row of } A_2$</td>
</tr>
<tr>
<td>$E$</td>
<td>$\sum \text{first column of } A_2$</td>
</tr>
<tr>
<td>$F$</td>
<td>$\sum \text{Sub diagonal of } A_2 \text{ above Principal diagonal}$</td>
</tr>
<tr>
<td>$G$</td>
<td>$\sum \text{Sub diagonal of } A_2 \text{ below Principal diagonal}$</td>
</tr>
</tbody>
</table>

Table 5.9: Measures for five orientations of edge based on DFRST coefficients.

<table>
<thead>
<tr>
<th>Edge orientation (in degrees)</th>
<th>Measure</th>
</tr>
</thead>
<tbody>
<tr>
<td>No edge</td>
<td>$\theta_{no_edge} = B$</td>
</tr>
<tr>
<td>0</td>
<td>$\theta_0 =</td>
</tr>
<tr>
<td>90</td>
<td>$\theta_{90} =</td>
</tr>
<tr>
<td>45</td>
<td>$\theta_{45} = C$</td>
</tr>
<tr>
<td>$-45$</td>
<td>$\theta_{-45} =</td>
</tr>
</tbody>
</table>
For $45^0$ edge orientation, the ideal binary edge has ones in the upper triangle and zeros in the lower triangle. Another pattern of $45^0$ edge orientation, has ones in the lower triangle and zeros in the upper triangle. The pattern of the real part of DFRST coefficients is as shown in Fig. 5.12 (b). Both $45^0$ and $-45^0$ have nearly same value of coefficients, however, they differ in their signs. This is taken into consideration while arriving at the empirical formula for $45^0$ orientation as $C$, which is the sum of absolute values of sub diagonal above and below principal diagonal of matrix of real coefficients. The other measures are formulated in a similar way. They are listed in Table 5.9. These measures remain the same even when the block sizes of $7 \times 7$ and $5 \times 5$ are considered.

In order to determine the edges in an image, it is divided into several blocks and DFRST is found for each block. $A1, A2$ and $B$ to $G$ are computed for each block as explained above. Measures of orientations $\theta_i$ are computed with $i$ = no edge, $0^0$, $90^0$, $45^0$ and $-45^0$ using the relations in Table 5.9. The value of $i$ with the maximum measure of orientation, $\theta_i$, is considered as the orientation of that block. The corresponding edge label shown in Fig. 5.4 is stored for each of the block in the original image, forming another image showing only the edges.

In the case of DFRST based block edge detection, the presence of an edge and its orientation are determined in terms of real and imaginary values of DFRST coefficients, taking into consideration the signs of these values.
also. The measures of five different orientations listed in Table 5.9 are independent of \( a \). There is no need to compute inverse transform to determine the edges. Though signs of DFRST coefficients are considered, the edge detection is irrespective of which side of the edge is dark.

Results

Three different block sizes, 9×9, 7×7 and 5×5 are considered. Figure 5.13 shows the edges of cameraman image as determined by the method discussed, for block size of 9×9, 7×7 and 5×5. Similarly, Fig. 5.14 shows the results for coins image. The results indicate that the measures of orientations are robust and stable even for nonideal edges present in images. There is no need to employ threshold to decide the presence of the edge. Preprocessing such as image blurring to widen the edge is not required.
Figure 5.13: Edge detection based on DFRST for
(a) Block size of 9x9.
(b) Block size of 7x7.
(c) Block size of 5x5.
Figure 5.14: Edge detection based on DFRST for
(a) Block size of $9 \times 9$.
(b) Block size of $7 \times 7$.
(c) Block size of $5 \times 5$.

Number of edges determined in the image in DFRST domain with
different block sizes is compared with that obtained in spatial domain. Table
5.10 and table 5.11 lists the number of edges as a percentage of total edges
respectively in cameraman and coins image. The maximum difference in the
number of edges detected in DFRST domain in comparison with the edges
in spatial domain is 2.1%.
Table 5.10: Number of edges detected in DFRST and spatial domain as a % of total blocks for cameraman image.

<table>
<thead>
<tr>
<th>Edge orientation</th>
<th>9x9 DFRST</th>
<th>9x9 Spatial</th>
<th>7x7 DFRST</th>
<th>7x7 Spatial</th>
<th>5x5 DFRST</th>
<th>5x5 Spatial</th>
</tr>
</thead>
<tbody>
<tr>
<td>No edge</td>
<td>72.8</td>
<td>70.7</td>
<td>77.6</td>
<td>77.1</td>
<td>78.9</td>
<td>78.5</td>
</tr>
<tr>
<td>0°</td>
<td>5.5</td>
<td>6.0</td>
<td>2.4</td>
<td>3.1</td>
<td>2.5</td>
<td>2.3</td>
</tr>
<tr>
<td>90°</td>
<td>4.7</td>
<td>4.8</td>
<td>4.1</td>
<td>5.7</td>
<td>5.9</td>
<td>5.7</td>
</tr>
<tr>
<td>45°</td>
<td>7.8</td>
<td>8.7</td>
<td>9.3</td>
<td>8.7</td>
<td>6.3</td>
<td>6.5</td>
</tr>
<tr>
<td>-45°</td>
<td>9.2</td>
<td>9.8</td>
<td>6.6</td>
<td>5.4</td>
<td>6.3</td>
<td>6.9</td>
</tr>
</tbody>
</table>

Table 5.11: Number of edges detected in DFRST and spatial domain as a % of total blocks for coins image.

<table>
<thead>
<tr>
<th>Edge orientation</th>
<th>9x9 DFRST</th>
<th>9x9 Spatial</th>
<th>7x7 DFRST</th>
<th>7x7 Spatial</th>
<th>5x5 DFRST</th>
<th>5x5 Spatial</th>
</tr>
</thead>
<tbody>
<tr>
<td>No edge</td>
<td>83.4</td>
<td>82.0</td>
<td>88.1</td>
<td>86.9</td>
<td>90.3</td>
<td>89.7</td>
</tr>
<tr>
<td>0°</td>
<td>4.0</td>
<td>3.4</td>
<td>2.9</td>
<td>2.9</td>
<td>2.3</td>
<td>2.2</td>
</tr>
<tr>
<td>90°</td>
<td>3.9</td>
<td>2.9</td>
<td>1.4</td>
<td>1.7</td>
<td>1.2</td>
<td>1.5</td>
</tr>
<tr>
<td>45°</td>
<td>4.3</td>
<td>6.3</td>
<td>3.6</td>
<td>4.2</td>
<td>3.7</td>
<td>3.5</td>
</tr>
<tr>
<td>-45°</td>
<td>4.4</td>
<td>5.4</td>
<td>4.0</td>
<td>4.2</td>
<td>2.5</td>
<td>3.0</td>
</tr>
</tbody>
</table>
5.5 Comparison of performance of DFRFT, DFRCT and DFRST in block edge detection

In determining the edges in an image using DFRFT, DFRCT and DFRST, individual blocks of the image are transformed to respective domains. The absolute, real and imaginary values of coefficients are considered in further computation. In DFRFT based method, absolute values of coefficients are summed up by grouping them into 4 sub blocks. Empirical formulae are found by considering these values. In the case of DFRCT based edge detection, all the coefficients are not required to decide the edge orientation. Instead, only first row, first column, principal diagonal elements and the diagonal elements above and below the principal diagonal of the block are considered. Real values and imaginary values of coefficients are employed along with absolute values in the case of DFRCT based method. And in the case of DFRST based edge detection, the signs of the coefficients are also required to decide the edge orientation. All the three fractional transforms result in orientation measures which are independent of order parameter $a$. The measure of orientations are different for different fractional transforms.

Ideally, the number of edges found by all the methods should be equal. However, there is a difference in their performance. Maximum difference in the number of edges as a percentage of total blocks, when compared with that of spatial domain, is observed in the case of DFRST based method. It
can be observed from the images in Fig. 5.6, 5.8, Fig. 5.10, 5.11 and Fig. 5.13 and 5.14 that the performance of DFRFT is better in distinguishing $45^0$ and $-45^0$ edge orientations.

5.6 Conclusions

A new scheme for block edge detection based on three fractional transforms is presented. The schemes are simple and require less computation when the images are in the fractional transform domain. The edge detection is performed in the transformed domain itself. There is no need to employ threshold to decide the presence of an edge and preprocessing such as image blurring to widen the edge. In all the cases, the measures of edge orientations are independent of value of $a$ and they are robust and stable for nonideal edges present in images. Five different orientations and three different block sizes have been considered, all the methods have unique orientation measures. Percentage of differences in the number of edges determined by these methods in comparison with spatial domain method is not more than 2.1%.
REFERENCES


