CHAPTER - I

INTRODUCTION

1.1 Nature and Importance of Mathematics

Mathematics is abstract science which investigates both inductively and deductively. It is a science of logical thinking and systematic reasoning. Since its inferences, results and conclusions are based on a definite process of logical thinking and reasoning, it is also called 'Science of necessary conclusions'.

In dictionary, the subject mathematics is defined as follows. 'It deals with number and space'. But according to mathematicians it is a body of knowledge acquired through one's inherent flair for recognition of patterns, relations, and structures. In other words, it requires the ability to establish connections. One's level of intelligence can even be judged by the sweep and speed seen in one's spelling out of connections not only among objects but more so among ideas generated by experiences one gets in life situations in general and handling of objects in particular. So it is said that mathematics is in the mind: but to realise it, external stimulation is essential. The degree and duration of provoca-
tion varies of course from person to person.
Mathematics like art and music, dance and sports requires exercise of one's intuition and imagination which are characteristics of the functioning of the right lobe of the brain.

Mathematics uses only deductive logic and there is no place for inductive logic. Bertrand Russel stated that pure mathematics consists of statements of the type: 'if P then Q, where P and Q are sets of statements and here we do not worry whether statements P are true or not'.

This view almost equates mathematics with deductive logic, but mathematics is certainly more than deductive knowledge, there we do worry about the truth of sets of statements P.

Every mathematician obtains his results inductively and deductively. Every student of mathematics should use inductive and deductive logics in a balanced manner.

There are certain views regarding the nature and scope of mathematics.

1. Mathematics is as much an experimental science as physics, chemistry or biology. The only difference is that the experimental material here consists of numbers and geometrical forms. Earlier the equipment used was only paper and pencil, but now we require calculators, possibly those with graphic capabilities. In fact the advent of calculators
and computers is going to bring out forcefully the experimental nature of mathematics.

The progress of mathematics is going to be considerably accelerated by exploring the nature of mathematics. The conception that mathematics is a pure theoretical science has done a great deal of harm to mathematical education. There is need for mathematics laboratory as in the case of science subjects. Mathematics has come closer in nature to other sciences now, than it has ever been before.

2. Mathematics is an abstract subject. It is true that mathematics deals with abstract concepts. It is also however true that mathematics is an abstracting science. The derivation of abstract concepts is as important as the abstract concepts themselves. Mathematics education suffers when only the abstract concepts are taught, but the process of derivation of abstract concepts is hidden from the students.

Good mathematics education has to place emphasis on the process of abstraction, since this will take away some of the mystery of abstract concepts. Students also have to understand the need for subtraction and the power of abstraction, otherwise an over emphasis on the abstract nature of mathematics can only serve to drive the student away from mathematics.
3. Mathematics modelling is a new discipline in mathematics. Mathematics modelling was the first scientific activity of mankind. Thales, in some sense the first scientist in modern times, used it to find the radius of earth. Archimedes used it to destroy enemy ships and astronomers in India, used it to model the motion of planets.

Mathematics modelling created the need for mathematics and mathematics makes mathematical modelling meaningful. It is unfortunate that mathematical modelling was not earlier given its rightful place in mathematics education. The truth is that mathematics and modelling have to be done simultaneously at every level of education.


This statement is against the whole history of mathematics. Pure and applied mathematics are two sides of the same coin of mathematics. To say that one side of the coin is the real coin and the other side is not, it is not correct.

Pure and applied mathematics supplement each other. Pure mathematics alone will be a fine art and applied mathematics by itself may become a technical art. Together they represent the great edifice of mathematics.

5. Mathematics is a sequential subject.

It means that in a mathematical argument all steps are in a sequence and one must follow the earlier step before
One goes to later steps. The myth of sequentialism has however hindered the real modernisation of mathematical courses. It has delayed the introduction of mathematical modelling and applications into the curriculum. In fact it was possible to introduce some of the concepts developed in eighties of the present century into the curriculum.

6. Mathematical education is a good preparation for logical thinking in life.

Since in mathematics, we argue logically, precisely, and quantitatively, it is presumed that we also imbibe these qualities and attitudes in life.

If we want habits of mathematical thinking to percolate into life, then these have to be discussed explicitly and examples of deductive reasoning have to be drawn from everyday life situation. Mathematics will have to be brought much nearer to life than it is today.

There are several objectives which could be thought of or outcomes which could be expected to derive from teaching or learning mathematics. The following are some of the commonly accepted objectives, though one can add objectives, the inclusion of which depends upon factors such as age, and grade level, mathematics as optional subject or compulsory subject, mathematics as part of general curriculum versus mathematics as an area of specialization, mathematics as an
adjunct to learning other subjects like physical and biological sciences and so on.

The student should -

i. Have a knowledge and understanding of mathematical processes, facts, concepts and principles.

ii. Have skill in computing with understanding, accuracy and efficiency.

iii. Have the ability to use a general problem solving technique, understand the logical structure of mathematics and the nature of proof.

iv. Use mathematical concepts and processes to discover new generalizations and applications.

v. Recognize and appreciate the role of mathematics in society, develop study habits for independent progress in mathematics.

vi. Develop reading skill and vocabulary essential for progress in mathematics.

vii. Demonstrate mental traits such as creativity, imagination, curiosity and visualisation.

viii. Develop attitudes that lead to appreciation, confidence, initiative and independence.

Mathematics is a subject included in the curriculum and taught at all levels including pre-school, school and college levels in our country and elsewhere. Even the first
lesson the child learns from his/her mother in its early years of childhood and impressionistic period that includes mathematical problems. In fact in that stage the children are generally enthusiastic in playing with number games. The counting board with multicoloured beads (Abacus) perhaps adds further charm to learning of numbers and their relations in the young minds. The number puzzles which are given to the children further invoke their intellectual curiosity and they feel thrilled once they come out with solution. Learning of numbers and their relations during childhood years which form the rudimentary aspects of mathematics facilitates the intellectual development of children and experience of learning of numbers is generally palatable to them. Thus the study of mathematics as a means of providing intellectual stimulation, apart from its function of making the person efficient in coping with the demands of the quantitative aspects of his day today life can hardly be underestimated. In other words learning of mathematics forms a fundamental aspect of overall learning and growth process of the individual.

Many scholars have commented on the importance of mathematics. For instance, Roger Brown said "Mathematics is the gate and key of the Sciences... Neglect of mathematics works injury to all knowledge. Since he who is ignorant of it cannot know the other sciences or the things of the world".
Comte remarked "All scientific education which does not commence with mathematics is of necessity defective at its foundation" (1:7-10).

Napoleon put it succinctly "the progress and improvement of mathematics is linked to the prosperity of the state".

In ancient India when there were no external aids or electronic gadgets available for computation purposes such as we have today, pupils still had attached much importance in acquiring and developing mathematical skills, reasoning bent of mind as an integral part of learning and growth. Somewhere in the course of our long history, we seem to have lost the traditional taste for mathematics and other substantial aspects of our daily living. We can see even today that some of our people who are octogenarians can calculate difficult mathematical problems mentally without the need for the use of paper and pencil, the problems which the younger generations will find difficult to solve. Thus in short mathematics enjoyed a place of pride in traditional learning and wisdom. But as economy expands and becomes market-oriented with spurt in scientific technological developments as we witness today, the scope of mathematics becomes enormous. A couple of decades ago even students of advanced science could afford to ignore mathematics and its various subject fields and still engage in scholarly pursuits in their fields, but today this cannot happen. Unless they have a fairly good knowledge in mathematics, they will be
handicapped even to grapple with the current developments in their fields, which have come about as a result of applications of mathematics, let alone pursue work at higher levels. We find today when developments in science and technology have taken a quantum jump, there is invariably an interface between science and mathematics. The new discoveries in physical, chemical and biological sciences that have come up and those that are in offing are all the outcome of investigations guided by mathematical thinking and computer modelling and the applications. The invention and use of computers which makes the present age to be called as 'computer age' is itself a testimony to the unique place mathematics enjoys in realm of human knowledge.

Richard Courant has pointed out that "the scope of mathematical research and teaching have been greatly extended in the present period and mathematical knowledge to field outside the mathematical sciences, such as physics, into new realms of technology, into biological sciences and even into economics and other social sciences. Computing machines and computing techniques have stimulated areas of research enormously (6:41-49).

1.2 Present Position of Teaching Mathematics

Despite the importance of mathematics, not sufficient attention is paid to teaching and learning of mathematics. For instance in the words of Kuppuswamy Aiyengar "the amount
of mathematics that Madras matriculate is expected to know is not as much as what a boy in elementary school in England or a boy of junior high school in America actually learns. The situation is not different in the case of teaching of mathematics at the secondary school level in other parts of India. Pupils generally show distaste or aversion towards mathematics. Aiyengar put it this way "students consider mathematics as 'a bug bare'." Mathematics is considered as a difficult subject requiring special aptitude and higher level of intelligence which is not always true. Only average level of mental ability is required for learning mathematics which is a prerequisite for learning any subject. We can trace a number of reasons for the unfavourable attitudes towards mathematics on the part of pupils. Aiyengar has succinctly described some of the reasons as follows:

(1) **Difficulty**: The view that mathematics is difficult subject is illfounded because in mathematics the premises are obvious and the reasoning is simple. It is only in mathematics they can solve problems by applying the principles learned. In other subjects it is comparatively more difficult to arrive at definite and concrete conclusions. Unlike in other subjects, rote learning does not help in mathematics, on the other hand the students are to think for themselves. Only the dull pupil who are deficient in thinking and pupils who are left out in the present day mode of instructions find mathematics difficult.
(2) **Sequencing nature of the subject:** Mathematics consists of a set of principles and patterns in a number system each of which is organised just as the arrangement of bricks one over the other in building a wall. The implications of this hierarchical organisation is that a student can grasp mathematical facts at a particular level and find learning interesting and meaningful only if he has adequate understanding of mathematical knowledge at the lower levels. In other words a good foundation or background or footing in mathematics is required for learning higher mathematics. Weakness in the knowledge of mathematics at one level of learnings will carry over to other levels with the result, the students lose interest in mathematics unless those weaknesses are remedied. Lapses at earlier stages cannot be fully corrected at later stages. So what is required is that right learning should take place at all levels which is possible when the student learns mathematics through understanding rather than by solving the problems mechanically, memorising mathematical symbols and formulae. Ineffective teaching or wrong approaches to teaching mathematics can score away even the so called 'brilliant' students or those who have required abilities, aptitude and initial enthusiasm. The real danger lies in making the pupils to solve the problems mechanically by using set norms and formulae rather than encouraging them to think through the problems and come out with the solutions. There is a stereotypic view that
the mathematics is a tough subject and require superior intelligence, which in fact is a 'myth'. This notion get circulated on from people to people making their beliefs stronger, this in turn demotivates them in pursuing studies in mathematics.

1.3 Aims of Teaching Mathematics

Universally recognised aims or large purposes of teaching mathematics in secondary schools are as follows:

i. To give the individual an understanding of ideas and operations in number and in quantity needed in daily life by the citizens of country as individuals,

ii. To develop in the individual an awareness of the mathematical principles and operations which will enable the individual to understand and participate in the general, social and economic life of his community.

iii. To provide through mathematical ideas, aesthetic and intellectual enjoyment and satisfaction and to give an opportunity for creative expression,

iv. To provide the basis of mathematical skills and processes which will be needed for vocational purpose, and

v. To help the child to develop skills and attitudes to meet the demands of (a) daily life, and (b) future mathematical work in the related field of knowledge.
Thus mathematics has disciplinary, utilitarian and cultural values.

1.4 Some Other Advantages of Mathematical Study

(a) Development of the Power of Concentration

Very few young people seem to be able to concentrate their minds for even a few minutes upon an idea. This is a faculty which can be acquired and mathematical study is admirably adapted to develop it.

(b) Development of the Constructive Imagination or the Intuitive Faculty

Far from being a dry science requiring pedantic accuracy and little imagination, true mathematical work consists in inventing, in finding something that is unknown to the worker and in this success is impossible without the use of creative power of mind.

To the student, the solving of a difficult problem is a discovery and constant training in such work develops those faculties that lead to discovery and invention.

(c) Growth of Mental Self-Reliance

Young students, as a rule, rely too much on facts taken from books or some other authority, and too little
upon their own faculties, a trait which shows that they have no confidence, in their own mental powers. Their former training has led them to believe utterly in authority, and especially to think that all knowledge depends upon authority. Mathematics however trains students to develop self-confidence.

(d) Development of Character

Mathematics study trains the students in systematic and orderly habits and the pleasure connected with the successful conquering of a difficulty stimulates the will power. It has also been claimed that dealing with a subject, that is absolutely true, that rejects and shows up any error is bound to increase respect for truthfulness and honesty.

(e) Power of Expression

The habit of expressing oneself clearly and accurately is a great asset in practical life. In mathematics, one has always to be very careful while using appropriate words and terms. The habit of clearness, brevity, accuracy, precision and certainty in expression are formed and strengthened by the study of this subject. Its concepts and symbolism provide a means of concise expression which is elegant in its simplicity and exactness.
(f) **Increase in General Culture**

An acquaintance with the fundamental facts and methods of mathematics seems to be necessary for general culture. A science that is closely interwoven with most mental achievements of the race, that is found in all civilizations, that represents the most finished types of exact thinking cannot be ignored by the man of culture.

A person unfamiliar with elements of mathematics cannot fully comprehend the simplest facts of astronomy, he is not able to read to grasp the accounts of the wonderful discoveries and inventions of our time.

In conclusion, mathematics is primarily taught on account of the mental training it affords, and only secondarily on account of the knowledge of facts it impacts. The true end of mathematical teaching is power and knowledge.

### 1.5 Objectives of Teaching Secondary School Mathematics

The following objectives are to be achieved by the teaching of mathematics at the secondary school stage, as presented in the guidelines and syllabi for secondary stage, 1988 by the N.C.E.R.T., New Delhi.

The teaching learning of mathematics should enable the child to:
i. consolidate the mathematical knowledge and skills acquired at the upper primary stage,

ii. acquire knowledge and understanding of the terms, symbols, concepts, principles, processes, proof, etc.,

iii. develop mastery of basic algebraic skills,

iv. develop drawing skills,

v. apply mathematical knowledge and skills to solve real mathematical problems by developing abilities to analyse, to see inter-relationship involved, to think and reason,

vi. develop the ability to articulate logically,

vii. develop skill in the use of mathematics for problem solving,

viii. develop ability to write and interpret algorithms for problem solving,

ix. develop necessary skills to work with modern technological devices such as calculators, computers, etc., where available and develop understanding of the cause effect relationships and the interplay of variables,

x. develop interest in mathematics and participate in mathematical competitions and other mathematics club activities in the school.

xi. develop appreciation for mathematics as problem solving tool in various fields for its beautiful structures and patterns, etc., and,
xii. develop reverence and respect towards great mathematicians particularly towards the Indian mathematicians for their contributions to the field of mathematics, astronomy, etc.

1.6 Specifications of Instructional Objectives of Mathematics

The broad objectives presented above have been translated into specific instructional objectives in behavioural terms as follows:

(1) **Objective - 1** : The pupil acquires knowledge of terms, symbols, formulae, definitions, concepts, facts, processes and principles in mathematics.

**Specifications**

The pupil -

- recalls terms, symbols, formulae, definitions, concepts, processes and principles,
- recognises symbols, formulae, concepts, etc., and
- carried out proofs successfully.

(2) **Objective - 2** : The pupil develops understanding of terms, symbols, formulae, definitions, concepts, processes, principles and facts in mathematics.
Specifications

The pupil -

- explains mathematical concepts in his own words,

- illustrates mathematical terms, formulae, definitions, etc.,

- translates verbal statement into symbols and vice-versa,

- detects errors in mathematical definitions, processes, formulae, etc.,

- corrects errors in mathematical definitions, processes, formulae, etc.,

- identifies relationship among the given data,

- compares and contrasts various concepts,

- discriminates between closely related concepts,

- classifies the data,

- estimates the results, and

- verifies mathematical results.

(3) Objective - 3: The pupil applies his knowledge and understanding of mathematics to unfamiliar situations.
Specifications

The pupil -

- analyses the data into parts,
- judges the sufficiency or insufficiency, consistency or inconsistency and the relevancy of data,
- establishes relationship among the data,
- suggests different methods for solving a problem,
- selects the most appropriate method or line of attack,
- draws conclusions or inferences (or reasons deductively), and
- generalises (reasons inductively).

(4) **Objective - 4**: The pupil acquires skill in -

- computation,
- handling mathematical instruments, and
- reading charts, graphs, and tables.

(i) **Computation Skills**

Specifications

The pupil -

- does oral calculations correctly and quickly, and
- does written calculations correctly, quickly and legibly.

(ii) Skill in handling mathematical instruments -

Specifications

The pupil -

- handles mathematical instruments with ease, speed and accuracy, and
- works with computers.

(iii) Drawing Skills

Specifications

The pupil -

- draws fairly, accurately, freehand figures,
- draws figures to given specifications,
- draws figures neatly and speedily, and
- labels the figures or graphs appropriately.

(iv) Skill in reading graphs, charts or tables

Specifications

The pupil -

- reads and interprets graphs, charts or tables correctly for problem solving.
(5) **Objective - 5:** The pupil develops interest in mathematics.

**Specifications**

The pupil -
- engages himself in mathematical activities during his leisure time,
- collects graphs, charts, puzzles, etc., and interprets,
- frames puzzles or new problems in mathematics,
- suggests new methods of solving a problem,
- makes mathematical designs or models,
- approaches the teacher with problems outside the class,
- contributes articles on mathematics in the school magazines, and
- becomes a member of the mathematics club.

(6) **Objective - 6:** The pupil acquires positive attitude towards mathematics.

**Specifications**

The pupil -
- develops reverence and respect towards great mathematicians,
- likes teachers of mathematics and mathematical literature,
- feels at home in the company of students of mathematics,
- helps students who are weak in mathematics, and
- likes to take tests in mathematics.

(7) **Objective - 7**: The pupil develops scientific attitude through the study of mathematics.

**Specifications**

The pupil -

- does not jump to conclusions,
- does not accept an argument unless logically convinced,
- owns his mistakes,
- points out errors of others boldly and tries to correct them, and
- is not disappointed by his failures.
1.7 Need of the Study

It is a matter of common experience of many teachers of mathematics that the actual achievement of most of the students in the mathematics subject is less when compared with their abilities. Hence, the present study is undertaken with a view to identify the cases of underachievement in mathematics subject at the secondary school stage and suggest remedial measures.
REFERENCES


