The concept of a fuzzy subset was introduced and studied by L. A. Zadeh in the year 1965. The concept of a fuzzy subset is a generalization of that of a subset. Fuzzy subsets are the classes of objects with grades of membership ranging between zero and one.

The introduction of this concept opened the floodgates of research work in many areas of Mathematical and Computer Sciences. The subsequent research activities in the area of fuzzy subsets and its related areas have found applications in many branches of science and engineering including areas such as Artificial Intelligence, Neural Networks, Medical diagnosis, Robotics etc. Some of the new areas emerging from this area are Fuzzy logic, Softcomputing, Genetic Algorithms, Neurocomputing etc, which are playing a key role in designing and constructing intelligent machines.

Fuzzy Topology is one such new area of Mathematics that has emerged consequent upon the introduction of fuzzy subsets. In the year 1968, C. L. Chang applied fuzzy subsets to general topology and thus introduced the structure of fuzzy topology. Subsequently many researchers like J. A. Goguen, C. K. Wong, R. H. Warren, S. E. Rodabaugh, R. Lowen, B. Hutton, K. K. Azad, A. K. Srivastava, D. M. Ali, M. N. Mukherjee and many others have
contributed to the development of fuzzy topological spaces. The theory of
general topological spaces is a special case of the theory of fuzzy
topological spaces (fts).

Functions play a very important role in Mathematical Sciences. Functions can be regarded as special types of relations. The fuzzification of functions has been attempted by several investigators including M. Sasaki, K. A. Dib, N. L. Youssef and M. A. Ghamdi. The concept of a fuzzy function, introduced in 1991 by K. A. Dib and N. L. Youssef, is a generalization of Zadeh’s functions.

The present thesis deals mainly with fuzzy functions in fuzzy topology.

This thesis is organized into six chapters. The first chapter provides a brief introduction to fuzzy subsets and fuzzy topology. The concept of a fuzzy subset, various operations on fuzzy subsets such as union, intersection and complementation of fuzzy subsets are discussed. A list of related properties is included. The image and the inverse image of fuzzy subsets under Zadeh’s functions and their properties proved by C. L. Chang and R. H. Warren are included. Further, fuzzy topological spaces are discussed. Some basic concepts and results on fuzzy topological spaces from the works of C. L. Chang, R. H. Warren,
J. A. Goguen, S. R. Malghan and S. S. Benchalli and C. K. Wong are presented, which are required in the subsequent chapters.

The second chapter of this thesis deals with fuzzy functions, continuous fuzzy functions, open fuzzy functions, closed fuzzy functions etc. The concept of fuzzy functions due to K. A. Dib and N. L. Youssef and their elementary properties are discussed. Recently, M. A. Ghamdi introduced continuity of fuzzy functions. In this chapter several characterizations of such fuzzy functions have been obtained. It is proved that connected fts are invariant under such fuzzy functions. Further in this chapter, open, closed and homeomorphic fuzzy functions have been introduced and several characterizations have been obtained. The invariance of regular and normal fts under fuzzy functions is also discussed.

In the year 1970, N. Levine introduced and studied the concept of generalized closed (g-closed) sets in general topology. Consequently, several results in general topology were improved. In this context, S. R. Malghan introduced generalized closed (g-closed) maps in topology. Later, g-open maps and g-continuous maps were also investigated in general topology.

The concepts of g-closed fuzzy sets and g-open fuzzy sets in fts were introduced and studied in 1993 by P. Sundaram. Further
G. Balasubramanian and P. Sundaram investigated g-continuous functions in fts, in 1997.

The chapter three deals with g-continuous fuzzy functions, g-open fuzzy functions and g-closed fuzzy functions. It is observed that every continuous fuzzy function is g-continuous fuzzy function but not conversely. Characterizations of such fuzzy functions have been obtained. The composition of g-continuous fuzzy functions, g-closed fuzzy functions and g-irresolute fuzzy functions have been discussed. The earlier results on invariance of regular and normal fts have been improved. The last section of this chapter deals with g-compactness, countable g-compactness and g-Lindelöf property in fts. It is observed that every g-compact fts is countably g-compact fts. Hereditary property for countable g-compactness of fts is obtained. It is also proved that countable g-compactness of fts is invariant under g-irresolute surjective fuzzy functions. This chapter contains several counterexamples.

Weakly generalized closed (wg-closed) and weakly generalized continuous (wg-continuous) functions in general topology were investigated by P. Sundaram and N. Nagaveni and then were extended to fts by them.

The chapter four deals with wg-continuous fuzzy functions, wg-closed fuzzy functions, wg-open fuzzy functions, wg-irresolute fuzzy
functions and wg-compact and its related fts. Such fuzzy functions have been characterized. It is observed that every continuous fuzzy function is wg-continuous fuzzy function but not conversely and that every g-continuous fuzzy function is wg-continuous fuzzy function but not conversely. Similarly it is observed that every closed (open) fuzzy function is wg-closed (wg-open) fuzzy function but not conversely and that every g-closed (g-open) fuzzy function is wg-closed (wg-open) fuzzy function but not conversely. Further it is proved that regularity of fts is invariant under continuous, open, wg-closed, surjective fuzzy functions. The last section of this chapter deals with wg-compact fts and related concepts in fts. It is proved that every wg-closed crisp subspace of a wg-compact fts is wg-compact, every wg-compact fts is a compact fts but not conversely and that wg-compact fts is g-compact fts but not conversely. Further the invariance of wg-compactness, countably wg-compactness and wg-Lindelöf fts under fuzzy functions has been discussed. Several counter examples have been constructed and illustrated throughout this chapter.

The chapter five deals with semicontinuous, semiclosed, semiopen, irresolute, presemiclosed and presemiopen fuzzy functions.
K. K. Azad introduced and studied semiopen fuzzy sets and semi continuous functions in fts in the year 1981. This was further investigated by M. N. Mukherjee, S. P. Sinha, B. Ghosh and T. H. Yalvac.

In this chapter semicontinuous fuzzy functions, semiopen fuzzy functions, semiclosed fuzzy functions and irresolute fuzzy functions have been introduced and studied. Several characterizations of such fuzzy functions have been obtained. It is observed that every continuous (resp. open, closed) fuzzy function is a semicontinuous (resp. semiopen, semiclosed) fuzzy function but not conversely. Further the composition of such fuzzy functions is also discussed.

Presemiclosed functions in topology were studied by G. L. Garg and D. Sivaraj. Presemiopen functions in fts were studied by T. H. Yalvac. In this chapter presemiopen and presemiclosed fuzzy functions have been introduced and studied. It is observed that a presemiclosed fuzzy function need not be a closed fuzzy function. Three characterizations of presemiclosed fuzzy functions have been obtained. Also composition of such fuzzy functions has been discussed.

In general topology, s-regular and s-normal topological spaces were introduced and studied by S. N. Maheshwari and R. Prasad. In the last section of this chapter, s-regular and s-normal fts have been introduced and studied. It is observed that every regular fts is s-regular fts.
but not conversely. Further s-regular and s-normal fts have been characterized. It is also proved that s-regularity is invariant under continuous, presemiopen and presemiclosed surjective fuzzy functions and that s-normality is invariant under continuous, presemiclosed, surjective fuzzy functions. Several counterexamples have been provided throughout this chapter.

The sixth and the last chapter deals with almost continuous fuzzy functions, almost open fuzzy functions, almost closed fuzzy functions and almost semicontinuous fuzzy functions.

Almost continuous functions in fts were studied by K. K. Azad and this was continued by M. N. Mukherjee, S. P. Sinha, S. Nanda and many others.

In this chapter it is observed that every continuous fuzzy function is almost continuous fuzzy function but not conversely and that almost continuous fuzzy functions and semicontinuous fuzzy functions are independent of each other. Further two characterizations of almost continuous fuzzy functions have been obtained. Further it is observed that every open fuzzy function is an almost open fuzzy function but not conversely and that almost open fuzzy functions and semiopen fuzzy functions are independent of each other. A characterization of such fuzzy functions is obtained. Further it is also proved that regular fts are
invariant under continuous, open, almost closed, surjective fuzzy functions.

In general topology almost semicontinuous functions were introduced and studied by S. P. Arya and M. P. Bhamini. In this chapter almost semicontinuous fuzzy functions have been introduced and studied. It is observed that every almost continuous fuzzy function is almost semicontinuous fuzzy function but not conversely. Two characterizations of such fuzzy functions have been obtained. The composition of such fuzzy functions is also discussed. In section 5 of this chapter, almost regular fts have been introduced and studied. It is observed that every regular fts is almost regular fts but not conversely and that almost regularity of fts and s-regularity of fts are independent of each other. A characterization of almost regular fts is obtained. Further it is proved that almost regularity of fts is invariant under almost continuous, almost open and closed fuzzy functions. This chapter also contains several counter examples.

The research work carried out and presented in this thesis is motivated mainly by the work done by K. A. Dib, N. L. Youssef and M. A. Ghamdi on Fuzzy functions and continuity of fuzzy functions.