CHAPTER - III
THEORIES OF EXHAUSTIBLE RESOURCES
A CRITICAL REVIEW

3.1 Introduction

Since Quarrying activity is concerned with the mining of exhaustible resources, it is worthwhile to study the theory of mining and evolve conditions of optimum utilization of exhaustible resources keeping in view inter generational equity. Natural resources require a separate study, though the same economic tools are employed here too. Because, (i). A large set of policy issues, today concern natural resources perse. An understanding of economic principles behind resources use is an invaluable aid to informal discussion of these practical problems, (ii). Natural resources have unique features, not found in other resources. Important among these are non-renewability and common property, and (iii). Decision taken in the case of natural resources involves inter-temporal considerations.

Exhaustibility introduces some new problems and issues in to the analysis of production from mining (or Quarrying) that do not arise in the production of reproducible goods, such as agricultural crops.

A mine owner must determine not only as to how to combine variable factor inputs such as labour and materials with fixed capital as does the farmer but also how quickly to run-down the fixed stock of ore reserves through extraction of mineral. A unit of ore extracted today means less in the total available for tomorrow. Time plays an
important role in the analysis. Each period is different because the stock of the resource remaining is different in size. What we are concerned with an economic analysis of non-renewable resources is how quickly the mineral is extracted, what the flow of production is over time and when the stock will be exhausted.

3.2 The Theory of Mine (Quarry):

Let us take an example of an individual mine (quarry) which operates under perfectly competitive conditions. Suppose the mine owner is interested in maximizing profit by extracting ore and selling it in the same way as the producer of a reproducible good. Since the extraction of ore is spread over years, he has to maximize the difference between the present value of the stream of revenue and the stream of cost. The general rule for maximizing the profit is equation of marginal stock which is fixed in quantity modifies the revenue and marginal cost. The fact that the mineral maximization conditions are in three ways. The owner of the mine is faced with an opportunity cost, which an ordinary producer of a reproducer and or a reproducible good or crop does not face, it is the cost of using the fixed stock at any point of time or being left with smaller remaining reserves. To maximize profits the mine owner must cover this opportunity cost of depletion. For a common producer the profit maximization rule is MR=MC=P. (Marginal Revenue=Marginal Cost=Price). For the mine owner it is the price P=MC+opportunity cost of depletion, where the opportunity cost is measured as the value of unextracted resource and it is called "resource rent".
The second fundamental difference between the mine owner and the ordinary producer is concerned with the value of the resource rent over time. This can be illustrated with a simple example of investing money (or fixed sum of money). It depends on expectation of the investor about the rate of return on that amount and the increase in its value over time. He invests in that asset which will yield to him highest return. However, under perfectly competitive conditions and with no uncertainty, all assets must earn the same return when in equilibrium situation.

Consider that there are two assets, A and B. A's value increases by 5% every year, while B's value increases by 10% every year. Then no one would invest in A. Every one would want asset B. The demand for asset B increases, hence its price rises, while the price of A would fall until their rates of return are equalized.

Here the rate of return to an exhaustible resource i.e., rate of return to the mine, is the resource - rent. When there is a positive discount rate the rent is positive and rises in nominal value as depletion occurs. If the resource rent does not increase in value over time, no one would purchase the mine because the rate of returns on alternative assets would be more valuable. In addition, the owner of an existing deposit would attempt to extract all the ore as quickly as it is technically feasible. He will hold on to ore in the ground only when that increases in value at a rate less than that can be earned on, say, a savings account. Alternatively if the value of the ore is growing at a rate in excess of what one could earn in an alternative investment there is no incentive to extract at all, ore left in the ground
is then more valuable to the mine owner than ore extracted. To have mineral extraction then, the rental value of the mineral must be growing at the same rate as that of the alternative assets.

There is one final condition imposed on the mine owner that does not occur with the reproduction goods. The total amount of the natural resource extracted over time cannot exceed its total stock of reserves. This is called the stock constraint.

3.3 “GRAY’S Theory of Mine” :

One of the earliest economic analyses of the mineral extraction appeared in 1914 in an article by L.C. Gray. In his model, he has shown how the owner of a small mine has to decide as to how much ore to be extracted and for how long a period of time. To solve this problem, Gray made an attempt with a number of simplifying assumptions, viz;

1. Market price of a unit of the mineral remains constant over the life of mine.

2. The producer knows the exact amount of reserves in the mine prior to extraction.

3. All the ore is of uniform quality.

4. The extraction costs then depends only on the quantity removed.

Gray considered the mine of a huge block of pure copper, where the price per ton is constant forever, while the marginal cost of cutting of a piece of copper rises with the size of piece cut off. If one ton of copper is cut-off, it will cost Rs.500/- to remove, If 10
tons are cut-off at once, the extraction costs could be Rs. 10,000/-. The economic problem is to cut-off appropriate quantities in each period in order to maximise the present value of profits available from the stock of the mineral. The model has practical appeal because in many mineral markets relatively constant prices have prevailed over long periods of time.

To determine the efficient extraction path for the mine, Gray starts with a simple illustration. Suppose, the mine will operate for two periods only. The mine owner must determine how much copper to chop-off the block today and tomorrow, using the three conditions identified earlier for the two period case. These conditions can be stated as;

1. Price = MC + rent in each period.
2. Rent today = the present value of rent tomorrow.
3. Extraction today + extraction tomorrow = total stock of reserves

The solution is shown in the figure 3.1 Assume that the mine
has U-shaped average cost curve (AC) and upward sloping marginal cost curve (MC) over some output range. The constant price is shown as “P”. Output today is designated as Q(o), output tomorrow is Q(T), where T signifies the end of the mining operation the length of time the mine operates (in this case, two periods). Given these curves and the total stock of ore, there will be a unique solution to the extraction problem that satisfies all three conditions.

The mine owner must pick up an initial output level where P=MC+ rent. The resource rent obtained at the output level Q(o) is R(o), This is the condition 1. Here, that condition 1 defines rent as the difference between price and marginal cost. In the next period extraction must equal Q(T) and the rent will be R(T). It must be the case that R(o)=R(T)/(1+r) where r is the market rate or discount rate, the rate of the return on any alternative assets. This is the condition 2. If the rents did not rise at the rate of interest, extraction would not occur in both the periods, if the rent rose more slowly than the interest rate, the entire stock of ore would be extracted in the initial period and the proceeds of the sale invested in some other assets whose value would rise at the rate of interest (eg. a saving account). If the rent rose faster than the rate of interest the entire stock of ore would be held in the ground until the last moment in time and then extracted. In this case the mine is worth more, unextracted, because the rate of the return in holding ore in the ground exceeds the return on alternative investments. Unless the rental value of the mine is growing at exactly the same rate as the value of other assets,
extraction will either be as fast as possible or deferred as long as possible.

Finally, output today and tomorrow must be chosen such that $Q(o)+Q(t)=S$, where $S$ is the stock of mineral reserves. This is the condition 3. For a given $S$, $R$, and $p$, there will be only one level of initial output and hence final output that satisfies all these conditions.

It can be extended to many periods of operation but the same three conditions must be met. In addition, we can also know as to when the mine will cease its operation, how long $T$ is from figure-1, it is not a coincidence that $Q(T)$ is at the point where $MC=AC$. This point is called "terminal condition" for the non-renewable resource extraction problem.

Average cost and marginal cost are equal at $Q(T)$, and $Q(T)$ is also that output which combined with $Q(o)$ exhausts the mine. In the many periods case the time for depletion will be such that all the three extraction conditions are satisfied plus the terminal condition.

3.4 Profit Maximization for the Mine:

Profit maximization involves the method of making revenues larger in relation to costs of production. There is a series of revenues minus costs each year or period in to the future. Each instant in time is slightly different. Since depletion of the stock is occurring year by year, discount with the current interest rate makes each annual profit value comparable to others, till the date from the
beginning of extraction. In the absence of discounting profit in year say 8, in the future would be not comparable with profit in year say 11. Each nominal value is different at any one point in time in the absence of discounting.

Even if the assumption of homogenous quality of the ore is relaxed the result remains the same. In that case, extraction cost increases not only with the quantity of ore extracted but also with the quality of ore.

If we assume that best quality is found on the top layer and the inferior quality is found in the bottom, to extract the inferior quality ore, the price has to rise. Otherwise extraction will stop. Once the best quality is extracted, continued extraction reduces the stock. Extraction is a state whose extraction becomes zero. This is difference between Physical extraction and Economic extraction. If price is very low as not to cover even cost of extraction, activity stops, though the ore is not completely exhausted this is called economic exhaustion. This is what is highlighted by the Gray's theory.

3.5 HOTELLING’S Theory:

In 1931, HARALD HOTELLING, wrote a classical paper which examined the optimal extraction of non-renewable resource from the view point of a social planning agency that had as its goal the maximization of social welfare from the production of minerals. The model was at the industrial level rather than that of the single mine owner. Hotelling also arrived at the same condition for the efficient extraction of the mineral. When he dealt with an industry rather
than a single mine, the mineral price can no longer be treated as constant. Rather assumed that the industry faces a negatively sloped demand curve. The greater the industrial output the lower the price to be. Hotelling assumed that price would adjust the mineral market at any given point of time. It must be in equilibrium. Supply must be always equal demand. The present value of unit of a homogeneous but finite stock of mineral must be identical regardless of when it is extracted. This principle reflects the conditions 1 and 2 which may be called flow condition, together with the stock condition and terminal condition, the optimal extraction plan for the non-renewable resource can be determined at the industry level as well as for the single mine.

Hotelling viewed the problem of how to extract a fixed stock of a natural resource from the advantage point of government social planning agency. He showed that a competitive industry facing the same extraction costs and demand curve as the government and having perfect information about the resource prices will arrive at exactly the same extraction path determined by each firm acting independently in the competitive industry and will yield the same social optimal extraction path.

Hotelling assumed that prices would adjust so that a mineral market would be in equilibrium at every point in time; supply must always equal demand. As before,

1. The stock of mineral reserves is of known size

2. All units of the minerals are homogeneous,
3. we assume a unit of stock costs “C” rupees to extract and refine and that this cost is constant for all units of the stock in the reserves endowment “S”. Once again it is as if we had a huge block of copper that cost “C” per ton to “chip-off”

We want to find the rate of extraction that maximises social welfare and completely exhausts the stock. If c=0 the analysis is qualitatively unchanged. If “c” increases with the quantity extracted, we are back with Gray’s cost assumption.

The crucial distinction between Hotelling’s model and Gray’s Model is that we must examine the demand curve explicitly and derive a unique price of the resource in each period (Q(0), Q(1), Q(2), Q(3), .................Q(t).) when the mineral industry is in operation.

The planner will want to measure the change in the social surplus as one more unit extracted is simply the difference between the market price and the marginal cost of extraction “c”.

![Diagram of Hotelling's and Gray's models with price and quantity for period t and t+1 showing the relationship between market price and marginal cost of extraction.]

Fig 3.2
The above figure 3.2 illustrates one pair of outputs for the two periods which will satisfy this condition, given "D", the demand curve and c and r. Here the flow condition implies that the mineral price must rise over time. In this model, with a stationary demand curve, the only way the price and hence the rent will rise, as if the quantity extracted declines over time. Therefore extraction in period (t+1) must be less than that in period (t) to ensure that the price rises.

As price rises the rent per ton grows over time at a rate equal to the rate of interest. This is often referred to as Hotelling's simple rule.

We have sketched the price paths in figure. How do we know that the path shown is the one that maximises social welfare? All that Hotelling's rule says is that "rent must grow at the rate of interest". Might there be dozens of different paths all of which satisfy Hotelling's rule? Yes, but a unique path of output can be derived with the help of stock constraint and terminal condition.

Fig. 3.3
If the costs are constant the planner will want to ensure that all the extraction mineral is removed. If any ore is left in ground the mine owner will be foregoing rents. The constant cost assumption is crucial in this argument. Each unit of ore costs the same for extraction.

We can also see in the figure 3.3 the linear demand curve. There is some price say P. at which no one is willing to buy more of the mineral. The price P is often called the choke price meaning that demand for the good is choked off at this point. Ideally the planner would seek to have the stock of the mineral go to zero at exactly the point that demand goes to zero. Therefore, the planner would seek to have the last unit of output extracted at P. To do otherwise would deprive the society of maximum benefits. We can then work back word from P, given the fixed stock S, to find just that initial output Q(0) which will over time decline so that rent increases at rate “r” and outputs sum to the stock of reserves. Only one such extraction and hence rent path exists. It will yield the largest amount of social surplus available to society and hence be the optimal plan. In addition, we can now determine the length of time the mine operates. The point at which the price path intersects price P will determine the unique duration of the extraction profile T. Once the price reaches P, there will be no more demand for the mineral, so extraction will cease. Extraction ends at time T.
3.6 Optimal Utilization under Different Market Structures:

Perfect competition, Monopoly and oligopoly are the different kinds of market structures. Market structure affects the level of profits that firms attain, which in turn determine the total amounts produced and the prices obtained in the market. To maximize profits a firm usually sets its marginal revenue (M.R.) equal to its Marginal cost (M.C.). What differs among the market structures is the number of firms in the industry and their ability to engage in strategic actions.

The number of firms will determine the MR function perceived by the firm and whether strategic actions are meaningful. In the case of perfect competition there are large number of relatively small firms and each firm cannot affect the market price of the good it produces. In the case of Monopoly there is only one firm and the monopolist thus determines the price it charges consumers.

The oligopolistic market structure is the most complex and applicable to real world market structures. Oligopoly means few sellers. Each seller has some share of the market demand but its share and its pricing actions are dependent on the actions of its rivals.

a) Monopoly:

In the monopoly market monopolist can select any price it wishes on the market demand schedule, because the monopolist is the only supplier of the good. The quantity supplied determines the price. The monopolist maximises its profits when quantity is chosen
so as to equate MR and MC. A competitive firm also equates MR to MC, but its MR differs from that of the monopolist. The difference between the two market structures is that the MR for the monopolist is continuously declining because the monopolist views the entire demand curve as its own. For each additional unit sold by the monopolist the price fall a bit, and thus the change in the revenue received (MR) declines. Each firm operating in a perfectly competitive market has such a small share of the market, that it can sell as much as it can produce at the equilibrium price determined in the market. The demand curve facing the competitive firm is thus “perfectly” elastic at the equilibrium price, MR is constant and equal to the price. The profit maximizing competitive firm sets price equal to M.C.

b) Market Structure in a Dynamic Environment Monopoly:

A monopolist that controls the entire stock of a non-renewable natural resource in the industry will act to maximize the present value of its profits over time. The monopolist will choose a time path of quantities extracted from the stock to achieve this result. We now illustrate the conditions the monopolists must satisfy to maximize its profits over time and see how the extraction paths which satisfies these conditions differs from that of the resources managed optimally under perfect competition. The monopolist must satisfy the conditions for an inter temporal profit maximum that are very similar to Hotelling’s rule, and terminal condition established for the competitive form.
Suppose extraction costs are zero if \( pt(0) \) is the inverse demand curve in period \( t \) then the present value of profit in period \( t \) will be

\[
\frac{[pt(q_t)q_t]}{(1+r)}
\]

The present value of the monopolists profits for a sequence of extractions over time \( Q_0, Q_1, \ldots, Q_T \).

As in the case of perfect competition the sum of all the "QS" must not exceed the known stock of reserves \( S_0 \). We assume that the demand curve remains stationary over time. The monopolist maximize its profits by choosing an extraction paths \( Q_0, \ldots, Q_t \), that uses up its entire stock of reserves. The flow condition equation

\[
\left(\frac{1}{1+r}\right)^tMR(Q_t) = \text{constant} \quad [a]
\]

from this equation we can derive for any two periods \( t \) and \( (t+r) \) that

\[
\frac{MR(Q_{t+1}) - Mr(Q_t)}{MR(Q_t)} = r \quad [b]
\]

Equation [b] requires that the percentage charge in MR over time equal the rate of interest. This is simply Hotelling rule for a monopolist facing zero cost of extraction.

The difference between the equation [b] and Hotelling's rule for the competitive firm is that MR replaces the price.
The intuition behind this condition for an intertemporal profit maximum is exactly the same as in the figure Hotelling rule of the flow condition require output in period t and period +1 requires to be chosen so that MRt(1+r)=MR+1 as show in the figure.

The monopolist must also satisfy a terminal condition as does the competitive firm to determine the actual path of MR. Without the terminal condition any sequence of "qs" that satisfies Hotelling's rule is acceptable. The terminal condition pins down the final output level and hence all subsequent outputs. The flow condition requires that the MR per ton extracted to be constant in present value. To meet this condition output must fall over time so that the current value of MR will rise.

The last ton extracted will thus yield the highest undiscounted value of MR. [shown in figure as the amount ab.] Hence the terminal condition is satisfied when qt=0 as MR (qt)=ab. Formally this outcome derives from the condition that marginal profit on the last
ton should equal to average profit on that ton, (the analogous condition as for perfect competition) this is satisfied only when qt equals to zero and MR.

However, because the MR curve is steeper than demand curve, the monopolist will have to decrease output in each subsequent period it operates by less than a competitive industry would. Think of the monopolist moving up its MR curve and the competitive industry moving up the demand curve in each succeeding period. Both regimes must satisfy their respective versions of Hotellings rule facing the same discount rate. Thus when the demand curve is linear and extraction costs are zero, the monopolist will extract its ore reserves more slowly than the competitive industry. The monopolists will therefore take longer to extract a fixed stock of ore than a competitive industry.

A monopolist will take exactly twice as long to extract the same stock of reserves, as would competitive industry. The difference in relative speeds of extraction between the two market structure led Hotelling (1931) to remark that the monopolist was a “friend” of the conservationist that is the monopolist would run down the stock more, slowly than the planner. This also implies that for the same stock size, the monopolist’s initial price will be higher and rise at a slower rate than for the competitive regime. The diagram below illustrates this result.
In the figure (a) there are two smooth price paths, one for the monopolist $M$ and other for the competitive industry "C". Each regime has an identical stock of ore reserve with zero extraction cost (for simplicity). The initial price for the monopoly regime is "PM" while "PC", is the initial price for the competitive regime. Suppose both begin extraction at time "to" and "P" is the choke price.

In the figure (b) there are two extraction (quantity) paths corresponding to the price paths. The monopolists output is initially lower than the competitive industry's so the initial monopoly price must be higher, but because MR is less than the price, the path of the monopolist is flatter than that of competitive industry (it rises more slowly). Recall that Hotelling's rule for the monopolist require MR to grow at the rate of interest while for the competitive industry, it is price that grows at the rate of interest (when extraction costs are zero). The monopolist then extracts less in early periods than the competitive industry but more in the later periods when the
If the resource market is characterised by Oligopoly the results with regard to price and output extraction, depend upon various assumptions regarding the characteristic of Oligopoly, such as a) dominant firm b) cartel c) price leader d) market sharing etc. The outcomes are numerous depending upon the characteristics of Oligopoly market providing determinate or in determinate price and output results.

Mining under uncertainty:- Uncertainty is prevalent in decision making regarding non-renewable resources extraction. There is uncertainty about stock extraction cost possibilities of research and development, development of substitutes etc. So under uncertainty the mine owner tries to maximise the expected net present value of the rent. Therefore, existence of uncertainty is equivalent to an increase in the discount rate for the owners, which implies that the price of the resource must rise more rapidly and the depletion is accelerated.

What would happen if the world were to run out of nonrenewable resource one day? What does the Hotelling model tell us about this occurrence? The impact of complete exhaustion on society depends on the technology of producing and using the resources in production and can be reflected in the demand curve for the resource. A crucial question is whether the resource is so
necessary to the production process of other goods that once it is depleted, the other goods will also cease to be produced. Our model with the linear demand curve and choke price, says that price at which the uses of the good will switch entirely to the use of the substitute good. This substitute may be another non-renewable resource such as oil shale as a substitute for conventional crude oil, or solar energy. If the substitute exists society and economic system will not collapse. When the non-renewable resource runs out, they will shift to substitute non-renewable resource or renewable resource.

From the foregoing discussion, we can conclude that the exposition of the received theory regarding Exhaustible resources suggests that, uncontrolled exploitation as under competitive condition leads to rapid exhaustion with no consideration about its future availability, on the other hand, the monopolistic control of resources would lead to lower rate of exploitation and the resource would last for a longer period from the point of view of conservation. Monopoly would be better than competition. A public monopoly would be still better.

Quarrying deals with the exhaustible resources. With finance available everywhere, housing boom has started. This has created huge demand for building materials. Quarrying is one of the industries which provides amply building materials, including Hyder, Parcy, Bandi, Khadi, Chavani, Katagal etc.

The above theoretical background helps us to know as to (1) whether quarry resources are recklessly exploited, (2) What has
been the income to the quarry owners and workers, (3) Whether it is going to exhaust in a near future, and (4) Whether there is need for substituting public monopoly in place of competition. Further it also aims at finding empirical proof for the existing theory regarding the exhaustible resource.