Introduction

One of the most interesting problems in the area of Graph Theory is that of labeling of graphs. A labeling or valuation or numbering of a simple graph $G$ is an one-to-one mapping from its vertex set into a set of non-negative integers which induces an assignment of labels to the edges of $G$. Labeled graphs serve as useful models for a wide range of applications. The typical problem in this field is the conjecture of G. Ringel [50], which states that “All trees are graceful”. A large number of families of graphs have been numbered gracefully. (See [1], [3], [8], [15], [16], [17], [18], [19], [31], [35], [36], [37], [38], [45], [47], [55]).

Apart from the graceful labeling, several kinds of labelings have been defined and developed. Graham and Sloane [27] introduced harmonious labeling. Maheo [45] investigated the strongly graceful labeling, while P. J. Slater ([53], [54]) dealt with $k$-graceful labelings. T. Grace [26] identified the sequential labeling and Acharya and Hegde [6] introduced the arithmetic numbering. We come across $k$-sequential labeling ([9],[52]), sequentially additive numbering
INTRODUCTION

Numerous classes of graphs have been labeled with such labelings (see [2], [44], [46], [49], [52], [59]). Some of these are specializations and some are generalizations. For a detailed survey of labeling see Chapter I and for results in this field, see Section 1 of each chapter.

This thesis generalizes some of the results on certain classes of labeling of graphs. In this thesis we give more attention to sequential labeling and in particular to arithmetic numbering. Unless mentioned otherwise, we consider only simple, finite graphs with no isolated vertices. Unless otherwise specified all terminology and notations in Graph Theory are from [14] and [28]. In Chapter I, a brief survey on labelings of graphs and some basic definitions and results needed in the sequel are given.

Chapter II deals with constructions of new (bigger) arithmetic graphs from a given arithmetic graph. In the first theorem we give a method of constructing bigger arithmetic trees from the given arithmetic tree. Let $T(n)$ be an arithmetic tree with $n$ vertices, $n \geq 3$, and having a vertex, say $v$ of zero valuation. We consider isomorphic copies of $T(n)$ and prove that, the tree obtained by adjoining a new vertex to the isomorphic image of $v$ in each isomorphic copy of $T(n)$ is also arithmetic.
We show that any tree of radius 2 with exactly one centre is arithmetic and also obtain certain conditions for a banana tree obtained from a family of stars, with a unique centre and radius 3 to be arithmetic. Finally, we prove that any tree $T$ with $p$ vertices and maximum degree greater than or equal to $p - 5$, is arithmetic.

Chapter III contributes Borne results on connected $(k, d)$-arithmetic graphs. First, we show that the join of the path $P_n$ with $K_t$ is sequential. Next, we prove $\gcd(k, d)$ divides each vertex valuation of a connected $(k, d)$-arithmetic graph, if 0 is a vertex valuation. On non-bipartite connected $(k, d)$-arithmetic graph property of $d$ in terms of $k$ and minimum vertex value of $G$ and finding an equivalent $(t, 1)$-arithmetic numbering for the same graph are brought out in a result. For a balanced bipartite graph, the criteria for existence of $(k, d)$-arithmetic numbering is given.

Further, we study the role of regularity and factorization on $(k, d)$-arithmetic graphs. On $r$-regular bipartite graphs and Hamiltonian $r$-regular graphs, conditions for non-existence of $(k, d)$-arithmetic graphs are given in two different cases. Finally, we prove that the union of two weakly arithmetic graphs, union of one arithmetic and one arithmetic bipartite graph are also
arithmetic.

In Chapter IV, we discuss the various labelings of the following classes of graphs.

(1) Sequential labeling of the double crown $C_n \odot K_2$.

(2) Sequential labeling of the cycle $C_{2n+1}$ with a chord joining 2 vertices at a distance 3.

(3) Graceful labeling of the prisms $P_n \times C_3$.

(4) Arithmetic numbering of prisms $P_n \times C_m$.

(5) Sequential labeling of the prisms $P_n \times C_m$.

(6) Sequential labeling of the ladder $L_n$.

(7) Arithmetic numbering of the ladder $L_n$.

(8) Arithmetic numbering of subdivision of the ladder $S(L_n)$.

Also, we study the nature of $k$ on $(k,1)$-arithmetic numbering of an odd cycle $C_{2n+1}$.

Chapter V is devoted to study of various labelings of the join of two complete graphs given below.

(1) Sequential labeling of $K_{1,n} + K_2$. 
(2) Simply sequentially additive numbering of $K_{1,n} + \overline{K}_t$.

(3) Sequential labeling of $K_{1,n} + \overline{K}_t$.

(4) Simply sequential labeling of $K_{1,n} + \overline{K}_2$.

Further, we provide a property of the star in a result. Finally, the complete bipartite graph $K_{a,b}$ is considered and we determine all possible arithmetic numbering of $K_{a,b}$, where $2 \leq \min (a, b)$. 