Preface

Fixed point theory concerns itself with a very simple, and basic, mathematical setting. For a function $f$ that has a set $X$ as both domain and range, a fixed point of $f$ is a point $x$ of $X$ for which $f(x) = x$. Banach's and of Brouwer's theorems are the two fundamental theorems concerning fixed points. In Banach's theorem, $X$ is a complete metric space with metric $d$ and $f : X \to X$ is required to be a contraction, that is, there must exist $\alpha < 1$ such that $d(f(x), f(y)) = \alpha d(x, y)$ for all $x, y \in X$. The conclusion is that $f$ has a fixed point, in fact exactly one of them. Brouwer's theorem requires $X$ to be the closed unit ball in a Euclidean space and $f : X \to X$ to be a continuous function. Again we can conclude that $f$ has a fixed point. But in this case the set of fixed points need not be a single point, in fact every closed nonempty subset of the unit ball is the fixed point set for some map. The theorems of Banach and Brouwer illustrate the difference between the two principal branches of fixed point theory: metric fixed point theory and topological fixed point theory.

The fundamental idea of applying fixed point results to produce theorems in analysis is due to famous French mathematician H. Poincare(1854-1912) and this idea was developed further in the work of Brikhoff(1913), Brikhoff Kellogue(1922) and then Schauder(1927), (1930). Systematic applications of the Banach's principal to various existence theorem in analysis were initiated by Cacciopoli(1930). An expository account of many such applications may be found in Niemytzki(1936) and Miranda(1949).

In this thesis, fixed point theorems and related results in metric spaces, probabilistic metric spaces or Menger spaces and fuzzy metric spaces and some more theorems satisfying implicit relations have been discussed and many concepts are generalized. We have also worked on the concept of absorbing maps, introduced by Ranadive et al [42].
The chapter wise details are given below:

In Chapter I, we include some basic definitions and fundamental results of the theory. The main aim of this chapter is to present the required material in a sequence. Proof of the famous Banach contraction principle is given in the starting section of this chapter. In the next section of this chapter we have given statements of most important theorems related to our work, specially on metric spaces, probabilistic metric spaces, Menger spaces and fuzzy metric spaces.

In Chapter II, some interesting results on classical metric spaces are obtained. In the first section of this chapter we generalize the result of Imdad and Khan [52], for rational inequality. This paper is published in Int. Review of Pure & Appl. Math., Vol.3 (1), (2007) pp 137-141. In the next section we have generalized a theorem on Mier-Keeler contractive mappings in ordinary metric spaces, given by Jha [58]. Examples are given in the support of results. This paper was presented in the national conference, at SSCET, Bhilai (C. G.) and is communicated for publication. In the last section of this chapter we prove an interesting theorem which is based on near-hybrid contraction. In this result we have used the concept of absorbing maps given by Ranadive et al [42]. This result is the generalization of results of Mbarki [84].

Chapter III, deals with Menger spaces, introduced by Menger [86] in 1951. In the first section, we give a common fixed point theorem by replacing continuity condition with a weaker condition called reciprocal continuity. Using the notion of reciprocal continuity of mappings we can widen the scope of many interesting fixed point theorem on Menger spaces as well as fuzzy metric spaces. The work of this section has been presented in The International Conference of CONIAPS- X (2008), at G. G. University, Bilaspur, C. G.. This paper is communicated for the publication.
In the next section of this chapter we obtain a common fixed point theorem assuming minimal commutativity condition and a suitable condition on the range of mappings. Our theorem extends the scope of study of common fixed point theorems from the class of compatible continuous mappings to a wider class of mappings, which includes non-compatible and discontinuous mappings. This result has been published in Int. J. Math. Sci. & Appl., 1(1) 2007, 91-99.

Chapter IV, is based on fuzzy metric spaces. In the first section of this chapter we prove a common fixed point theorem in fuzzy metric spaces by using the notion of absorbing maps, which is neither a subclass of compatible maps nor a subclass of non-compatible maps. We establish that it is not necessary that absorbing maps commute at their coincident points however if the mapping pair satisfy the contractive type condition then point-wise absorbing maps not only commute at their coincident point but this becomes a necessary condition for obtaining a common fixed point of mapping pair. The results of this section have been published in The Journal of Fuzzy Mathematics, U.S.A., vol 16, No 2(2008) pp 613-620.

In the next section we have proved some theorems with different contractive conditions while other conditions remain same as in our above result and give an example in the support of our theorem. This paper has been published in Thai Journal of Mathematics, vol 6(1) (2008) pp 49-60.

In the third section of this chapter, we have proved a common fixed point theorem for reciprocally continuous semi-compatible maps and give a proposition to show that in the context of reciprocal continuity the notion of compatibility and semi-compatibility of maps becomes equivalent. This paper has been published in Tamkang journal of Mathematics, vol. 39 (4) (2008) pp 309-316.
In the next section, we remark that the Theorem 2.1 given by Balasubramaniam et al [6] appears to be wrong. This is so as the contractive condition involved in this theorem is self contradictory at fixed point. However, we can get rid of this problem by altering the contractive condition of the proposed Theorem 2.1 of Balasubramaniam et al [6]. This paper is accepted for the publication in The journal of Fuzzy Mathematics, vol. 17, No. 3 (2009), Los Angeles U.S.A..

In Chapter V, we obtain some fixed point theorems satisfying different implicit relations and replacing the commutativity condition (namely weakly compatible maps) by absorbing maps. Moreover we give some examples for different implicit functions and give an example in support of our result. This paper is communicated for the publication.

Throughout in the thesis by R we mean a set of real numbers, by $R$ we mean any real number, $N$ stands for the set of natural numbers.

The thesis ends with a detailed Bibliography.