Chapter 5

DEFICIENT VALUES OF MEROMORPHIC FUNCTIONS AND THEIR DERIVATIVES
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AND THEIR DERIVATIVES.

5.1 INTRODUCTION AND DEFINITIONS.

An entire function can assume all values infinitely many times except possibly one, which has been proved by Picard in 1879, has been the starting point for all the subsequent research in the field of exceptional values. Picard has proved the theorem by using elliptic modulus function. Because of the publication of the classical theorem of Picard, much work has done in this direction. Since then many alternative proof of this theorem are given, indicating different direction in which the subject has developed. For an entire function of finite order, Borel proved in 1896, that

\[ \lim_{r \to \infty} \frac{\log n(r, a)}{\log r} = \lim_{r \to \infty} \frac{\log \log M(r, a)}{\log r} = \sigma(a) \]

exact possibly for one value of ‘a’, and this exception can arise only when a is an integer. This includes the Picard Theorem for entire function of finite order.

It is well known [4, PP78-107, 5, PP 254-269] that ‘a’ is said to be \( N \) defect or Nevanlinna
deficient value (e.v.N) if $\delta(a) > 0$ where

$$\delta(a) = \delta(r, a) = \lim_{r \to \infty} \frac{m(r, a)}{T(r, f)} = 1 - \lim_{r \to \infty} \frac{N(r, a)}{T(r, f)}$$

Further let

$$\Theta(a) = \Theta(r, a) = 1 - \lim_{r \to \infty} \frac{\overline{N}(r, a)}{T(r, f)}$$

and

$$\theta(a) = \theta(r, a) = \lim_{r \to \infty} \frac{N(r, a) - \overline{N}(r, a)}{T(r, f)}.$$ 

$\theta(a)$ is called the index of multiplicity of $f$.

**Remark 5.1.1**

$$\Theta(a) \geq \delta(a)$$

We have

$$\overline{N}(r, a) \leq N(r, a)$$

Therefore

$$\lim_{r \to \infty} \frac{\overline{N}(r, a)}{T(r, f)} \leq \lim_{r \to \infty} \frac{N(r, a)}{T(r, f)}$$

$$1 - \lim_{r \to \infty} \frac{\overline{N}(r, a)}{T(r, f)} \geq 1 - \lim_{r \to \infty} \frac{N(r, a)}{T(r, f)}$$

$$\Theta(a) \geq \delta(a)$$

Also it is known that $'a'$ is e.v.V (in the sense of Valiron) if $\Delta(a) > 0$, where

$$\Delta(a) = \Delta(r, a) = \lim_{r \to \infty} \frac{m(r, a)}{T(r, f)} = 1 - \lim_{r \to \infty} \frac{N(r, a)}{T(r, f)}.$$ 

We know that $\delta(a) \leq 1$ and

$$\sum_{i=1}^{\infty} \delta(a_i) \leq 2.$$ 

If $\delta(a) = 1$ we say that $'a'$ is e.v.N (in the sense of Nevanlinna) with total defect and if

$$\sum_{i=1}^{\infty} \delta(a_i) = 2.$$ 

we say that the function $f(z)$ has the maximum defect.

A value 'o' is defined to be e.v.E for entire function as follows:

Let $E$ denote the set of positive, non-decreasing functions $\phi(x)$ such that

$$\frac{dx}{x\phi(x)}$$

is convergent. It is known that for functions of non-integral order and zero order and for a class of functions of integral order, including all functions of maximal or minimal type we have

$$\lim_{r \to \infty} \log \frac{M(r, f)}{n(r, a)\phi(r)} = 0,$$

for every 'a'.

Hence if

$$\lim_{r \to \infty} \log \frac{M(r, f)}{n(r, a)\phi(r)} = 0,$$

for some $\phi \in E$. We call 'a' as an e.v.E.

**Definition 5.1.1** Let $f(z)$ and $a(z)$ be meromorphic functions in the complex plane. If

$T(r, a(z)) = S(r, f)$, then $a(z)$ is called a small function of $f(z)$.

**Definition 5.1.2** Suppose that $f(z)$ is a meromorphic function in the complex plane and $a(z)$ is a small function of $f(z)$. Let $n$ be non-negative integer. We denote by

$M(f) = a(f^n)$, the differential monomial in $f$ of degree $n$.

**Definition 5.1.3** Let $f(z)$ be a meromorphic function in the complex plane $C$, and $M(f)$ be differential monomial in $f$ of degree $n$. Then

$$P(f) = M(f) = a(f^n) = a(f^{(m)})^n$$

where $m = 1$ is said to be differential polynomial in $f$, and $n$
5.2 PRELIMINARIES AND LEMMAS.

In this section we present some lemmas which will be needed in the sequel.

Lemma 5.2.1 [29] If \( P(f) \) is a homogeneous differential polynomial of degree \( n \geq 1 \), then
\[
m \left( r, \frac{P(f)}{f^n} \right) = S(r, f)
\]

Lemma 5.2.2 [29] If \( P(f) \) is a differential polynomial in \( f \) of degree \( n \) and \( m \) be highest derivative of \( f \) occurring in \( P(f) \), then
\[
m(r, P(f)) \leq nm(r, f) + S(r, f),
\]
\[
N(r, P(f)) \leq nN(r, f) + mnN(r, f) + S(r, f),
\]
\[
T(r, P(f)) \leq nT(r, f) + mnT(r, f) + S(r, f),
\]
\[
T(r, P(f)) \leq (n + mn)T(r, f) + S(r, f).
\]
Therefore
\[
S(r, P(f)) = S(r, f).
\]

Lemma 5.2.3 [Clunie] Let \( f(z) \) be a transcendental meromorphic function, \( P(f) \) and \( Q(f) \) be differential polynomials in \( f \). If degree of \( Q(f) \) is at most \( n \) and \( f^n P(f) = Q(f) \), then
\[
m(r, P(f)) = S(r, f).
\]

Lemma 5.2.4 [29] Let \( P \) be a homogeneous differential polynomial in \( f \) of degree \( n \) and suppose that \( P \) does not involve \( f \). That is \( P \) is a homogeneous differential polynomial of degree \( n \) in \( f^{(1)}, f^{(2)}, \ldots, \) with its coefficients of the form \( a(z) \). If \( P \) is not a constant and \( b_1, b_2, \ldots, b_q \) are distinct elements of \( C \) (where \( q \) is any positive integer), then
\[
n \sum_{i=1}^{q} m(r, b_i, f) + N \left( r, \frac{1}{P} \right) \leq T(r, P) + S(r, f). \quad (5.2.1)
\]
Lemma 5.2.5 [29] Let $f$ be a meromorphic function and $P$ a homogeneous differential polynomial in $f$ of degree $n$. If $P$ is not a constant and $a \in C - \{0\}$, then

$$nT(r,f) < N(r,f) + nN\left(r, \frac{1}{f}\right) + N\left(r, \frac{1}{P-a}\right) - N_0(r,0,p') + S(r,f) \quad (5.2.2)$$

where $N_0(r,0,p')$, only zeros of $p'$ which are not zeros of $P-a$ are to be considered.

5.3 STATEMENT AND PROOF OF MAIN THEOREMS.

In this section we list some known theorems and present the main results of this chapter.

Theorem 5.3.A

$$\delta(a) + \theta(a) \leq \Theta(a)$$

Theorem 5.3.B Let $f(z)$ be meromorphic in $C$. Then the set of values $\{a\}$ for which $\Theta(a) > 0$ is countable and

$$\sum_{a \in \mathbb{C}} \Theta(a) \leq 2,$$

or

$$\sum_{a \in \mathbb{C}} (\delta(a) + \theta(a)) \leq \sum_{a \in \mathbb{C}} \Theta(a) \leq 2,$$

or

$$\sum_{a \in \mathbb{C}} \delta(a) \leq 2.$$

S.K. Singh and V.N. Kulkarni [21] proved the following theorem and corollaries.

Theorem 5.3.C [21] Let $f(z)$ be meromorphic function having $\{a_i\}$ as e.v.N where all $a_i$'s are different from each other such that

$$\sum_{\{a_i\} \neq \infty} \delta(a_i) = \alpha \quad \text{and} \quad \sum \delta(a_i) = 2, \quad (5.3.1)$$

then

$$T(r,f') \sim \alpha T(r,f) \quad (5.3.2)$$
Corollary 5.3.A [21] If $a$ and $\infty$ are e.v.E (or)

$$\sum_{i=1}^{j} \delta(a_i) = 1$$

and $\delta(\infty) = 1$. Then

$$T(r, f') \sim T(r, f)$$  \hspace{1cm} (5.3.3)

Corollary 5.3.B [21] If $a_1$ and $a_2$ are e.v.E (or) where $|a_i| < \infty$, $i = 1, 2, \ldots$, (or)

$$\sum_{i=1}^{j} \delta(a_i) = 2.$$ 

Then

$$T(r, f') \sim 2T(r, f)$$  \hspace{1cm} (5.3.4)

Subhas S Bhoosnurmath, proved the following Theorem.

**Theorem 5.3.D [29]** Let $f$ be a meromorphic function of finite order.

(a) If $P$ is a homogeneous differential polynomial in $f$ of degree $n$, then

$$\lim_{r \to \infty} \frac{T(r, P)}{T(r, f)} \leq n \{m + 1 - m\Theta(\infty, f)\}$$  \hspace{1cm} (5.3.5)

where $f^{(m)}$ is the highest derivative of $f$ occurring in $P$ ($m \geq 0$. As usual, $f^{(0)}$ stands for $f$).

(b) If $P$ is a homogeneous differential polynomial in $f$ of degree $n$ and if $P$ does not involve $f$, then

$$n \sum_{b \in C} \delta(b, f) \leq \delta(0, P) \lim_{r \to \infty} \frac{T(r, P)}{T(r, f)},$$  \hspace{1cm} (5.3.6)

and

$$n \sum_{b \in C} \delta(b, f) \leq \Delta(0, P) \lim_{r \to \infty} \frac{T(r, P)}{T(r, f)},$$  \hspace{1cm} (5.3.7)

provided that $P$ does not reduce to a constant.

Statement and Proof of Our Main Theorems.
Theorem 5.3.1 Let $f(z)$ be meromorphic function having $\{a_i\}$ as e.v.N where all $a_i$'s are different from each other such that
\[
\sum_{|a_i|\neq \infty} \delta(a_i) = \alpha, \quad \Theta(\infty, f) = 2 - \alpha \quad \text{and} \quad \sum \delta(a_i) = 2.
\]

(a) If $P$ is a homogeneous differential polynomial in $f$ of degree $n$, then
\[
\lim_{r \to \infty} \frac{T(r, P)}{T(r, f)} \leq n \{1 + m(\alpha - 1)\}
\]
where $f^{(m)}$ is the highest derivative of $f$ occurring in $P$ ($m \geq 0$. As usual, $f^{(0)}$ stands for $f$).

(b) If $P$ is a homogeneous differential polynomial in $f$ of degree $n$ and if $P$ does not involve $f$, then
\[
n\alpha \leq \delta(0, P) \lim_{r \to \infty} \frac{T(r, P)}{T(r, f)},
\]
and
\[
n\alpha \leq \Delta(0, P) \lim_{r \to \infty} \frac{T(r, P)}{T(r, f)},
\]
provided that $P$ does not reduce to a constant.

Proof of Theorem 5.3.1

(a)
\[
m(r, P) \leq m \left( r, \frac{P}{f^n} \right) + m(r, f^n),
\]
\[
m(r, P) \leq nm(r, f) + S(r, f), \quad (5.3.8)
\]
since by Lemma 5.2.1

At a pole of $f$ of order $p$, which is not a pole of any of the co-efficients $a(z)$ of $P$, $P$ has a pole of order at most $pn + mn$.

So,
\[
N(r, P) \leq nN(r, f) + mn\overline{N}(r, f) + S(r, f) \quad (5.3.9)
\]
By adding (5.3.8) and (5.3.9), we get

\[ T(r, P) \leq nT(r, f) + mnN(r, f) + S(r, f). \]

Divide by \( T(r, f) \) on both sides of the above inequality, we get

\[ \frac{T(r, P)}{T(r, f)} \leq n + mn \frac{N(r, f)}{T(r, f)} + \frac{S(r, f)}{T(r, f)}. \]

Therefore

\[
\lim_{r \to \infty} \frac{T(r, P)}{T(r, f)} \leq n + mn (1 - \Theta(\infty, f)),
\]

\[
\lim_{r \to \infty} \frac{T(r, P)}{T(r, f)} \leq n + mn - mn\Theta(\infty, f),
\]

\[
\lim_{r \to \infty} \frac{T(r, P)}{T(r, f)} \leq n + mn - mn(2 - \alpha),
\]

\[
\lim_{r \to \infty} \frac{T(r, P)}{T(r, f)} \leq n + mn - 2mn + mna,
\]

\[
\lim_{r \to \infty} \frac{T(r, P)}{T(r, f)} \leq n - mn + mna,
\]

\[
\lim_{r \to \infty} \frac{T(r, P)}{T(r, f)} \leq n \{1 - m + m\alpha\},
\]

\[
\lim_{r \to \infty} \frac{T(r, P)}{T(r, f)} \leq n \{1 + m(\alpha - 1)\},
\]

(5.3.10)

where \( 1 \leq \alpha \leq 2. \)

(b) Suppose \( P \) does not involve \( f \) and is not a constant. Let \( \{b_i\} \) be an infinite sequence of distinct elements in \( C \) which includes every \( b \in C \) for which, \( \delta(b, f) > 0. \) Let \( q \) be a positive integer. Then by Lemma 5.2.4,

\[
\sum_{i=1}^{q} m(r, b_i, f) + N \left( r, \frac{1}{P} \right) \leq T(r, P) + S(r, f),
\]

Adding \( m \left( r, \frac{1}{P} \right) \) on both sides of above inequality and using

\[ T \left( r, \frac{1}{P} \right) = T(r, P) + O(1), \]

we get

\[
\sum_{i=1}^{q} m(r, b_i, f) + N \left( r, \frac{1}{P} \right) + m \left( r, \frac{1}{P} \right) \leq T(r, P) + m \left( r, \frac{1}{P} \right) + S(r, f),
\]

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\begin{align*}
n \sum_{i=1}^{q} m(r, b_i, f) + T(r, \frac{1}{p}) &\leq T(r, P) + m\left(r, \frac{1}{p}\right) + S(r, f), \\
n \sum_{i=1}^{q} m(r, b_i, f) + T(r, P) &\leq T(r, P) + m\left(r, \frac{1}{p}\right) + S(r, f).
\end{align*}

Hence
\begin{align*}
\lim_{r \to \infty} \frac{n \sum_{i=1}^{q} m(r, b_i, f)}{T(r, f)} &\leq \lim_{r \to \infty} \frac{m\left(r, \frac{1}{p}\right)}{T(r, f)}, \\
\lim_{r \to \infty} \frac{n \sum_{i=1}^{q} m(r, b_i, f)}{T(r, f)} &\leq \lim_{r \to \infty} \frac{m\left(r, \frac{1}{p}\right)}{T(r, P)} \lim_{r \to \infty} \frac{T(r, P)}{T(r, f)},
\end{align*}

and
\begin{align*}
\lim_{r \to \infty} \frac{n \sum_{i=1}^{q} m(r, b_i, f)}{T(r, f)} &\leq \lim_{r \to \infty} \frac{m\left(r, \frac{1}{p}\right)}{T(r, P)} \lim_{r \to \infty} \frac{T(r, P)}{T(r, f)},
\end{align*}

i.e.,
\begin{align*}
\lim_{r \to \infty} \frac{n \sum_{i=1}^{q} m(r, b_i, f)}{T(r, f)} &\leq \delta(0, P) \lim_{r \to \infty} \frac{T(r, P)}{T(r, f)},
\end{align*}

and
\begin{align*}
\lim_{r \to \infty} \frac{n \sum_{i=1}^{q} m(r, b_i, f)}{T(r, f)} &\leq \Delta(0, P) \lim_{r \to \infty} \frac{T(r, P)}{T(r, f)}.
\end{align*}

i.e.,
\begin{equation}
na \leq \delta(0, P) \lim_{r \to \infty} \frac{T(r, P)}{T(r, f)}, \tag{5.3.11}
\end{equation}

and
\begin{equation}
na \leq \Delta(0, P) \lim_{r \to \infty} \frac{T(r, P)}{T(r, f)}. \tag{5.3.12}
\end{equation}

From (5.3.10) and (5.3.12), we get
\begin{equation}
na \leq \Delta(0, P) \lim_{r \to \infty} \frac{T(r, P)}{T(r, f)} \leq \Delta(0, P) \lim_{r \to \infty} \frac{T(r, P)}{T(r, f)} \leq \Delta(0, P)n \{1 + m(\alpha - 1)\} \tag{5.3.13}
\end{equation}
Case (i) If \( \alpha = 1 \), then (5.3.13) becomes
\[
- \Delta(0, P) \leq \frac{T(r, P)}{T(r, f)} \leq \Delta(0, P) \frac{T(r, P)}{T(r, f)} \leq \Delta(0, P)n,
\]
Therefore
\[
\lim_{r \to \infty} \frac{T(r, P)}{T(r, f)} = n
\]
with \( \Delta(0, P) = 1 \).

Case (ii) If \( \alpha = 2 \), and \( m = 1 \), then (5.3.13) becomes
\[
2n \leq \Delta(0, P) \frac{T(r, P)}{T(r, f)} \leq \Delta(0, P) \frac{T(r, P)}{T(r, f)} \leq \Delta(0, P)2n,
\]
Therefore
\[
\lim_{r \to \infty} \frac{T(r, P)}{T(r, f)} = 2n
\]
with \( \Delta(0, P) = 1 \).

Corollary 5.3.1 If \( f \) is a meromorphic function of finite order with
\[
\sum_{b \in C} \delta(b, f) = \alpha, \quad \Theta(\infty, f) = 2 - \alpha
\]
and if \( P \) is a homogeneous differential polynomial in \( f \) of degree \( n \) and not involving \( f \), then
\[
\delta(0, P) = \Delta(0, P) = 1, \quad \text{(5.3.14)}
\]
and
\[
T(r, P) \sim n\alpha T(r, f), \quad \text{(5.3.15)}
\]
provided that \( P \) does not reduce to a constant.

In particular (5.3.14) and (5.3.15) hold if \( f \) is an entire function and
\[
\sum_{b \in C} \delta(b, f) = \alpha.
\]
Corollary 5.3.2 If $f$ is a meromorphic function of finite order with $\Theta(\infty, f) = 2 - \alpha$ and if $P$ is a homogeneous differential polynomial in $f$ and not involving $f$, then
\[
\sum_{b \in C} \delta(b, f) \leq \delta(0, P),
\]
provided that $P$ is not a constant.

If $f$ is an entire function then (5.3.16) holds since $\Theta(\infty, f) = 2 - \alpha$.

We have proved Theorem 5.3.1 (b) when $P$ is a homogeneous differential polynomial in $f$ and $P$ does not involve $f$. If $P$ involves $f$, then Subhas S Bhoosnurmath [29] proved the following Theorem:

**Theorem 5.3.E** [29] Let $f$ be a meromorphic function of finite order and $P$ a homogeneous differential polynomial in $f$ of degree $n$. Let
\[
N(r, f) + N(r, \frac{1}{f}) = \lim_{r \to \infty} \frac{T(r, f)}{r^{n(1 - m\beta)}},
\]
then
\[
n(1 - m\beta) \leq \lim_{r \to \infty} \frac{T(r, P)}{T(r, f)} \leq \lim_{r \to \infty} \frac{T(r, P)}{T(r, f)} \leq n(1 + m\beta),
\]
where $f^{(m)}$ is the highest derivative of $f$ occurring in $P$, provided that $P$ does not reduce to a constant.

For $a \in C$, we define, with Milloux
\[
\delta^P_r(a, f) = 1 - \lim_{r \to \infty} \frac{N(r, a, P)}{T(r, f)}
\]
where $P$ is defined as before, a differential polynomial in $f$ (Milloux considered only the special case $P = f'$). $\delta^P_r(a, f)$ is called the relative defect of $a$ for $P$ with respect to $f$.

We investigate the conditions under which the defect and relative defects are equal. In this connection, we prove the following theorem.

**Theorem 5.3.2** Let $f$ be a meromorphic function of finite order,
(a) If $P$ is a homogeneous differential polynomial in $f$ of degree $n$, then, for $a \in \mathbb{C}$,

$$1 - \delta_f^P(a, f) \leq (1 - \delta(a, P)) n \{1 + m(a - 1)\}$$  \hspace{1cm} (5.3.18)

$$1 - \delta_f^P(a, f) \leq (1 - \delta(a, P)) n \{1 + m\beta\}$$  \hspace{1cm} (5.3.19)

and

$$1 - \delta_f^P(a, f) \geq (1 - \delta(a, P)) n \{1 - m\beta\}$$  \hspace{1cm} (5.3.20)

where $\beta$ is as in Theorem 5.3.E and

$$\sum_{b \in \mathbb{C}} \delta(b, f) = \alpha.$$

(b) If $P$ is a homogeneous differential polynomial in $f$ of degree $n$, not involving $f$, then

$$\Delta(0, P) (1 - \delta_f^P(a, f)) \geq (1 - \delta(a, P)) n \alpha,$$

for $a \in \mathbb{C}$, provided $P$ does not reduce to a constant.

Proof of Theorem 5.3.2

(a) Now, by the definition of relative defect and by Theorem 5.3.1, we have

$$1 - \delta_f^P(a, f) = \lim_{r \to \infty} \frac{N(r, a, P)}{T(r, f)},$$

$$1 - \delta_f^P(a, f) \leq \lim_{r \to \infty} \frac{N(r, a, P)}{T(r, P)} \lim_{r \to \infty} \frac{T(r, P)}{T(r, f)},$$

$$1 - \delta_f^P(a, f) \leq (1 - \delta(a, P)) n \{m + 1 - m (2 - \alpha)\},$$

$$1 - \delta_f^P(a, f) \leq (1 - \delta(a, P)) n \{m + 1 - 2m + ma\},$$

$$1 - \delta_f^P(a, f) \leq (1 - \delta(a, P)) n \{1 - m + ma\}.$$

Therefore,

$$1 - \delta_f^P(a, f) \leq (1 - \delta(a, P)) n \{1 + m(\alpha - 1)\}.$$
Thus (5.3.18) is proved.
Similarly by using (5.3.17), we can prove (5.3.19) and (5.3.20).

(b) 
\[
\Delta(0, P) (1 - \delta^P(a, f)) = \lim_{r \to \infty} \frac{N(r, a, P)}{T(r, f)} \Delta(0, P),
\]
\[
\Delta(0, P) (1 - \delta^P(a, f)) \geq \lim_{r \to \infty} \frac{N(r, a, P)}{T(r, P)} \lim_{r \to \infty} \frac{T(r, P)}{T(r, f)} \Delta(0, P),
\]

by the definition of $\delta(a, P)$ and Theorem 5.3.1 (b), we get,
\[
\Delta(0, P) (1 - \delta^P(a, f)) \geq (1 - \delta(a, P)) n\alpha.
\]

Thus (b) is proved.

We then have the following corollaries.

Corollary 5.3.3 If $f$ is a meromorphic function of finite order with
\[
\sum_{b \in C} \delta(b, f) = \alpha,
\]
$m = 1, \Theta(\infty, f) = 2 - \alpha$ and if $P$ is a homogeneous differential polynomial in $f$ of degree
$n$ and not involving $f$, then
\[
(1 - \delta^P(a, f)) = (1 - \delta(a, P)) n\alpha,
\]
for $a \in \mathbb{C}$, provided that $P$ does not reduces to a constant.

Proof of Corollary 5.3.3

Since
\[
\sum_{b \in C} \delta(b, f) = \alpha,
\]
and $\Theta(\infty, f) = 2 - \alpha$, by Corollary 5.3.1, $\Delta(0, P) = 1$.

From (5.3.18) and (5.3.21) respectively, we get
\[
(1 - \delta^P(a, f)) \leq (1 - \delta(a, P)) n\alpha, \quad (5.3.22)
\]
and

\[(1 - \delta^p_t(a, f)) \geq (1 - \delta(a, P)) n \alpha. \quad (5.3.23)\]

Therefore from (5.3.22) and (5.3.23) we get,

\[(1 - \delta^p_t(a, f)) = (1 - \delta(a, P)) n \alpha,\]

**Remark 5.3.2** If in the above Corollary 5.3.3, the degree of \( P = 1 \), and \( \alpha = 1 \), then

\[\delta^p_t(a, f) = \delta(a, P)\]

Again from (5.3.19) and (5.3.20), we obtain the following

**Corollary 5.3.4** If \( f \) is a meromorphic function of finite order with

\[\Theta(\infty, f) = \Theta(0, f) = 2 - \alpha\]

and if \( P \) is a homogeneous differential polynomial in \( f \) of degree \( n \), then

\[(1 - \delta^p_t(a, f)) \geq n (1 - \delta(a, P)),\]

for \( a \in \mathbb{C} \), provided that \( P \) does not reduce to a constant.

In particular, if \( n = 1 \), then

\[\delta^p_t(a, f) = \delta(a, P).\]