Preface

The representation of functions and operators by means of their simpler counterparts is essential for solving problems in mathematics, physics and applied sciences. In many applications, we encounter functions which are much more complicated than the standard functions of classical analysis. Some of these functions cannot be expressed explicitly. This means that, it is difficult or impossible to extract information about the function $f$, especially, if we need to perform some calculations on $f$. In such cases, it is important to be able to approximate $f$ with simple functions.

Every continuous function on a closed and bounded interval can be approximated arbitrarily well with a polynomial. This is the famous Weierstrass theorem. In practice, however, it is not of much use, as it is only an existence theorem. It does not tell how one can choose a polynomial that approximates a given function. Under additional assumptions, we can have, a much more useful result called Taylor's theorem accompanied by Taylor's series (power series). Unfortunately, not all functions can be represented (approximated) with the help of Taylor's series. So we often need to seek other tools to approximate functions. Fourier series and Fourier transform are such tools. But, what is missing in Taylor’s and Fourier series is a method for analyzing local behavior of functions that are not smooth and periodic, and wavelets arise naturally in this way. Thus wavelet series and wavelet transform are new mathematical tools to improve Fourier and other classical series (expansions) in a simpler, more compact and thus more efficient manner.

As one can see from the history, the solution of differential and integral equations was one of the motivations for the development of wavelet theory from the onset. Because of the similarities between Fourier basis and Finite element basis and wavelet basis lying midway between the two, wavelets are used as a tool in scientific computing and numerical solution of differential and integral equations. Over the last few years substantial progress was made concerning the numerical treatment of these problems. However, we are far from the ultimate goal, still needed is the development of fast,
accurate and robust numerical solvers for a whole range of linear and nonlinear differential and integral equations. This thesis can be viewed as one step towards this goal. It is organized into six chapters.

Chapter 1 is introductory in nature. We describe the fundamental ideas, a brief history, the essence of wavelet theory and some applications, that are needed to understand the remainder of the thesis. It mostly addresses development since the definition of multiresolution analysis by Mallat and Meyer (approximately around 1985). It also deals with properties of wavelets, construction of wavelets, wavelet-packets, multidimensional wavelets and applications of wavelets to data and operator analysis.

Chapter 2 describes the use of oldest and perhaps the simplest wavelet called Haar wavelet in solving problems from the calculus of variations. Exact solutions for variational problems are available for relatively simple problems, because the Euler differential equations are generally a non-linear second-order differential equations. Fortunately, accurate approximate solutions often suffice for many applications. One approach to approximate solution is to solve approximately the Euler differential equation. We can also obtain an approximate solution by solving a discrete analogue of the variational problem by using the classical direct method. The key to a viable direct method is the proper choice of basis functions. Haar wavelets with properties viz. locality, orthogonality and symmetry are the right candidates for solving variational problems using direct method as proposed for the first time by Hasio. An attempt is made to extend the method to eigenvalue problems. We present a novel method for the computation of eigenvalues and solutions of Sturm-Liouville eigenvalue problems (SLEP) arising in mathematical physics using truncated Haar wavelet series. In recent years, there has been renewal of interest in SLEP's in view of their importance in mathematics and their applications in physics and engineering. The nice properties of Haar wavelets combined with the direct method of calculus of variations discussed in standard texts reduce a regular SLP into a system of algebraic
equations. The most distinguishing feature of the method is that, it gives eigenvalues and the solution simultaneously, which is not possible by classical methods. To demonstrate the efficiency and accuracy of the method, various celebrated SLP's are analyzed for their eigenvalues and solutions.

Many physical systems such as rapid chemical reactions give rise to ordinary differential equations either single or coupled. These equations are characterized by a property of having solutions varying on different time scales. It is common to refer to such equations as stiff. Chapter 3 presents Single Term Haar Wavelet Series (STHWS) approach to the solution of linear and nonlinear stiff differential equations. The unique feature of the method is, it avoids totally computation of operation matrix of integration used in chapters 2 there by reducing the computation effort enormously. The other advantages of STHWS are: computations can be carried to any desired length of time without having to worry about stability. Secondly, it gives the option to choose between two types of solutions viz. discrete and block-pulse. Numerical solutions of some model equations encountered in chemical kinetics and nonlinear dynamics are obtained to establish the validity and applicability of the proposed method.

Chapter 4 discusses a more advanced and innovative alternative to Walsh series and single term methods for solving linear and nonlinear integral equations. A simple but powerful generalization of wavelets and the associated multiresolution analysis is wavelet packets. The wavelet packet method is an extension of wavelet decomposition that offers a richer range of possibilities for approximation of functions and operators. The wavelet packet transform generalizes the discrete wavelet transform and provides a more flexible tool for the time-scale analysis of functions. Our approach consists of expanding the kernel of the integral equation by a truncated double Haar wavelet packet series and approximating the integral operator using 2D-Haar wavelet packet transform together with operational matrix of integration. The proposed method eliminates integration totally and converts it into matrix-vector multiplication, thereby reducing computation time substantially. A good
agreement between computed results and the exact solutions of test problems is established with the help of numerical tables.

In chapter 5 we describe how wavelets may be used to solve partial differential equations. These problems are currently solved by techniques such as finite differences, finite elements, Galerkin, collocation and multigrid. The use of wavelet bases, however, offers several advantages over traditional methods. The hierarchical nature of wavelets makes them particularly appealing to the techniques of solving partial differential equations, providing a logical means of obtaining solutions at different levels of resolutions, which makes them useful for engineering applications. Furthermore, compactly supported wavelets such as those due to Daubechies are localized in space, which means that the solution can be refined in regions of high frequency. In this chapter, we present a wavelet-based multigrid approach to solve elliptic boundary value problems. The most attractive feature of the method is it works as both solver and preconditioner. As a consequence, it avoids instability, minimizes error and accelerates the rate of convergence. The simplicity and robustness of the method are demonstrated by the numerical solution of test problems. Chapter 6 deals with application of the method discussed in Chapter 5 to a real life problem. The effects of surface roughness on a squeeze film behavior of poroelastic bearings with Newtonian fluid as lubricant is investigated. Modified and realistic forms of constitutive relations of the viscoelastic matrix are incorporated in the derivation of relevant partial differential equation. The modified Reynolds equation is derived on the basis of stochastic theory, describing roughness structure of the articular cartilage and the constitutive equation of the poroelastic material. Recently developed fast wavelet-multigrid method is used for the solution of modified Reynolds equation, which has many advantages over classical methods. The influences of roughness on poroelastic material provides increase in the squeeze film pressure and hence load carrying capacity; that is sustained by articular cartilage during normal conditions. This helps design bio-medical engineers in choosing suitable parameters while manufacturing the artificial human knee-joint.