Determination of the anomalous scattering factors of high-Z atoms using bremsstrahlung radiation

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Abstract
The anomalous scattering factors ($f'$ and $f''$) of tungsten, gold and lead atoms have been determined using external bremsstrahlung (EB) photons. The EB photons are produced by the interaction of a beta particle from a beta source with a nickel target. These photons are allowed to pass through thin targets of tungsten, gold and lead. The transmitted photons have been measured by using a GMX-type HPGe detector coupled to an 8K multichannel analyser. The transmitted spectra show a sharp decrease in intensity at the K shell binding energies of the target atoms. The regions around the decreased portion have been used to determine the anomalous scattering factors. The experimentally measured values are compared with the available theoretical values.

1. Introduction
The study of elastic scattering of x-rays and gamma rays from atoms, molecules and solids has been of experimental as well as theoretical interest for several years as it provides information about the inner shell structure of atoms and macromolecules [1–3]. Several processes such as Rayleigh scattering, Thomson scattering, nuclear resonance scattering and Delbruck scattering may contribute to the elastic scattering. Rayleigh scattering is an important process which occurs when an x-ray or gamma ray photon is elastically scattered by the bound electron. This process is predominant in the low energy region (<100 keV) and for high-Z atoms. Thomson scattering refers to elastic scattering from a free electron or nucleus. Delbruck scattering is elastic scattering of photons by the Coulomb field of a nucleus. The cross sections of these processes are very small in the low energy region. Kissel et al [4] have calculated the Rayleigh scattering amplitude using a second-order $S$ matrix. But this numerical method requires considerable computational time especially for heavy atoms and for high photon energies. However, using the form factor approximation, the Rayleigh scattering cross section can be estimated. But these form factors are derived with the assumption that the photon energy is large compared with the binding energy of the electron in the target. Therefore, when the photon energy is comparable to or less than the binding energy of the inner shell electrons, the form factor begins to fail. The deviation of the form factor from the exact forward scattering amplitude is known as the anomalous scattering factor or anomalous dispersion correction.

Therefore, when the energy of the incident photon approaches the absorption edge of an atom, the form factor is given by

$$f = f_0 + f' + if''$$

(1)

where $f_0$ is the Thomson scattering factor, $f'$ and $f''$ are respectively the real and imaginary parts of the anomalous scattering factors. The real part ($f'$) corresponds to the dispersion and the imaginary part ($f''$) corresponds to the absorption.

The imaginary part ($f''$) of the form factor at the photon energy $E_S$ is related to the photoelectric absorption cross section ($\sigma_{PE}$) through the relation [4, 5]

$$f'' = \frac{E_S}{2\hbar c r_0} \sigma_{PE}$$

(2)

where $\hbar$ is Planck's constant, $c$ is the velocity of light, $r_0$ is the classical radius of the electron and $E_S$ is the photon energy of interest. Thus by determining the photoelectric cross section at various photon energies, the imaginary form factor $f''$ of the
atom can be determined. The real and imaginary parts of the anomalous scattering factor are connected by the dispersion relation

\[ f'(E_2) = f'(\infty) - \frac{2}{\pi} P \int_0^{\infty} \frac{E' f''(E') dE'}{(E_2^2 - E^2)} \]  

(3)

where \( f'(\infty) \) is the high energy limit, \( P \) is the Cauchy principal value of the dispersion integral and \( E_2 \) is the energy of interest. Thus by determining the imaginary form factor \( f'' \) at various photon energies, the real form factor \( f' \) of the atom can be determined. It is to be noted from equation (2) that in the vicinity of the absorption edge the imaginary part \( f'' \) of the dispersion correction is strongly energy dependent; so is \( f' \), as is evident from equation (3).

Various researchers have calculated \( f' \) and \( f'' \) with great precision at various photon energies and extensive tabulations have been published [6–14]. Cromer [6] has developed a Fortran program and a photoelectric cross-section data file for calculating the anomalous scattering factors at arbitrary x-ray wavelengths. Cromer and Libermann [7] have made extensive tabulations of the dispersion correction factors using relativistic Dirac–Slater wavefunctions. Bremer [8] has calculated the energy dependence of the anomalous atomic scattering factors at the K absorption edge for free atoms and molecules. Kissel and Pratt [9] have calculated the energy-dependent and energy-independent correction factor for the real anomalous scattering factor \( f' \) and the differential elastic scattering factors at the K absorption edge for neon ions, Bergstrom et al [11] have investigated the angle dependence of the elastic photon–atom scattering amplitude using full relativistic \( S \) matrix calculations. Cullen et al [12] have provided photon data library for all elements with atomic numbers ranging from \( Z = 1 \) to 100 for the energy range from 1 eV to 100 GeV, which includes the entire photoelectric cross sections, and \( f' \) and \( f'' \) values. Kefi et al [13] have developed an analytical method to determine the anomalous scattering factors and tested their calculation for Pd, Ag, Cd, In, Sn, I and Xe elements around the K edge of low-Z elements and to the L edge of medium-Z elements. Kefi et al [13] have determined the anomalous x-ray scattering factors of Ni, Cu, Zn and Zr close to K absorption edges and of Ta, W, Pt and Au close to L edges by applying the dispersion relation to the absorption spectra. Using the \( S \) matrix approach, Rao et al [18] measured the Rayleigh scattering cross sections and the anomalous dispersion for Pd, Ag, Cd, In, Sn, Sb, Pt, Au and Pb at an angle of 90° in the x-ray region 3.41–8.04 keV. Chantler et al [19, 20] and Tran et al [21] have determined the imaginary form factor \( f'' \) for Cu and Si respectively in the energy range up to 20 keV using synchrotron radiation. Also, de Jonge et al [22] have measured the atomic form factor of tin using synchrotron radiation. By adopting the gamma ray attenuation method, Appajigowda and Umesh [23] have determined the anomalous scattering factors for Ta and Pb near the L edge in the energy range 6.4–24.14 keV. In the case of synchrotron radiation, a monochromatic beam has to be selected using a crystal monochromator. It is to be noted that in the case of the currently available synchrotron sources, the intensity of the synchrotron radiation decreases rapidly above 50 keV and hence the measurements are limited to the K edge of low- and medium-Z elements and to the L edge of high-Z elements. However, third-generation synchrotron radiations are capable of producing large fluxes of photons even above 100 keV and hence the measurements can be extended to high-Z elements. On the other hand, the gamma ray attenuation method can be used for determining the form factors of high-Z elements, but it requires a large number of monoenergetic gamma sources whose energies should be around the K shell binding energy of the target atoms.

In view of this, we have developed in the present work a simple and novel method for measuring the real and imaginary form factors for high-Z elements which requires a single beta source and a single elemental target. We report our results for tungsten, gold and lead atoms. We have already shown in our earlier papers [24, 25] that imaginary form factors and oscillator strength of high-Z elements such as Gd, Hf and Ta atoms can be determined accurately using the bremsstrahlung radiation produced by a weak beta source. By extending the same method, we have determined the real form factors along with the imaginary form factors of W, Au and Pb. We have compared our measured values with the theoretical values predicted by Chantler [14], relativistic Hartee–Fock–Slater anomalous scattering factors [26] and WinXCOM [27]. We find that there is good agreement between our experimental values and the theoretical values.

2. Experimental details

The experimental arrangement used in the present investigations is as shown in figure 1. A weak \(^{90}\text{Sr–}^{90}\text{Y}\) beta source (S) of strength about 10 \( \mu \)Ci is used to produce external bremsstrahlung (EB) photons in the nickel converter foil (C) of thickness 41.5 mg cm\(^{-2}\). The \(^{90}\text{Sr–}^{90}\text{Y}\) beta source being long lived (~28 years), its strength is constant throughout the experiment. So the intensity of the bremsstrahlung spectrum produced by these beta particles in the nickel converter will also remain constant throughout the experiment. As the thickness of the nickel converter is less than the range of the beta particles, some energetic beta particles may get transmitted through the converter. These transmitted beta
particles may produce unwanted EB photons in the detector. In order to prevent these beta particles from reaching the detector, a Perspex absorber of 10 mm thickness is placed in between the converter foil and the target (T). By placing the target in between the perspex absorber and the detector, the EB photons transmitted through the target are recorded with an HPGe detector (ORTEC) of the type GMX 10P. It has a window of 49.6 mm in diameter and 47.1 mm in length with a 0.5 mm thick beryllium window. The detector has an energy resolution of 700 eV at 88 keV. The efficiency of the detector is 100% in our energy range of interest.

The output of the detector was coupled to an 8K multichannel analyser which had MAESTRO software for data acquisition and analysis. The detector was calibrated in the energy range from 10 to 140 keV by using monoenergetic gamma sources of $^{241}$Am (59.536 keV), $^{109}$Cd (88.034 keV) and $^{57}$Co (14.411, 122.061 and 136.473 keV). The least-squares fit calibration constant was found to be 19.98 eV per channel. The peak position of the 88.034 keV gamma photon was checked every time before and after the acquisition of data. It was found that the peak position was constant, indicating that the detector spectrometer was stable throughout the experiment. Pure elemental targets of tungsten (W), gold (Au) and lead (Pb) used in the present work were obtained from Advent Research Material, UK. These foils were rolled to make a thin and uniform foil of required thickness. We used a tungsten target of thickness 100.66 mg cm$^{-2}$, gold of 46.76 mg cm$^{-2}$ and lead of 110.10 mg cm$^{-2}$ in our experiment.

The uniformity of these targets was checked by measuring the transmission of 59.54 keV gamma radiations at different regions of the target using a good geometry setup. First we recorded the background spectrum for 20 h by placing all the radioactive sources far away from the detector. Such a background spectrum is shown in figure 2(a). The incident spectrum of EB photons produced in the nickel converter (C) is recorded by placing a Perspex absorber in position P; a typical incident spectrum recorded for 20 h is shown in figure 2(b). Then we recorded the transmitted spectrum of EB photons by placing the tungsten target in between the Perspex absorber and the detector; the transmitted spectrum recorded for 20 h is shown in figure 2(c). From the transmitted spectrum we note that the intensity of the EB photons suddenly decreases at the K shell binding energy of the target atom. This decrease in the intensity is attributed to the onset of the K shell photoelectric cross section. Below the sudden drop region, we observe $K_{\beta 1}$, $K_{\beta 2}$, $K_{\beta 1}$ and $K_{\beta 2}$ x-ray peaks; these x-ray peaks are due to the K shell vacancies created by the EB photons in the target atoms. Due to the low energy resolution of the detector, $K_{\beta 2}$ and the sudden drop at the K edge are not well separated. From figure 2(c), we note that there are three important regions in the present investigations, namely a lower energy region (over which K x-ray peaks occur), a sudden drop region (around the K edge) and an upper energy region (beyond the K edge). The background corrected incident and transmitted spectra over these regions are used for determining the anomalous scattering factors of W, Au and Pb.

3. Results and discussion

3.1. Determination of the imaginary part ($f''$) of the anomalous scattering factor

The experimental K shell photoelectric cross section $\sigma_{PE}$ at the photon energy ($E$) is determined using the relation

$$\sigma_{PE} = \left(\frac{\ln(I_o/I_i)A \times 10^{24}}{tN_a}\right)$$

where $A$ is the mass number of the target atom, $t$ is the thickness of the target foil in g cm$^{-2}$, $N_a$ is the Avogadro number, $I_o$ is the intensity of the incident photon at the photon energy $E$ and $I_i$ is the transmitted intensity at $E$. The total photoelectric cross section, $\sigma_{PE}$ (barn/atom), can be calculated by knowing the intensity of the incident photons (in the absence of the target foil) and that of the transmitted photons (in the presence of the target foil) at various photon energies. The background spectrum is subtracted from the incident and the transmitted spectra and such corrected spectra are used for the calculation of $\sigma_{PE}$.

From background subtracted incident spectra, we have noted the incident intensity $I_o$ at various photon energies below and above the K edge of the target atom. In the
case of background subtracted transmitted spectra, we have deleted the x-ray peaks lying below the K edge by fitting a Gaussian distribution function and then we have noted the transmitted intensity at various photon energies. By knowing \( I_0 \), the incident intensity, and \( I_t \), the transmitted intensity, the photoelectric cross section \( (\sigma_{\text{PE}}) \) is calculated at various photon energies using equation (4). The plot of the \( \sigma_{\text{PE}} \) versus photon energy around the K edge for the W target is shown in figure 3. From the figure, we note that there is a sudden increase in \( \sigma_{\text{PE}} \) at the K edge of the target atom which is attributed to the onset of the K shell photoelectric cross section. Below the K edge, the cross section is mainly due to \( \sigma_{\text{PE}} \) corresponding to the L and higher shells; above the K edge, the cross section is essentially due to the K and higher shells. The midpoint of the sudden drop gives the K edge value of the target atom. Also, in the figure we have compared our experimentally determined photoelectric cross section values with the WinXCOM values of the photoelectric cross section. Since there is close agreement between the experimental photoelectric cross section and theoretical photoelectric cross section, we can categorically say that in our experiment, attenuation of EB photons in the target is essentially due to the photoelectric absorption process. We have also estimated the attenuation due to Rayleigh scattering and Compton scattering at various energies using the experimental values of incident photon intensity and the theoretical values of the Rayleigh and the Compton scattering cross sections which were taken from the WinXCOM software. It is found that these two contributions are less than 3%. Also, we have estimated the yields due to the presence of the air medium between the converter and the detector, treating air as a composition of nitrogen (75%), oxygen (23%), argon (1.2%) and carbon (0.012%). In this calculation, we have used the experimental EB photon intensity and the theoretical values of the cross section for air from the WinXCOM code. From the calculation it is found that the contribution of the air medium in the present experimental setup is less than 1.3%. Thus the present experiment gives
with the values predicted by Chantler [14], relativistic Hartee–Fock–Slater anomalous scattering factors [26] and WinXCOM [27]. From figures 4(a), 5(a) and 6(a), we note that our experimentally determined form factors closely agree with the values predicted by Chantler [14], relativistic Hartee–Fock–Slater anomalous scattering factors [26] and WinXCOM [27] within an error of 4–5% in the case of W, Au and Pb. From these comparisons we may conclude that the present method can be used for determining the imaginary form factors of high-Z atoms of W, Au and Pb.

3.2. Determination of the real part of the anomalous scattering factor ($f''$)

Using the $f''$ values determined by equation (2), the real part ($f'$) of the anomalous scattering factor is calculated by the numerical evaluation of the dispersion integral in equation (3). To evaluate the integral, the lower limit of integration was chosen to be 1 keV and the upper limit of integration was chosen to be 2 MeV. For the energies in the range 1–25 keV, the XCOM values of $\sigma_{pe}$ are used and above 25 keV our experimental values are used. Again from 110 keV to 2 MeV, the theoretical $\sigma_{pe}$ values of XCOM are used. Further, to evaluate the integral numerically, the energy region of integration is divided into a large number of small intervals of 20 eV each. Within each interval $E_i$ and $E_{i+1}$, the energy dependence of $f''(E)$ is determined using a linear function:

$$f''(E) = (a_i + b_i E).$$

(5)

For this interval, the dispersion integral is given by

$$I_{i,i+1}(E) = \frac{2}{\pi} \rho \int_{E_i}^{E_{i+1}} \frac{E' f''(E')}{E'^2 - E'^2} dE'$$

(6)

where $E_s$ is the energy of our interest.

Mathematically, equation (6) can be expressed as

$$I_{i,i+1}(E_s) = \left( \frac{2}{\pi} \right) \frac{a_i}{2} \ln \left| \frac{E_s^2 - E_{i+1}^2}{E_s^2 - E_i^2} \right| + b_i \left( E_{i+1} - E_i - \frac{E_s}{2} \ln \left| \frac{(E_{i+1} + E_s)(E_i - E_s)}{(E_{i+1} - E_s)(E_i + E_s)} \right| \right).$$

Therefore, using the coefficients $a_i$ and $b_i$, the dispersion integral in equation (6) is calculated for each interval using equation (7). The final value of the dispersion integral (equation 3) is obtained just by adding all the values of $I_{i,i+1}$ calculated using equation (7). Finally, the value of the real part of the anomalous scattering factor $f'$ is obtained using equation (3), for each target of our interest by separately adding $f''(\infty)$ values of Cromer and Liberman [7] and Kissel and Pratt [9]. It is needless to mention that the theoretical values of relativistic Hartee–Fock–Slater anomalous scattering factors [26] incorporate the relativistic Kessl and Pratt correction term, and that the theoretical values of Chantler [14] incorporate the Crommer and Liberman correction term. In order to compare our experimental values with the values predicted by relativistic Hartee–Fock–Slater anomalous scattering factors and by Chantler, we have added the relativistic term given by Kissel and Pratt [9] and Crommer and Liberman [7] to our experimental values. Therefore, the experimental values of $f'$ determined by involving the Kissel and Pratt correction are in good agreement with the theoretical values given by relativistic Hartee–Fock–Slater anomalous scattering factors [26], whereas the $f'$ values determined by incorporating Crommer and Liberman correction term are in good agreement with the theoretical values given by Chantler [14]. The experimental values so determined are compared with theoretical values of Chantler [14] and relativistic Hartee–Fock–Slater anomalous scattering factors [26]. These are given in figures 4(b), 5(b) and 6(b).

4. Conclusions

In the present paper, we have reported our measurements of the real and imaginary parts of the anomalous scattering factors for pure elemental targets of W, Au and Pb using external bremsstrahlung photons produced by a weak beta source in a nickel foil. The form factors have been determined by measuring the imaginary and transmitted spectra with the HPGe detector spectrometer. Measured values are compared with theoretical values. Good agreement between the measured form factors and the theoretical values indicates that the present method can undoubtedly be adopted for measuring the real and imaginary form factors of the atom. To the best knowledge of the authors, the anomalous scattering factors have been determined experimentally for W, Au and Pb for the first time using bremsstrahlung radiation produced by a weak beta source. This is a cost-effective technique as compared to the others. Researchers who do not have synchrotron radiation facilities or strong radioactive sources and many thin foils can adopt this method for measuring the anomalous scattering factors. This method can also be extended to medium-Z atoms.
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References

[12] Cullen D E, Dermott E, Hubbell J H and Kissel L 1997 The Evaluated Photon Data Library, 97 Version (Lawrence Livermore National Laboratory)
Determination of the K shell oscillator strengths and the imaginary form factors of atoms using a weak beta source

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Abstract
The K shell oscillator strengths and the imaginary form factors of Gd, Hf and Ta atoms have been determined using a novel method. In this method, bremsstrahlung photons produced by beta particles from a weak beta source of $^{89}$Sr-$^{90}$Y in a nickel foil are incident on an elemental target and the transmitted spectrum of photons emerging from the target is measured using an ORTEC make HPGe detector coupled to 8 K multichannel analyser. The recorded spectrum shows a sudden drop at the K shell binding energy of the target atom and an exponential decrease in the intensity above the K shell binding energy. These portions have been used to determine the K shell binding energy, photoelectric cross-section at the K edge, the K shell oscillator strength and the imaginary form factor of the elements Gd, Hf and Ta. Good agreement between the experimental and the theoretical values is observed.

1. Introduction
The interaction of gamma radiation with matter has been of experimental and theoretical interest in recent years in the view of its application in various fields such as radiation biology, medical physics, material science and industry [1-3]. It is well known that gamma radiation, having energy in the range from a few keV to a few MeV, can interact predominantly with matter through three processes, namely the photoelectric effect, Compton scattering and pair production. In the low-energy region the photoelectric effect is predominant. In the photoelectric absorption process, an incident gamma photon is totally absorbed by a tightly bound electron, resulting in emission of the photoelectron. For a photon energy greater than the K shell binding energy, the K shell electron is ejected and the process is known as the K shell photoelectric effect.

So the plot of the cross-section versus photon energy gives a saw-tooth structure. The steep increase in cross-section at the K shell binding energy is due to the onset of the K shell photoelectric absorption process. The saw-tooth structure has two branches, namely a lower energy branch and an upper energy branch. The cross-section corresponding to the lower energy branch is attributed to the photoelectric effect in the L and higher shells, and the cross-section corresponding to the upper energy branch is due to the K shell and higher shells. The difference between the cross-sections at the upper energy and the lower energy gives a K shell photoelectric cross-section at the K edge. The interaction of photons with atoms in the neighbourhood of the K shell binding energy has attracted the attention of researchers to understand various other processes such as extended x-ray absorption fine structure (EXAFS) and resonance Raman scattering (RRS) [4-13].

In the photon energy region of a few keV, the total cross-section, $\sigma_{\text{tot}}$, for the interaction of gamma photon with an atom is given by

$$\sigma_{\text{tot}} = \sigma_{\text{PE}} + \sigma_{\text{inccoh}} + \sigma_{\text{coh}},$$

where $\sigma_{\text{PE}}$ is the atomic photoelectric absorption cross-section, $\sigma_{\text{inccoh}}$ is the incoherent (the Compton) scattering cross-section and $\sigma_{\text{coh}}$ is the coherent (the Rayleigh) scattering cross-section. In the case of a large perfect crystal-like silicon as a target the coherent scattering contribution may also come from co-operative solid-state effects of Laue-Bragg scattering and thermal diffusion scattering. But as we have used polycrystalline foils as targets in our experiment the effect of these contributions is negligible. The photoelectric cross-section, $\sigma_{\text{PE}}$, is related to the dispersion correction to the forward-angle Rayleigh scattering cross-section through the
The scattering factor, $f$, of an isolated atom relative to that of a free electron is given by

$$f = f_0 + f' + f''$$

where $f_0$ is the atomic scattering factor, $f'$ and $f''$ are the real and imaginary parts of the dispersion corrections or the anomalous scattering factors. The real part corresponds to dispersion whereas the imaginary part corresponds to absorption. In the vicinity of the absorption edges, $f'$ and $f''$ are strongly energy dependent. The imaginary part of form factor $f''$ is related to the photoelectric absorption cross-section, $\sigma_{PE}$, through the relation [15]

$$f'' = \left( \frac{E}{2mc} \right)^2 \sigma_{PE},$$

where $h$ is the Planck constant, $c$ is the velocity of light, $r_0$ is the classical electron radius and $E$ is the photon energy of interest. From equation (3), we note that the imaginary form factor of the atom near and above the K edge can be determined by knowing the values of photoelectric cross-section at various photon energies.

The K shell oscillator strength $g_K$ is the probability of transition of the K shell electrons to all permissible states and it is given theoretically [16] by

$$g_K = \frac{mc^2\tau_K}{2\pi\hbar^2(n-1)},$$

where $m$ is the mass of the electron and $n$ is the slope of ln$(\sigma_{PE})$ versus ln$(E)$ plot above the K edge. From equation (4), we note that $g_K$ can be determined accurately by knowing the K edge energy $E_K$, the K shell photoelectric cross-section $\sigma_{PE}$ at the K edge and the slope $n$.

Several investigators have calculated the atomic form factors for various atomic numbers of the target at various photon energies [17-24]. Cromer and Liberman [17] have tabulated the dispersion correction factors using self-consistent field relativistic Dirac-Slater wavefunctions. Cromer [18] has made a FORTRAN program and photoelectric cross-section data file for calculating anomalous scattering factors $f'$ and $f''$ at arbitrary x-ray wavelengths. Creagh and McAuley [19] have given the tables for atomic form factors by making use of relativistic correction factors. Chantler [20, 21] has tabulated atomic form factors, attenuation co-efficients, elastic and inelastic scattering for the interaction of photon with an isolated atom for the atomic number $Z = 1-92$ in the energy range 1-10 eV to 0.4-10 MeV. Cullen et al [22] have provided a photon data library for all elements with atomic number between $Z = 1$ and $100$ for the energy range 1 eV-100 GeV. Berger and Hubbell [23] have developed a program to calculate the photon cross-section for scattering, photoelectric absorption and pair-production as well as total attenuation coefficients in any element, compound or mixture at energies from 1 keV to 100 GeV. Gerward et al [24] have developed a window version of XCOM-WinXCOM to calculate the photon cross-section.

Similarly, several investigators have experimentally determined atomic form factors for various elements at various energies [25-36]. Normally, two methods are adopted for measuring the form factors, namely the direct method and the attenuation method. The direct method is based on (a) measurement of reflection and refraction [25, 26], (b) determination of the intensities of the Bragg reflected rays [27, 28] and (c) x-ray interferometry [29, 30]. However, the measurement of atomic form factors using the attenuation method is more reliable [31-36] and does not suffer from the problems associated with the other methods. The other advantage of the attenuation method is that the atomic form factors can be determined for a larger range of atomic numbers over a wider range of x-ray or gamma-ray energies. In the attenuation method, the form factors are determined by using either monochromatic gamma sources or synchrotron radiation. Chantler et al [31] have adopted the x-ray extended range technique and determined the imaginary form factor of copper in the energy range of 8.85-20 keV using synchrotron radiation. Tran et al [32, 33] have determined the imaginary form factor of silicon and silver using synchrotron radiation. In the case of silicon, they have used synchrotron radiation of energy from 5 keV to 20 keV and for silver the energy range is 15-50 keV. Appaji Gowda et al [34] have determined the anomalous scattering factors for La, Ce, Pr, Nd, Sm, Gd, Dy, Ho and Er in the energy range from 6 keV to 85 keV using the gamma-ray attenuation method. Here, they have used monochromatic gamma sources for measuring the attenuation co-efficient. Recently, Appaji Gowda and Umesh [35, 36] have determined the atomic form factors of tantalum, mercury and lead using the gamma-ray attenuation method. Recently, de Jonge et al [5] have measured the atomic form factor of tin over the energy range of 20-60 keV using synchrotron radiation. In the case of synchrotron radiation, monochromatic beam is to be selected using crystal monochromator. The intensity of synchrotron radiation decreases rapidly above 50 keV and hence investigations are mostly confined to medium-Z elements. However, using third generation synchrotrons such as EPS, APS and Spring-8, the investigations can be extended to high-Z elements. The gamma-ray attenuation method can also be used for measuring the atomic form factors of medium and high-Z elements. But this method requires a number of monoenergetic gamma sources to cover the energy range in the neighbourhood of the K shell binding energy of the target atoms.

We have developed an alternative, inexpensive and simpler method for measuring the K edge and K shell photoelectric cross-section at the K edge using bremsstrahlung photons from a week (~10 $\mu$C) beta source [8, 9]. In the present experiment, we have shown for the first time that the K shell oscillator strength $g_K$ and the imaginary part of form factor $f''$ of high-Z atoms around and above the K edge can be determined accurately using bremsstrahlung photons. The advantages of the present method are that we use a single weak beta source and a single elemental target for each Z. In this paper we report our experimental results for Gd, Hf and Ta. Measured K shell oscillator strength values are compared with the theoretical values obtained from tables of Cullen [22] and WinXCOM [23], and the imaginary form factor values are compared with the theoretical values predicted by Chantler [21] and WinXCOM [23].
Then, the elemental target of our interest is placed between the perspex absorber of thickness 41.5 mg cm$^{-2}$. Some of the beta particles from the source S produce continuous external bremsstrahlung (EB) photons in the nickel converter of thickness 41.5 mg cm$^{-2}$. Some of the beta particles may transmit through the converter and produce unwanted EB photons in the detector; to prevent these beta particles from reaching the detector, a perspex absorber of thickness 10 mm is placed between the converter and the detector. Then, the elemental target of our interest is placed between the perspex absorber and the detector. The EB photons from the nickel converter are made to pass through the elemental target T whose K shell oscillator strength and the imaginary form factor are to be determined. The spectrum of photons transmitted through the target T is recorded with a GMX 10P HPGe detector, which has an energy resolution of 700 eV at 88 keV. The efficiency of the detector in the energy range of our interest is 100%.

The detector is calibrated in the energy range from 10 keV to 140 keV by using monoenergetic gamma sources of $^{241}$Am (59.536 keV), $^{109}$Cd (88.034 keV) and $^{51}$Co (14.411, 122.061 and 136.473 keV). From the least-square fit, the calibration constant is found to be 19.98 eV per channel. The stability of the detector spectrometer was checked before and after data acquisition by observing the peak position of 88.034 keV gamma radiations. From this least-square fit, we have found that the detector spectrometer is stable throughout the experiment. We have used pure (99.9%) elemental targets of Gd, Hf and Ta. These foils are obtained from Advent Research, England. These foils are rolled to make thin and uniform foils of required size and thickness. The uniformity of the targets is checked by using the transmission of 59.54 keV gamma radiations at different regions of the target using a good geometry arrangement. We have observed almost the same count rate for transmitted photon intensity at different regions in the case of each target which indicates that the targets are uniform.

First, the background spectrum is recorded for 8 h by keeping all the radioactive sources far away from the detector. The background spectrum is recorded for 8 h by keeping the target T between the perspex sheet and the detector. Such recorded transmitted spectrum is recorded for 8 h by keeping the target T between the perspex sheet and the detector. Such recorded transmitted spectrum for Gd target is as shown in figure 3. From the figure, we note that the transmitted EB intensity suddenly decreases at the K shell binding energy of the Gd target. This sudden decrease in intensity over the energy range from 49.65 keV to 51.01 keV for Gd target is attributed to the onset of the K shell photoelectric cross-section; the finite energy range is due to the finite energy resolution of the detector. Below the sudden drop region, we observe K x-ray peaks $K_{a}$, $K_{p1}$ and $K_{p2}$ which are due to the creation of the K shell vacancies in the target by the incident photons. From figure 3, we note that there are three regions, namely a lower energy region, a sudden drop region and an upper energy region; these are the important regions in the present investigations. We have used these three regions for determining the K shell oscillator strength and the imaginary form factor of Gd. We have also recorded the background, the incident and the transmitted spectra for each of the two elemental targets of Hf and Ta. We have used these spectra for determining the oscillator strength and the imaginary form factor of Gd, Hf and Ta.

The beta particles from the source S produce continuous external bremsstrahlung (EB) photons in the nickel converter of thickness 41.5 mg cm$^{-2}$. Some of the beta particles may transmit through the converter and produce unwanted EB photons in the detector; to prevent these beta particles from reaching the detector, a perspex absorber of thickness 10 mm is placed between the converter and the detector. Then, the elemental target of our interest is placed between the perspex absorber and the detector. The EB photons from the nickel converter are made to pass through the elemental target T whose K shell oscillator strength and the imaginary form factor are to be determined. The spectrum of photons transmitted through the target T is recorded with a GMX 10P HPGe detector, which has an energy resolution of 700 eV at 88 keV. The efficiency of the detector in the energy range of our interest is 100%.

The detector is calibrated in the energy range from 10 keV to 140 keV by using monoenergetic gamma sources of $^{241}$Am (59.536 keV), $^{109}$Cd (88.034 keV) and $^{51}$Co (14.411, 122.061 and 136.473 keV). From the least-square fit, the calibration constant is found to be 19.98 eV per channel. The stability of the detector spectrometer was checked before and after data acquisition by observing the peak position of 88.034 keV gamma radiations. From this least-square fit, we have found that the detector spectrometer is stable throughout the experiment. We have used pure (99.9%) elemental targets of Gd, Hf and Ta. These foils are obtained from Advent Research, England. These foils are rolled to make thin and uniform foils of required size and thickness. The uniformity of the targets is checked by using the transmission of 59.54 keV gamma radiations at different regions of the target using a good geometry arrangement. We have observed almost the same count rate for transmitted photon intensity at different regions in the case of each target which indicates that the targets are uniform.

First, the background spectrum is recorded for 8 h by keeping all the radioactive sources far away from the detector. Then the spectrum of EB photons produced in the nickel foil, which is known as incident spectrum, is recorded by keeping the nickel converter C and perspex sheet P in between the source and the detector. Such background and incident spectra are shown in figure 2. The spectrum of transmitted EB photons is recorded for 8 h by keeping the target T between the perspex sheet and the detector. Such recorded transmitted spectrum for Gd target is as shown in figure 3. From the figure, we note that the transmitted EB intensity suddenly decreases at the K shell binding energy of the Gd target. This sudden decrease in intensity over the energy range from 49.65 keV to 51.01 keV for Gd target is attributed to the onset of the K shell photoelectric cross-section; the finite energy range is due to the finite energy resolution of the detector. Below the sudden drop region, we observe K x-ray peaks $K_{a}$, $K_{p1}$ and $K_{p2}$ which are due to the creation of the K shell vacancies in the target by the incident photons. From figure 3, we note that there are three regions, namely a lower energy region, a sudden drop region and an upper energy region; these are the important regions in the present investigations. We have used these three regions for determining the K shell oscillator strength and the imaginary form factor of Gd. We have also recorded the background, the incident and the transmitted spectra for each of the two elemental targets of Hf and Ta. We have used these spectra for determining the oscillator strength and the imaginary form factor of Gd, Hf and Ta.

3. Analysis of data

By knowing the incident intensity $I_{i}$ and the transmitted intensity $I_{t}$ at photon energy $E$, the photoelectric cross-section $\sigma_{pe}$ at $E$ can be determined using the relation

$$\sigma_{pe} = \frac{\ln \left( \frac{I_{i}}{I_{t}} \right) A \times 10^{24}}{t N_{A}}$$

where $t$ is the thickness of the target expressed in g cm$^{-2}$, $A$ is the mass number of the target atom and $N_{A}$ is the Avogadro number. To determine the total photoelectric cross-section.
σ_{PE}, we need to know the intensity of the incident photons and transmitted photons at various photon energies. We have subtracted the background spectrum from the recorded incident spectrum and the background spectrum from the transmitted spectrum to obtain background free incident and transmitted spectra. From such incident spectra we have noted the incident intensity I₀ at various photon energies around and above the K edge of the target atom. In the case of the transmitted spectrum, we note an unwanted K{sub 2} x-ray peak below the K edge. We have deleted the unnecessary counts due to the K{sub 2} x-ray peak from the transmitted spectrum. Now the transmitted spectrum is free from the K{sub 2} x-ray peak. We have noted the intensity I₀ of the transmitted photon at various photon energies around 10 keV above the K shell binding energy of the target atom. Using I₀ and I₀, we have calculated the total photoelectric cross-section at various photon energies using equation (4). The plot of total photoelectric cross-section versus photon energies around the K edge for the Gd target is shown in figure 3. From the figure we note that three regions are important; sudden increase in cross-section at the K edge of the target atom which is attributed to onset of the K shell photoelectric cross-section; below the K edge, the cross-section is almost constant over the energy range from 49.31 keV to 49.65 keV which is essentially due to photoelectric cross-section corresponding to the L and higher shells; above the K edge, the cross-section is almost constant over the region from 51.01 keV to 52.15 keV which is due to the K and higher shells. The mid-point of the sudden increase region is the K edge of the target atom. In order to determine the exact value of E_K, we have least-squares fitted a sigmoidal function to our measured cross-section data. The sigmoidal function is given by

\[ \sigma_{PE} (E) = \frac{\sigma_{PE} - \sigma_{PE}^{K}}{1 + \exp \left( \frac{E - E_{K}}{\Delta K} \right)} + \sigma_{PE}^{K} \]  

where \( \sigma_{PE} (E) \) is the total photoelectric cross-section at photon energy E, \( \sigma_{PE}^{K} \) is the total photoelectric cross-section value below the K edge which is due to all shells excluding the K shell and \( \sigma_{PE}^{K} \) is the photoelectric cross-section from all the shells including the K shell. It is interesting to note that the photoelectric cross-section above and below the K edge over a small energy range is almost constant; the difference between the values of \( \sigma_{PE}^{K} \) and \( \sigma_{PE}^{K} \) over this region gives the K shell photoelectric cross-section \( \tau_{K} \) at the K edge. To determine the E_K value accurately, we have taken the derivative of the least-squares fit sigmoidal function and this derivative for the Gd target is shown in figure 5; the peak position accurately gives E_K, the K edge of the target atom. Thus we have determined E_K and \( \tau_{K} \) values for the targets of Gd, Hf and Ta using the above procedure. The average of three trials of E_K and \( \tau_{K} \) values is presented in table 1. To determine g_K values, we need to know the slope of ln(σ_{PE}) versus ln(E) plot above the K edge. In figure 5, we have given the variation of experimentally determined values of ln(σ_{PE}) with ln(E) over the range from 1 keV above the K edge to 6 keV above the K edge in the case of Gd. We see that the experimental points lie close to the least-squares fit straight line. Using the experimental values of E_K, \( \tau_{K} \) and n, we have determined the value of g_K. The values of g_K are the average of three trials for Gd, Hf and Ta are given in column 6 of table 1.

It is to be noted that the theoretical values of g_K are not given in the literature. We have calculated the theoretical value of g_K by the following procedure. From the tables provided by Cullen et al [22] and WinXCOM [23], we have noted the K edge energy E_K and the K shell photoelectric cross-section \( \tau_{K} \) at the K edge for Gd target. Then from the same tables we have taken the total photoelectric cross-section values for various photon energies over the range from 1 keV above the K edge to 6 keV above the K edge. From the plot of ln(σ_{PE}) versus ln(E) above the K edge, we have determined the slope n. By knowing the theoretical values of E_K, \( \tau_{K} \) and n, we have calculated g_K values using equation (4). In column 7 of table 1 we have presented the theoretical values of g_K for Gd, Hf and Ta. In the same table, we have also given the value of g_K for Gd obtained by another researcher who has adopted the gamma-ray attenuation method. We note that our experimental values...
Table 1. Comparison of measured and theoretical values of the K shell oscillator strength.

<table>
<thead>
<tr>
<th>Element</th>
<th>Z</th>
<th>( t ) (mg cm(^{-2} ))</th>
<th>( E_K ) (keV)</th>
<th>( r_K ) (b/atom)</th>
<th>( g_K ) (( e/\text{atom} )) ( \times 10^5 )</th>
<th>Theoretical</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gd</td>
<td>64</td>
<td>46.75</td>
<td>50.29</td>
<td>3263.89</td>
<td>1.06 ± 0.05</td>
<td>1.101 (c)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1.077 (w)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1.112 (d)</td>
</tr>
<tr>
<td>Hf</td>
<td>72</td>
<td>67.14</td>
<td>65.41</td>
<td>2520.71</td>
<td>1.04 ± 0.05</td>
<td>1.057 (c)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1.040 (w)</td>
</tr>
<tr>
<td>Ta</td>
<td>73</td>
<td>69.76</td>
<td>67.51</td>
<td>2464.90</td>
<td>1.01 ± 0.06</td>
<td>1.046 (c)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1.023 (w)</td>
</tr>
</tbody>
</table>

The theoretical oscillator strength has been deduced from the works of (c) Cullen [22] and (w) WinXCOM [23]. The experimental result (d) is from [37].

closely agree with the theoretical values within an error of 3.8%. Our experimental values of \( g_K \) for Gd closely agree with the value obtained from [37]. This indicates that our method complements the other well-established method.

To determine the imaginary form factor \( f'' \) around and 6 keV above the K edge, we need to know the photoelectric cross-section values at various energies. Using the experimental photoelectric cross-section data, we have estimated the imaginary form factor values around and 6 keV above the K edge. These form factor values are plotted in figure 7(a) for Gd along with the theoretical values predicted by Chantler [21] and WinXCOM [23]. Similarly, we have determined imaginary form factor values for Hf and Ta and they are plotted in figure 7(b) and (c), respectively, along with theoretical values predicted by Chantler and WinXCOM.

We wish to mention that we have estimated the yields corresponding to the Rayleigh scattering and the Compton scattering at various energies using the experimental values of incident photon intensity and the theoretical values of the Rayleigh and the Compton scattering cross-sections which are taken from the WinXCOM software. It is found that these two contributions are less than 3%. We have also estimated the yields due to the presence of air medium between the converter and the detector, treating air as a mixture of nitrogen (75%), oxygen (23%), argon (1.2%) and carbon (0.012%). In this calculation, we have used the experimental EB photon intensity and the theoretical values of the cross-section for air from WinXCOM code. From the calculation, it is found that the contribution of air medium in the present experimental setup is less than 1.3%.

4. Results and discussion

We have determined the values of \( g_K \) and \( f'' \) for the elemental targets Gd, Hf and Ta using a weak beta source method. In table 1, we have presented the experimentally determined \( g_K \) values along with the theoretical values obtained from Cullen [22] and WinXCOM [23]. From the table we note that our measured values of \( g_K \) closely agree with the theoretical values, an error of 3.8%. This shows that the present method can be used for determining the K shell oscillator strengths of high-Z elements also. The imaginary form factor values have also been determined experimentally for Hf, Ta and Gd and compared with theoretical values predicted by Chantler [21] and WinXCOM [23]. Our \( f'' \) values are found to agree with the theoretical values predicted by Chantler within an error...
of 4%. However, our measured $f''$ values agree with the values predicted by WinXCOM by 6%. The error 4–6% in the measured imaginary factor is attributed to the statistical fluctuation in the measured counts and scattering effects. Either by using a strong beta source or by collecting the data for a longer period, the error in the measured value can be reduced.

5. Conclusion

We have determined the K-shell oscillator strengths and imaginary form factors of Gd, Hf and Ta elemental targets using bremsstrahlung radiation produced by a weak beta source. To the best knowledge of the authors, the K-shell oscillator strengths and the imaginary form factors have been determined for the first time using bremsstrahlung radiation produced by a weak beta source. This method can be extended for medium-Z atoms also. It uses a single beta source and a single target. The advantage of this method is that it can be adopted by researchers who have no access to synchrotron facilities.

Acknowledgments

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References


[22] Cullen D E, Dermott E, Hubbell J H and Kissel L 1997 The Evaluated Photon Data Library 97 Version (Lawrence Livermore National Laboratory)

[23] Berger M J and Hubbell J H 1999 XCOMNIST MD 20899


Determination of rest mass energy of the electron—an undergraduate laboratory experiment

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Abstract
We present a simple Compton scattering experiment to determine the rest mass energy of the electron which is unique for graduate and undergraduate laboratories. In the present experiment, we have measured the energies of the backscattered gamma photons with an NaI(Tl) gamma ray spectrometer coupled to a 1 K multichannel analyser. In order to enhance the backscattered gamma photons, a thick aluminium target is placed over the radioactive gamma source. The rest mass energy of the electron is determined by using $^{203}\text{Hg}$, $^{137}\text{Cs}$, $^{54}\text{Mn}$ and $^{60}\text{Co}$ radioactive gamma sources. The measured values are found to agree with the standard value.

1. Introduction

Many laboratory experiments based on the interaction of gamma radiation with matter have been published in the literature [1, 2, 8–11]. Gamma photons interact with matter through photoelectric absorption, Compton scattering and pair production processes. Compton scattering is an important phenomenon which demonstrates the particle nature of gamma radiation. In Compton scattering, the gamma photon interacts with free or bound electrons of the target and gets scattered with less energy. As the incident photons are scattered at different angles (from 0 to 180°), the Compton scattered electrons have continuum distribution of energy from zero to a maximum energy; the maximum energy of an electron is known as the Compton edge. The minimum energy of the scattered photon occurs in a head-on collision; the minimum photon energy is for the backscattered photon.

Using the gamma ray spectrometer, Badiger and Thontadarya [1] have demonstrated the important aspects of Compton scattering that the Compton shift in wavelength in any particular direction is independent of the energy of the incident photon and the Compton shift in energy is dependent on the incident photon energy. Bertsch and Nolen [2] have shown that the intensity
of the backscattered gamma photon can be enhanced by using a low Z absorber; they have demonstrated that the intensity of the backscattered photons is more pronounced using an aluminium target rather than lead. Sanjeevaiah and Venkataramaiah [10] have measured the variation of Compton shift in wavelength with the angle of scattering. Singhal and Burns [11] have measured the Compton scattering cross-section at various angles and verified the Klein–Nishina formula which predicts the Compton scattering cross-section.

It is interesting to note that the absorption of monoenergetic gamma photons in the thick NaI(Tl) detector leads only to a photopeak; the photopeak is due to the complete absorption of monoenergetic gamma photons entering within the detector. However, for the intermediate thickness of a NaI(Tl) detector, complete absorption of monoenergetic gamma photons leads to a photopeak; the absorption of only recoiled Compton electrons in the detector leads to the Compton continuum; and the absorption of only backscattered photons in the detector through the photoelectric absorption process leads to the backscattered peak. It is important to note that if a head-on collision takes place at the interface between the detector and a glass envelope of a photomultiplier, the forward moving electron escapes from the detector without depositing its energy in the detector and absorption of backscattered photons leads to the backscattered peak. If a head-on collision occurs on the edge or surface of the detector, the backscattered gamma photon may escape from the detector without depositing its energy, and the forward moving electron may deposit its energy to produce the Compton edge. If the head-on collision takes place in the middle of the detector, both the backscattered photon and the Compton electrons together produce photopeak at the incident photon energy.

In the present experiment, we have shown that the rest mass energy of the electron can be determined accurately by measuring the backscattered photon energies with a NaI(Tl) detector spectrometer. The intensity of the backscattered photon is enhanced by keeping the aluminium target over the radioactive gamma source.

2. Theory

When a photon of energy \( E_y \) is scattered by an electron through Compton scattering at an angle \( \theta \), the energy of the scattered photon \( E'_y \) is given by

\[
E'_y = \frac{E_y}{1 + \frac{E_y}{m_0c^2} (1 - \cos \theta)} \tag{1}
\]

where \( m_0c^2 \) is the rest mass energy of an electron.

The Compton shift in wavelength [3] at any particular angle \( \theta \) is given by

\[
\lambda' - \lambda = \frac{h}{m_0c} (1 - \cos \theta) \tag{2}
\]

The Compton shift in energy [3] of the photon is given by

\[
\frac{1}{E'_y} - \frac{1}{E_y} = \frac{1}{m_0c^2} (1 - \cos \theta) \tag{3}
\]

The Compton shift in energy [3] for a head-on collision (\( \theta = 180^\circ \)) is given by

\[
\frac{1}{E_b} - \frac{1}{E_y} = \frac{2}{m_0c^2} \tag{4}
\]

where \( E_b \) is the energy of the backscattered photon.
The energy of the backscattered photon ($\theta = 180^\circ$) is given by

$$E_b = \frac{E_\gamma}{1 + 2 \frac{E_b}{m_0c^2}}.$$  \hspace{1cm} (5)

Therefore, the rest mass energy of the electron in terms of $E_\gamma$ and $E_b$ can be written as

$$m_0c^2 = 2 \left[ \frac{E_\gamma \times E_b}{E_\gamma - E_b} \right].$$  \hspace{1cm} (6)

Hence, by measuring the backscattered photon energy $E_b$ for the known incident photon energy $E_\gamma$, the rest mass energy of the electron can be determined.

In our experiment, we have measured the backscattered energy $E_b$ using the scintillation detector gamma ray spectrometer. By knowing $E_\gamma$ from the literature and $E_b$ from the experiment, we have determined the rest mass energy $m_0c^2$ of the electron.

3. Experimental details

The experimental arrangement to determine the rest mass energy of the electron is shown schematically in figure 1. It consists of a radioactive gamma source, a thick aluminium target and the detector spectrometer. The radioactive gamma sources, $^{203}\text{Hg}$ (279.197 keV), $^{137}\text{Cs}$ (661.660 keV), $^{54}\text{Mn}$ (834.855 keV) and $^{60}\text{Co}$ (1252.873 keV, which is the weighted average energy of 1173.237 keV and 1332.501 keV), used in our experiment are obtained from the Board of Radiation and Isotope Technology (BRIT), Mumbai, India. The gamma radiation emitted from the radioactive source at S is detected with a 2 inch diameter and 11 inch thick NaI(Tl) crystal coupled to a photomultiplier of the type 9656 KL (EMI). The output of the photomultiplier is fed to a linear amplifier and then to a 1 K multichannel analyser. In order to reduce the noise, we have placed the detector in a lead castle; the lead K x-ray (~3 cm) can also produce a photopeak at 82 keV. To reduce the intensity of the lead x-ray, the inner portion of the lead shield is lined with a copper plate of the thickness 0.5 cm. It is interesting to note that for a large change in incident photon energy from 280 keV to 1200 keV, the backscattered photon energy changes from 130 keV to 220 keV. In view of
the change of the backscattered photon energy over a small energy interval, we have calibrated the detector spectrometer in the region from 80 keV to 300 keV, using monoenergetic gamma sources $^{109}$Cd (88.034 keV), $^{57}$Co (123.6 keV) and $^{203}$Hg (279.197 keV). The calibration constant is found to be 0.878 keV per channel and is shown in figure 2. Our main objective in the present experiment is to measure the backscattered photon energies for the four different monoenergetic gamma sources. A typical experimentally measured gamma ray spectrum for $^{137}$Cs (661.660 keV) is shown in figure 3(b). From the figure, we notice that the intensity of the backscattered photons produced in the detector is not pronounced. In order to obtain pronounced backscattered photon intensity for all the radioactive sources, we have placed a thick aluminium target (7.2 cm) over the source. In such a position, more gamma photons
Determination of rest mass energy of the electron

Figure 4. Typical backscattered peaks for $^{60}$Co, $^{54}$Mn, $^{137}$Cs and $^{203}$Hg with an aluminium target.

Table 1. Comparison of experimental values with standard value of the rest mass energy of the electron.

<table>
<thead>
<tr>
<th>Sources</th>
<th>Photopeak energy $E_y$ (keV)</th>
<th>Backscattered peak energy $E_b$ (keV)</th>
<th>Rest mass energy of the electron $m_e c^2$ (keV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{203}$Hg</td>
<td>279.197</td>
<td>130.8</td>
<td>492.3</td>
</tr>
<tr>
<td>$^{137}$Cs</td>
<td>661.660</td>
<td>187.8</td>
<td>524.8</td>
</tr>
<tr>
<td>$^{54}$Mn</td>
<td>834.855</td>
<td>201.0</td>
<td>529.6</td>
</tr>
<tr>
<td>$^{60}$Co</td>
<td>1252.873</td>
<td>212.9</td>
<td>512.9</td>
</tr>
</tbody>
</table>

* Reference [4].
* Equation (5).
* Reference [6].

incident on the target, get backscattered into the detector leading to a pronounced backscattered peak at the lower energy region of the Compton continuum; this is shown in figure 3(c).

We have measured the backscattered photon energies for other radioactive sources of $^{203}$Hg, $^{54}$Mn and $^{60}$Co by keeping the aluminium target over the sources. The experimentally observed backscattered peaks are shown in figure 4. In table 1, we give the list of radioactive sources used, the values of the incident gamma energies ($E_y$) taken from the literature [4], the values of experimentally measured backscattered photon energies and also the calculated backscattered photon energies using equation (5). Using $E_y$ and experimentally measured $E_b$, we have determined the rest mass energy of the electron using equation (6); for each incident gamma energy, we see from column 5, the values lie close to one another. We have determined the rest mass energy of the electron using equation (4). In this case, we have plotted $1/E_b$ versus $1/E_y$ which gives a straight line. From figure 5, we see that all the four points lie close to the least-square fit straight line. The rest mass energy of the electron is determined using the least-square fit value of the intercept. It is found to be 534.75 keV which is in agreement with the standard value of 511.003 keV within an error of 4.6%.
4. Conclusions

Here, we have set up a simple experiment for measuring the rest mass energy of the electron, as discussed above. We have measured the backscattered photon energy for determining the rest mass energy of the electron. From table 1, we notice that our experimental values of rest mass energy closely agree with the standard value within an error of 3.6%. Thus, we can say that this is an exclusively simple setup, which graduate and undergraduate laboratories can adopt, without much expense, to determine the rest mass energy of the electron, which is an important physical quantity. If the laboratories do not have four radioactive sources, they can determine the rest mass energy of the electron just by using a single radioactive source. Thus, in the present experiment, the student can also learn the important aspects of the interaction of gamma photons with a NaI(Tl) detector along with the determination of the rest mass energy of the electron.

Acknowledgments

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References

Determination of rest mass energy of the electron


