PREFACE

Topology is that branch of Mathematics, the purpose of which is to elucidate and investigate ideas of continuity and nearness within the framework of Mathematics. The study of topological spaces, their continuous mappings and general properties makes up one branch of topology known as 'general topology'. This thesis is an elaborate study of a new type of generalized closed sets in a topological space called $\omega\alpha$-closed sets, their respective continuous maps, closed maps, homeomorphisms, compactness, regular spaces, normal spaces and their extension to bitopological spaces.

The present thesis contains six chapters and mainly deals with weaker forms of closed sets namely $\omega\alpha$-closed sets and their related concepts in topological and bitopological spaces.

Njastad introduced the concept of $\alpha$-set in 1965. Levine, generalized the concept of closed sets to generalized closed (briefly g-closed) sets. Since then, a lot of work has been done using these notions and many interesting results have been obtained. Maki, Devi and Balachandran introduced and investigated the notions of generalized $\alpha$-closed and $\alpha$-generalized closed sets (briefly $g\alpha$-closed and $\alpha g$-closed respectively) which are generalizations of closed or $\alpha$-closed sets and introduced new separation axioms. Recently Sundaram and Sheik John introduced and studied new class of closed sets called $\omega$-closed sets through semi open sets.

This idea of Levine motivated us to generalize the concept of closed sets in topological spaces to the concept of $\omega\alpha$-closed sets using $\omega$-open sets.
The first chapter of this thesis deals with the $\omega\alpha$-closed sets, $\omega\alpha$-open sets, $\omega\alpha$-closure, $\omega\alpha$-interior, $\omega\alpha$-neighbourhoods, $\omega\alpha$-limit points, $\omega\alpha$-spaces and related concepts. Many new concepts have been introduced and studied in this chapter.

A subset $A$ of $(X,\tau)$ is said to be $\omega\alpha$-closed set if $\text{acl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is $\omega$-open in $(X,\tau)$. It is proved that every closed (resp. $\alpha$-closed) set is $\omega\alpha$-closed. We have proved that the class of $\omega\alpha$-closed sets is properly placed between the class of $\alpha$-closed sets and the class of $\alpha\gamma$-closed sets. It is observed that $\omega\alpha$-closed sets are independent of $g$-closed, $\omega$-closed and $g\alpha$-closed sets. It is proved that if a subset $A$ of $X$ is $\omega\alpha$-closed in $(X,\tau)$, then $\text{acl}(A) - A$ does not contain any non-empty $\omega$-closed set in $(X,\tau)$ but not conversely. The complement of $\omega\alpha$-closed set is called $\omega\alpha$-open set and some of their properties are studied. It is proved that a subset $A$ of $(X,\tau)$ is $\omega\alpha$-open if and only if $F \subseteq \text{aint}(A)$ whenever $F$ is $\omega$-closed and $F \subseteq A$. We introduce $\omega\alpha$-closure and $\omega\alpha$-interior of a subset $A$ of a topological space $(X, \tau)$ using $\omega\alpha$-closed sets which are denoted by $\omega\alpha\text{cl}(A)$ and $\omega\alpha\text{int}(A)$ respectively. Among many other results, it is observed that a subset $A$ of $(X, \tau)$ is $\omega\alpha$-closed if $\omega\alpha\text{cl}(A) = A$ but not conversely and $\omega\alpha$-closure is a Kuratowski's closure operator on $X$. We have proved that for any subset $A$ of $(X, \tau)$, the complement of $\omega\alpha$-interior of $A$ is the $\omega\alpha$-closure of the complement of $A$. We have shown that, if $A$ is a $\omega\alpha$-closed in $X$ and $x \in X - A$, then there exists a $\omega\alpha$-neighbourhood $N$ of $x$ such that $N \cap A = \emptyset$ and if $A$ is $\omega\alpha$-closed, then $\omega\alpha\text{cl}(A) \subseteq A$. Also we introduce four new spaces of $\omega\alpha$-closed sets namely, $T_{\omega\alpha}$-spaces, $\omega\alpha T$-
spaces, $\omega \alpha T_{ag}$ -spaces and $\omega \alpha T_{stg}$ -spaces and investigate some of their properties. Among other results, it is observed that, if a space $(X, \tau)$ is $\omega \alpha T_{stg}$ -space, then every singleton of $(X, \tau)$ is closed or strongly $g$-open but not conversely. Also we show that every $T_{1/2}$-space is $\omega \alpha T_{ag}$ -space.

In the second chapter, $\omega \alpha$-continuous, pre-$\omega \alpha$-continuous, $\omega \alpha$-irresolute, $\omega \alpha$-closed, pre-$\omega \alpha$-closed and some stronger forms of continuous maps namely strongly $\omega \alpha$-continuous, completely $\omega \alpha$-continuous, perfectly $\omega \alpha$-continuous and $\omega \alpha$-homeomorphisms and related concepts in topological spaces have been introduced and studied.

A map $f: (X, \tau) \to (Y, \mu)$ is called $\omega \alpha$-continuous if the inverse image of every closed set in $(Y, \mu)$ is $\omega \alpha$-closed set in $(X,\tau)$. Among many other results, it is observed that every continuous (resp. $\alpha$-continuous) map is $\omega \alpha$-continuous map and every $\omega \alpha$-continuous map is $\alpha g$-continuous map but not conversely. Characterizations of $\omega \alpha$-continuous maps are obtained. It is observed that the composition of $\omega \alpha$-continuous maps need not be a $\omega \alpha$-continuous map.

We introduce the notions of pre-$\omega \alpha$-continuous maps and proved that every pre-$\omega \alpha$-continuous map is $\omega \alpha$-continuous map but not conversely. It is observed that $\omega \alpha$-irresolute maps and irresolute maps are independent and the characterizations of $\omega \alpha$-irresolute maps are obtained. It is proved that the class of all pre-$\omega \alpha$-continuous maps lies between the class of all $\omega \alpha$-irresolute maps and the class of all $\omega \alpha$-continuous maps. It is shown that, if $f: (X, \tau) \to (Y, \mu)$ is bijective, $\omega^*\alpha$-open, pre-$\omega \alpha$-continuous, then $f$ is $\omega \alpha$-
irresolute, and a map \( f: (X,\tau) \rightarrow (Y,\mu) \) is \( \omega\alpha \)-closed if and only if for each subset \( S \) of \( (Y,\mu) \) and for each open set \( U \) containing \( f^{-1}(S) \) there is a \( \omega\alpha \)-open set \( V \) of \( (Y,\mu) \) such that \( S \subseteq V \) and \( f^{-1}(V) \subseteq U \). Also if a map \( f: (X,\tau) \rightarrow (Y,\mu) \) is bijective, \( \omega \)-irresolute and pre-\( \omega\alpha \)-closed and \( A \) is \( \omega\alpha \)-closed set of \( (X,\tau) \), then \( f(A) \) is \( \omega\alpha \)-closed set in \( (Y,\mu) \). It is observed that the composition of \( \omega\alpha \)-closed maps need not be a \( \omega\alpha \)-closed map. It is proved that every strongly \( \omega\alpha \)-continuous map is a \( \omega\alpha \)-continuous map and every perfectly \( \omega\alpha \)-continuous map is strongly \( \omega\alpha \)-continuous. Further a map \( f: (X,\tau) \rightarrow (Y,\mu) \) is strongly \( \omega\alpha \)-continuous if and only if for each \( x \in X \) and each \( \omega\alpha \)-open set \( V \) of \( (Y,\mu) \) containing \( f(x) \), there exists an open set \( U \) of \( (X,\tau) \) containing \( x \) such that \( f(U) \subseteq V \). It is proved that every homeomorphism is \( \omega\alpha \)-homeomorphism and thus it is \( \alpha\gamma \)-homeomorphism but not conversely and \( \omega\alpha \)-homeomorphisms are independent of \( g \)-homeomorphisms and \( \omega \)-homeomorphisms. It is shown that every \( \omega\alpha \)-homeomorphism is \( \omega\alpha \)-homeomorphism and every \( \omega\alpha \)-homeomorphism is \( \alpha\gamma \)-homeomorphism but not conversely.

The third chapter gives the notions of \( \omega\alpha \)-compactness and \( \omega\alpha \)-connectedness and some of their properties are obtained.

A topological space \( (X,\tau) \) is called \( \omega\alpha \)-compact if every \( \omega\alpha \)-open cover of \( (X,\tau) \) has a finite subcover. Among many other results, we observe that, every \( \omega\alpha \)-compact space is compact (resp. countably \( \omega\alpha \)-compact and \( \omega\alpha \)-Lindelöf) and a \( \omega\alpha \)-closed subset of a \( \omega\alpha \)-compact space \( (X,\tau) \) is \( \omega\alpha \)-compact relative to \( (X,\tau) \). We prove that the image of a \( \omega\alpha \)-compact space
under a strongly $\omega\alpha$-continuous map is $\omega\alpha$-compact and the image of a countably $\omega\alpha$-compact space under $\omega\alpha$-irresolute map is countably $\omega\alpha$-compact. We prove that if a space $(X, \tau)$ is both $\omega\alpha$-Lindelöf and countably $\omega\alpha$-compact then $(X, \tau)$ is $\omega\alpha$-compact and the image of a $\omega\alpha$-Lindelöf space under the strongly $\omega\alpha$-continuous map is $\omega\alpha$-Lindelöf.

Moreover we observe that every $\omega\alpha$-connected space is connected but not conversely and the image of a $\omega\alpha$-connected space under the $\omega\alpha$-continuous surjection is connected.

In the fourth chapter, the concepts of $\omega\alpha$-regular spaces, $\omega\alpha$-normal spaces and some separation axioms namely, $\omega\alpha_0$-spaces, $\omega\alpha_1$-spaces and $\omega\alpha_2$-spaces have been introduced and studied and some of their properties are obtained.

We observe that every $\omega\alpha$-regular space is regular and the image of a $\omega\alpha$-regular space is $\omega\alpha$-regular under open, $\omega\alpha$-irresolute bijective map. It is proved that every $\omega\alpha$-normal space is normal and a $\omega\alpha$-closed subspace of a $\omega\alpha$-normal space is $\omega\alpha$-normal. It is shown that the image of a $\omega\alpha$-normal space is $\omega\alpha$-normal under $\omega^*$-open, pre-$\omega\alpha$-continuous bijective open map. It is shown that, if $f: (X, \tau) \to (Y, \mu)$ is weakly continuous, $\omega\alpha$-closed injection and $(Y, \mu)$ is $\omega\alpha$-normal, then $(X, \tau)$ is normal and obtain some characterizations. Moreover we prove that every subspace of a $\omega\alpha_0$-space is $\omega\alpha_0$-space and every $\omega\alpha_1$-space is $\omega\alpha_0$-space. We also obtain some characterizations.
In the fifth chapter, we introduce and study new weaker forms of locally closed sets (briefly lc-sets) called $\omega\alpha\text{lc}$-sets, $\omega\alpha\text{le}^*$-sets and $\omega\alpha\text{le}^{**}$-sets, and $\omega\alpha\text{LC}$-continuous, $\omega\alpha\text{LC}^*$-continuous and $\omega\alpha\text{LC}^{**}$-continuous maps. Among many other results, it is observed that every lc-set is $\omega\alpha\text{lc}$-set (resp. $\omega\alpha\text{le}^*$-sets and $\omega\alpha\text{le}^{**}$-sets). We introduce the notion of $\omega\alpha$-submaximal and we prove that, a space $(X, \tau)$ is $\omega\alpha$-submaximal if and only if $P(X) = \omega\alpha\text{LC}^*(X, \tau)$.

We prove that every LC-continuous map is $\omega\alpha\text{LC}$-continuous, $\omega\alpha\text{LC}^*$-continuous and $\omega\alpha\text{LC}^{**}$-continuous but not conversely. It is observed that the composition of two $\omega\alpha\text{LC}$-continuous (resp. $\omega\alpha\text{LC}^*$-continuous and $\omega\alpha\text{LC}^{**}$-continuous) maps need not be $\omega\alpha\text{LC}$-continuous (resp. $\omega\alpha\text{LC}^*$-continuous and $\omega\alpha\text{LC}^{**}$-continuous) map and obtain some of their properties. It is proved that $\omega\alpha\text{LC}$-irresolute (resp. $\omega\alpha\text{LC}^*$-irresolute, $\omega\alpha\text{LC}^{**}$-irresolute) map is $\omega\alpha\text{LC}$-continuous (resp. $\omega\alpha\text{LC}^*$-continuous, $\omega\alpha\text{LC}^{**}$-continuous) map and we observe that, any map defined on a door space is $\omega\alpha\text{LC}$-continuous (resp. $\omega\alpha\text{LC}$-irresolute).

In the sixth chapter, we introduce the concepts of $\omega\alpha$-closed sets, $\omega\alpha$-open sets, $\omega\alpha$-closure, $\omega\alpha$-continuous maps and $\omega\alpha$-irresolute maps in bitopological spaces and obtain some of their properties.

A subset $A$ of a bitopological space $(X, \tau_1, \tau_2)$ is called $(\tau_i, \tau_j)$-$\omega\alpha$-closed if $\tau_j\omega\text{cl}(A) \subseteq G$ whenever $A \subseteq G$ and $G$ is $\tau_i$-$\omega$-open set and the family of all $(\tau_i, \tau_j)$-$\omega\alpha$-closed sets in $(X, \tau, \tau_2)$ is denoted by $B(\tau_i, \tau_j)$. Among many other results, we observe that, for each element $x$ of $(X, \tau_1, \tau_2)$,
{x} is $\tau_i$-\omega-closed or $\{x\}^c$ is $(\tau_i, \tau_j)$-\omega-$\alpha$-closed and also prove that a subset A of $(X, \tau_1, \tau_2)$ is $(\tau_i, \tau_j)$-\omega-$\alpha$-open if and only if $F \subseteq \tau_j$-\alpha-int(A) whenever $F$ is $\tau_i$-\omega-closed and $F \subseteq A$, for $i, j \in \{1, 2\}$. We introduce the new space namely, $(\tau_i, \tau_j)$- $\tau_{k\alpha}$-space and it is shown that every singleton set of $X$ is $\tau_i$-\omega closed or $\tau_i$-open. Moreover we define, $(\tau_i, \tau_j)$- $\omega\alpha cl^*(E)$ and it is observed that, $E \subseteq (\tau_i, \tau_j)$- $\omega\alpha cl^*(E) \subseteq \tau_j$- $\omega\alpha cl(E)$ for any subset $E$ of $(X, \tau_1, \tau_2)$ and $\omega\alpha$-closure of $\omega\alpha$-closed sets is the Kuratowski’s closure operator.

A map $f: (X, \tau_1, \tau_2) \rightarrow (Y, \mu_1, \mu_2)$ is called $B(\tau_i, \tau_j)$- $\mu_k$- continuous ($\omega\alpha$-continuous) if the inverse image of every $\mu_k$-closed set in $(Y, \mu_1, \mu_2)$ is $(\tau_i, \tau_j)$- $\omega\alpha$-closed set in $(X, \tau_1, \tau_2)$. Among other results, we prove that, a map $f: (X, \tau_1, \tau_2) \rightarrow (Y, \mu_1, \mu_2)$ is $\tau_j$-$\mu_k$-continuous and $(X, \tau_1, \tau_2)$ is $(\tau_i, \tau_j)$- $\tau_{k\alpha}$-space if and only if $f$ is $B(\tau_i, \tau_j)$- $\mu_k$-continuous. Further it is observed that $B(\tau_i, \tau_j)$- $\mu_k$-continuous map is independent of $D(\tau_i, \tau_j)$- $\mu_k$- continuous, $C(\tau_i, \tau_j)$- $\mu_k$- continuous and $D(\tau_i, \tau_j)$- $\mu_k$- continuous maps. We prove that every pairwise $\omega\alpha$- irresolute map is $B(\tau_i, \tau_j)$- $\mu_e$-continuous but not conversely and if a map $f: (X, \tau_1, \tau_2) \rightarrow (Y, \mu_1, \mu_2)$ is bijective, $\tau_i$-$\mu_k$-\omega-open and $\tau_i$-$\mu_k$-pre $\omega\alpha$-continuous, then $f$ is pairwise $\omega\alpha$-irresolute. Moreover we introduce the strong forms of $B(\tau_i, \tau_j)$- $\mu_k$- continuous maps namely, $\omega\alpha$-bi-continuous and $\omega\alpha$-strongly- bi-continuous maps and study some of their properties.