PREFACE

The expansion of functions into infinite series is one of the most powerful techniques of mathematical analysis. Taylor series provide precise local information about a function, but only if the function has many derivatives, which is not guaranteed even with large number of derivatives. Trigonometric series or Fourier series, Fourier Analysis (Fourier Transforms and Spectral Methods) is one of the most extensively studied topics with applications in Physical Sciences, Engineering and other disciplines. Inspite of its great success and beautiful Mathematics in it – FA has limitations in applications. Fourier series or integrals can be used for functions with no smoothness properties, yield lots of information about the global properties of the function, but they are inefficient for analyzing the detailed behavior of a function near a point. It is not well suited for the local analysis of functions/data, for the solution of nonlinear (ODE/PDE) equations etc. A pure Fourier basis diagonalises translation invariant linear operators. We look for a basis (of function spaces) that is well localized in frequency and nearly diagonalizes the operator i.e. their matrix entries decay rapidly away from the diagonal. Also, it is desirable a basis to be well localized in space for effective local analysis. What is missing is a method for analyzing the local irregular behavior of functions that aren’t smooth and this is where wavelets come into play. Haar’s (1910) research led to the simplest orthonormal wavelet basis, a set of rectangular basis functions of $L^2(0,1)$ with compact support and localized in space/time but not continuous. The work of early mathematicians like Calderon-
Zygmund, Paley-Littlewood, Balian-Low and Stein laid the groundwork for future development of the theory. Later, it was Gabor (1946) whose idea was to break the signal into segments/slices and then analyse the individual segments each of which will have well defined frequency band and position in time enabled for data analysis.

The synthesis of these theories leading to Wavelet analysis and the impetus to great interest was the result of research by Grossman and Morlet (1984) who called Littlewood–Paley theory as Wavelet theory. Meyer (1993) and his coworkers noticed that Calderon – Zygmund theory, in particular the Littlewood–Paley representation having discrete analog gave unified view of research in harmonic analysis. Lamarie and Meyer (1986) constructed new orthogonal Wavelet expansion. Mallet (1989) introduced the most useful functional analysis of multiresolution concept. Daubechies (1988), using Mallet’s ideas, constructed discrete orthonormal smooth wavelets with compact support. From this stage – from continuous signal processing, to discrete signal processing opened the flood gates in its theory and applications and since then several others have been contributing substantially (Daubechies, 1988, 1992, Strang, 1989), Bendetto and Frazier(1994), and many others).

The multiscale representation provided by wavelet analysis can be thought of as a representation that does for functions what positional notation does for numbers. Wavelet analysis provides a systematic and efficient universal representation for a wide class of functions. Indeed, the class of functions representable by wavelet
analysis is much larger than the class of square-integrable functions or finite energy signals.

The main objective of this thesis is to harness the fast, efficient, robust and accurate wavelet based algorithms and series for solving nonlinear and Isoperimetric Variational problems, nonlinear Integro–Differential Equations, nonlinear Stiff–Differential Equations arising in various disciplines. Also, these newer schemes are implemented for the solution of general Elliptic Partial Differential Equations arising in tribology. Specifically it focuses on Wavelet multigrid method with applications in computational fluid dynamics. This thesis is organized into seven chapters.

The FIRST CHAPTER is an introductory topic. It gives a motivation and Historical background of Wavelet Series used in the thesis and briefly gives the relevant Mathematical background of wavelet series. A brief introduction to Multiresolution Analysis, properties of scaling function, construction of orthonormal wavelets, Fluid Mechanics, Hydrodynamic Lubrication, Synovial Joint Lubrication, Couple-stress Fluids and Surface Roughness etc, are given.

In CHAPTER TWO, Haar wavelet series technique (Hsiao, 2004), is used to analyze several variational problems leading to extremization of functionals dependent on only one & several functions. In this chapter which is in some sense an extension of Hsiao’s schemes, Haar Wavelet Series Solution to nonlinear and Isoperimetric problems in the calculus of Variations is given. The technique has been generalized to a functionals depending on more than one function. Basic
principle involved is that of converting variational problems into a system of linear and nonlinear algebraic equations, using localization and orthogonolization properties of Haar wavelets and later taking the advantage of the operational matrix for integration. To demonstrate the validity and applicability of the method important problems of the calculus of variations with fixed boundaries are solved. Comparison of the results obtained with the exact solutions, confirm our view that, the method proposed is fast, easy and accurate. These are attributed to (i) localization of Haar wavelets (ii) orthogonality of Haar wavelets (iii) adaptability of the method to digital computers (iv) efficiency of the algorithm. Better accuracy can be achieved by taking more number of terms (larger m value). We have tested the applicability of the method, even to problems where continuous solution does not exist. The main, perhaps, the greatest advantages of the method is that, it gives estimation of the eigenvalues the least of which corresponds to the solution of the isoperimetric problems considered.

CHAPTER THREE presents An Application of Rationalized Haar Wavelet Series Solution to nonlinear Integro-Differential Equations. Nonlinear Integro-differential equations are one of the extensively studied problems which still generate interest because of their occurrence/applications in Engineering and Physical Sciences. Basic principle involved is that of converting nonlinear Integro-differential equation into nonlinear-Volterra-Hammerstein integral equation which is then solved using Rationalized Haar wavelets. The operational matrices of integration, the product of two rationalized Haar wavelet vector functions together
with Newton-Cote nodes are utilized to reduce integral equations into non-linear algebraic equations. These algebraic equations are solved using Brown’s method. The universal feature of the underlying matrices and their sparseness is fully exploited to reduce CPU time, storage requirements and to obtain fast and accurate solutions. The accuracy of the solutions is compared with those obtained from Wavelet-Galerkin method. These comparisons show that 3-4 digit accuracy is achieved by just taking $m = 8$ and 16. It is found from the literature that compared with polynomial approximation methods the RH wavelet series method is computationally convenient, consumes less CPU time and requires less memory space. More importantly, it retains accuracy to any desired level.

The aim of CHAPTER FOUR is to extend the previous Single Term Haar Wavelet Series (STHWS) approach to the solution of nonlinear Stiff-Differential equations arising in science and engineering applications. The properties of single term Haar wavelet series are given. The method of implementation is discussed. Some model equations are investigated for their stiffness and stability and numerical solutions are obtained to demonstrate the accuracy and effectiveness of the method. The results in the form of block-pulse and discrete solutions are given for non-linear stiff systems. Sensitivity of the solutions to initial conditions is demonstrated effectively in which the essential features of stiff systems have been captured. Hence regardless of the degree of stiffness, STHWS method is more efficient compared to conventional explicit methods.
CHAPTER FIVE contributes simplified mathematical model for understanding combined effects of poroelasticity and couple stresses on the performance of lubrication aspects of poroelastic bearings in general and that of synovial joints in particular. The modified form of Reynolds equation which incorporates the elastic nature of cartilage and Stokes couple-stress fluid as lubricant is derived and analyzed using a recently developed Daubechies wavelet series multigrid method. This method has greatest advantages of minimizing the errors using wavelet transforms in obtaining accurate solution as grid size tends to zero. It is found that, 6-7 cycles are required to obtain a reasonably accurate solution in the multigrid scheme, whereas, only one cycle is required to obtain the solution in the Daubechies wavelet series multigrid method. Also, matrix of DWT acts as a natural preconditioner producing rapid convergence. It is observed that, the poroelastic bearings with couple-stress fluid as lubricant provide enhancement in pressure and ensure the increased load carrying capacity compared with viscous fluids. This may be one of the reasons in the efficient lubrication and proper functioning of synovial joints.

In earlier chapter we have used the simple geometry to illustrate salient features of lubrication characteristics of poroelastic bearings by using Daubechies wavelet series multigrid method.

In CHAPTER SIX a more realistic bearing geometry (spherical form) is considered. Theoretical analysis of effects of poroelasticity on the squeeze film behavior of poroelastic bearings with couple-stress fluid as lubricant is given.
Proposed model is that a rigid sphere approaches poroelastic matrix and joint cavity is filled with couple stress fluid. The modified form of Reynolds equation with variable coefficients considering poroelasticity of the articular cartilage is derived. Again Daubechies wavelet series multigrid method is used for its solution. It is observed that the poroelastic bearings with couple-stress fluid as lubricant provide large pressure distribution and ensure significant load carrying capacity compared to classical case. Also, the role of elasticity is to enhance the load capacity of the poroelastic bearings. Predictions based on this simple model closely describe some salient features of lubrication aspects of synovial joints.

In previous chapters (fifth and sixth) we have observed the effects of couple stress and poroelasticity with suitable geometry and in CHAPTER SEVEN we attempt to analyze the effects of surface roughness and poroelasticity on the squeeze film behaviour of bearings in general and that of synovial joints in particular. More realistic constitutive equations describing poroelastic cartilage deformation and fluid flow in matrix are considered here. The modified form of Reynolds equation, which incorporates the randomized roughness structure as well as elastic nature of articular cartilage, is derived. Christensen stochastic theory describing roughness structure of cartilage surfaces is used by assuming the roughness asperity heights to be small compared to the film thickness. Daubechies wavelet series multigrid method is used for the solution of Reynolds equation. The influence of roughness and elasticity on bearing characteristics are discussed in some detail.