CHAPTER 6

Intersubband Absorption Linewidth in GaAs/AlGaAs Quantum Wells
due to Scattering by Confined LO Phonons and Interface Phonons
6.1 Introduction

Infrared absorption spectroscopy is a useful tool to study the electronic states in semiconductor space charge layers. Early intersubband transitions (ISBT) are observed in Si n-inversion layer [6.1-6.3] and in GaAs/AlGaAs QWs [6.4, 6.5]. These transitions are characterized by huge oscillator strength due to confinement of carriers and by small full width at half maximum (FWHM) of absorption spectra. ISBT are quantum well envelope state transitions in contrast to valence to conduction band transitions, which occur between Bloch states. ISBT are resonant in character corresponding to the transitions between two quantized states. Physics of ISBT in semiconductor nanostructures has attracted a great deal of interest largely because of their applications in quantum cascade lasers, quantum well infrared photo-detectors, modulators and non-linear optics [6.6,6.7]. In these devices, it has been demonstrated that, the microscopic physics process, first considered only of pure scientific interest, became critical issue of technology. The design of ISBT-based devices requires sophisticated theory and modeling capabilities involving bandstructure engineering and electron scattering by various processes. Electron scatterings are not only important for transport properties, they are also important for optical properties such as spectral line shapes, peak value of intersubband absorption and intersubband absorption line width.

In order to investigate the effects of various electron scattering processes, the intersubband absorption line widths have been measured as a function of temperature [6.8], well width, electron concentration, and alloy compositions [6.9], in a GaAs QW. It is found that linewidth has weak dependence of temperature and alloy composition. The observed strong well width dependence of the linewidth suggests dominance of interface roughness scattering. Linewidth is found to remain constant as carriers are removed from the quantum well (QW) apparently showing its little correlation with mobility.

Understanding of electron scattering processes will become important as it will affect detailed device design and optimization as the ISBT-based technology grows.
Apart from technological significance, the line shape and line width of any optical process have always been one of the central issues of fundamental importance. The intersubband absorption line width (ISBALW) in semiconductor quantum wells is related to the problems in the physics of optical transitions. ISBALW is an important factor in improving the performance of quantum cascade lasers and photo detectors.

Unama et al. [6.10, 6.11] have employed the Ando's theory [6.12] to calculate intersubband linewidth $2\Gamma_{\text{op}}$ in QWs due to scattering by interface roughness, LO phonons, LA phonons, alloy disorder and ionized impurities. This linewidth $2\Gamma_{\text{op}}$ is compared with the transport energy broadening $2\Gamma_{\text{tr}} = 2\hbar/\tau_{\text{tr}}$ where $\tau_{\text{tr}}$ is the transport relaxation time. The combination of the contributions with significantly different characteristics makes absolute values of $2\Gamma_{\text{op}}$ and transport broadening $2\Gamma_{\text{tr}}$ very different and leads to apparent lack of correlation between them. Interface roughness scattering contributes about an order of magnitude more to the linewidth $2\Gamma_{\text{op}}$ than to transport broadening $2\Gamma_{\text{tr}}$. LA phonon and alloy disorder scatterings make comparable contributions to the linewidth $2\Gamma_{\text{op}}$ and $2\Gamma_{\text{tr}}$. On the other hand LO phonon scattering and ionized impurity scattering are shown to contribute much less to the linewidth $2\Gamma_{\text{op}}$ than to the transport broadening $2\Gamma_{\text{tr}}$.

Unama et al. [6.11] have used the bulk LO phonon model for the calculations of linewidth. However, as discussed in Chapter 1 the LO phonons in GaAs/AlGaAs QWs are of confined nature. The widely used LO phonon confinement models are improved Huang and Zhu (IHZ) model [6.13, 6.14], Huang and Zhu (HZ) model [6.15] and Slab mode model [6.16, 6.17]. The electron interaction with confined LO phonons is discussed in Chapter 1. The confinement leads to the decrease of the scattering rate [6.18, 6.19]. Since at room temperature LO phonon scattering is the dominant scattering mechanism limiting the mobility, the confinement nature of LO phonons need to be taken while calculating $2\Gamma_{\text{op}}$ also. In the present work we study the effect of LO phonon
confinement on $2\Gamma_{op}$ and compare it with its value due to bulk LO phonons and transport energy broadening $2\Gamma_{tr}$. We also study the effect of interface (IF) phonons on the linewidth.

### 6.2 Theory for linewidth

We consider the case that only electrons occupy the ground subband with the photon energy $\hbar \Omega$ close to $E_{10}$, the energy difference of the ground $E_0$ and the first excited $E_1$ subbands. A general theory of intersubband absorption linewidth was formulated by Ando [6.20]. Full width at half maximum of the spectrum $2\Gamma_{op}(E)$ is given by [6.20]

$$
\Gamma_{op}(E) = \frac{1}{2}[\Gamma_{\text{intra}}(E) + \Gamma_{\text{inter}}(E)]
$$

(6.2.1)

where $\Gamma_{\text{intra}}(E)$ is the width due to the difference in intrasubband scattering matrix elements for the ground and the first excited subbands and $\Gamma_{\text{inter}}(E)$ is the width due to the intersubband scattering. They are given by [6.11,6.12]

$$
\Gamma_{\text{intra}}(E) = 2\pi \sum_{k'} \left[ \left\langle [0k']|H_1|0k\rangle - [1k']|H_1|1k\right\rangle^2 \right] \delta \left[ E(k) - E(k') \pm \hbar \omega_{\text{LO}} \right]
$$

(6.2.2)

and

$$
\Gamma_{\text{inter}}(E) = 2\pi \sum_{k'} \left[ \left\langle [0k']|H_1|k\right\rangle^2 \right] \delta \left[ E(k) - E(k') + E_{10} \pm \hbar \omega_{\text{LO}} \right]
$$

(6.2.3)

where $\pm$ indicates phonon absorption (+) and emission(-), $\left\langle \ell k \right|$ is the state vector of the electron with subband index $\ell$ and wave vector $k$, $E(k)$ is the electron energy, $H_1$ is the scattering potential and $<>$ denotes the average over distribution of scatterers. This theory assumes a parabolic conduction band or a constant effective mass for different subbands.
The transport relaxation rate $2\Gamma_{tr}(E)$ is expressed

$$
2\Gamma_{tr}(E) = 4\pi \sum_{k'} \left[ \delta \left(E(k) - E(k') \pm \hbar \omega_{LO} \right) \right] \left( |(0k'|H_{i}|0k)^{2} \right)
$$

(6.2.4)

### 6.2.1 Linewidth due to bulk LO phonons

The intrasubband relaxation rate for LO phonon scattering is obtained by substituting the scattering matrix element for bulk phonons, equation (1.6.4.2) in equation (6.2.2) and simplifying it by using the definition of form factor $F_{(k\ell)(mn)}(q)$, which is given as [6.12]

$$
F_{(k\ell)(mn)}(q) = \int dz \int dz' \chi_{k}(z) \chi_{\ell}(z) \chi_{m}(z') \chi_{n}(z') e^{-q|z-z'|}
$$

(6.2.1.1)

we get the intrasubband relaxation rate

$$
\Gamma_{int}(E) = \frac{m^{*}e^{2}\hbar \omega_{LO}}{\hbar e'} \int_{0}^{\pi} \frac{N_{LO}}{q_{a}} \left[ F_{00(00)}(q_{a}) + F_{11(11)}(q_{a}) - 2F_{00(11)}(q_{a}) \right] d\theta
$$

(6.2.1.2)

where $\Theta(E - \hbar \omega_{LO})$ is the Heaviside step function defined by

$$
\Theta(E - \hbar \omega_{LO}) = \begin{cases} 
1 & \text{if } E > \hbar \omega_{LO} \\
0 & \text{if } E < \hbar \omega_{LO}
\end{cases}
$$

(6.2.1.3)

and the scattering vectors $q_{e}$ and $q_{a}$ for emission and absorption, respectively, are given by

$$
q_{a}^{2} = \left[ 2k^{2} + \frac{2m^{*} \omega_{LO}}{\hbar} - 2k \sqrt{k^{2} + \frac{2m^{*} \omega_{LO}}{\hbar} \cos \theta} \right]
$$

(6.2.1.4)
and

\[ q_e^2 = \left[ 2k^2 - \frac{2m^* \omega_{LO}}{\hbar} - 2k \sqrt{k^2 - \frac{2m^* \omega_{LO}}{\hbar} \cos \theta} \right] \] (6.2.1.5)

The subscripts 'a' and 'e' represent absorption and emission of LO phonons.

Intersubband relaxation rate for bulk phonons is obtained by substituting equation (1.6.4.2) in equation (6.2.3),

\[ \Gamma_{\text{inter}}(E) = \frac{m^* e^2 \omega_{LO}}{\hbar c^2} \pi \left\{ \frac{N_{LO}}{q_a} \left[ F_{0\ell(10)}(q_a) \right]^{\dagger} \right\} d\theta \]

(6.2.1.6)

and the scattering vectors \( \tilde{q}_e \) and \( \tilde{q}_a \) for emission and absorption, respectively, are

\[ \tilde{q}_a^2 = \left[ 2k^2 + \frac{2m^* \omega_{LO}}{\hbar} + \frac{2m^* E_{10}}{\hbar^2} - 2k \sqrt{k^2 + \frac{2m^* \omega_{LO}}{\hbar} + \frac{2m^* E_{10}}{\hbar^2} \cos \theta} \right] \] (6.2.1.7)

and

\[ \tilde{q}_e^2 = \left[ 2k^2 - \frac{2m^* \omega_{LO}}{\hbar} + \frac{2m^* E_{10}}{\hbar^2} - 2k \sqrt{k^2 - \frac{2m^* \omega_{LO}}{\hbar} + \frac{2m^* E_{10}}{\hbar^2} \cos \theta} \right]. \] (6.2.1.8)

The linewidth is obtained by substituting \( \Gamma_{\text{intra}}(E) \) and \( \Gamma_{\text{inter}}(E) \) from equations (6.2.1.2) and (6.2.1.6), respectively, in equation (6.2.1)

The average linewidth is given by [6.12]

\[ \Gamma_{\text{op}} = \frac{\int_0^\infty \Gamma_{\text{op}}(E) f(E) dE}{\int_0^\infty f(E) dE} \] (6.2.1.9)

The transport relaxation rate \( 2 \Gamma_c(E) \) is obtained using equation (1.6.4.2) in equation (6.2.4). It is found to be
2\Gamma_{tr}(E) = \frac{2m^*e^2\omega_{LO}}{\hbar e'} \frac{1}{1 - f(E)} \sum_{l_1} \int \left\{ \frac{N_{LO}}{q_a} \left[ f_{00}(q_a) \right] \left[ 1 - f(E + \hbar \omega_{LO}) \right] + \right\} \Theta(E - \hbar \omega_{LO}) \times \left\{ \left[ 1 - f(E - \hbar \omega_{LO}) \right] \frac{(N_{LO} + 1)}{q_e} \left[ f_{00}(q_e) \right] \right\} \delta \left[ E_{1}(k) - E_{0}(k') + E_{10} + \hbar \omega_{LO} \right]

\text{(6.2.1.10)}

### 6.2.2 Linewidth due to confined LO phonons.

Using the matrix element for electron-confined LO phonon interaction, equation (1.6.5.5) in equation (6.2.2) and simplifying we obtain the intrasubband scattering rate

\[ \Gamma_{\text{int}}(E) = \frac{8\pi^2 e^2 \hbar \omega_{LO} \sum_{k,n} |t_{\text{LO}}(k)|^2}{\hbar e'} \delta \left[ E_{1}(k) - E_{0}(k') + E_{10} + \hbar \omega_{LO} \right] \delta \left[ \epsilon_{1}(k) - \epsilon_{0}(k') - \hbar \omega_{LO} \right] \times \left[ \left( N_{LO} + 1 \right) \left| G_{01}^{\text{na}} \right|^2 + 2 \left| G_{00}^{\text{na}} \right|^2 \right] \]

\text{(6.2.2.2)}

The intersubband scattering rate is obtained, by substituting the matrix element for electron-confined LO phonon interaction, equation (1.6.5.5) in equation (6.2.3) and simplifying.

\[ \Gamma_{\text{int}}(E) = \frac{8\pi^2 e^2 \hbar \omega_{LO} \sum_{k,n} |t_{\text{LO}}(k)|^2}{\hbar e'} \delta \left[ E_{1}(k) - E_{0}(k') + E_{10} + \hbar \omega_{LO} \right] \times \left[ \left( N_{LO} + 1 \right) \left| G_{01}^{\text{na}} \right|^2 + 2 \left| G_{00}^{\text{na}} \right|^2 \right] \]

\text{(6.2.2.1)}

The transport relaxation rate for confined phonons is obtained by substituting the matrix element for electron-confined LO phonon interaction, equation (1.6.5.5), in equation (6.2.4) and simplification gives
\[ \Gamma_{\text{intra}}^{\text{c}}(E) = \frac{16\pi^2 e^2 \hbar \omega_{LO}}{\sqrt{\varepsilon}} \sum_{\mathbf{q}_n} \sum_{n, \alpha} |t_{\alpha n}(\mathbf{q}_n)|^2 \left\{ N_{LO} \left[ G_{00}^{\text{intra}}(1 - f(E + \hbar \omega_{LO})) \right] \delta \left[ \varepsilon_0(\mathbf{k}) - \varepsilon_0(\mathbf{k'}) + \hbar \omega_{LO} \right] + \right. \\
\left. \Theta(E - \hbar \omega_{LO}) \times \right\} \left[ N_{LO} + 1 \right] \left[ G_{00}^{\text{intra}}(1 - f(E - \hbar \omega_{LO})) \delta \left[ \varepsilon_0(\mathbf{k}) - \varepsilon_0(\mathbf{k'}) - \hbar \omega_{LO} \right] \right]
\]

(6.2.2.3)

Substituting for \( G_{00}^{\text{intra}} \), \( G_{11}^{\text{intra}} \), \( G_{01}^{\text{intra}} \) and \( |t_{\alpha n}(\mathbf{q}_n)|^2 \) from Chapter 1, for the respective models intrasubband relaxation rate, intersubband relaxation rate and transport relaxation rate are determined from the equations (6.2.2.1), (6.2.2.2) and (6.2.2.3) for improved Huang and Zhu model, Huang and Zhu model and slab modes.

Linewidth and average linewidth are calculated from equations (6.2.1) and (6.2.1.9), respectively.

**Improved Huang and Zhu (IHZ) model**

We obtain the intrasubband relaxation rate for IHZ model from equation (6.2.1), by substituting for \( |t_{\alpha n}(\mathbf{q}_n)|^2 \) from equation (1.6.5.33).

\[ \Gamma_{\text{intra}}^{\text{c}}(E) = \frac{4m^* e^2 \omega_{LO}}{\varepsilon \hbar L} \sum_{n} \pi \left\{ \left[ \frac{2}{L} \int_{-L/2}^{L/2} q_n^2 (\phi_{n-}(z))^2 + \left( \frac{d\phi_{n-}(z)}{dz} \right)^2 \right] \right\}^{-1} \times \Theta(E - \hbar \omega_{LO}) \left\{ \frac{2}{L} \int_{-L/2}^{L/2} q_n^2 (\phi_{n-}(z))^2 + \left( \frac{d\phi_{n-}(z)}{dz} \right)^2 \right\} \times \left[ N_{LO} + 1 \right] \left[ G_{00}^{\text{intra}}(q_e) + G_{11}^{\text{intra}}(q_e) \right] - 2 \left| G_{00}^{\text{intra}}(q_e) \right| \left| G_{11}^{\text{intra}}(q_e) \right| \right]

(6.2.2.4)
where the overlap integrals $|G_{00}^{n}(q)|^2$ and $|G_{11}^{n+1}(q)|^2$ are obtained from equations (1.6.5.35) and (1.6.5.36),

Substituting for $|t_{n+1}(q)|^2$ from equation (1.6.5.34) in equation (6.2.2.2) we obtain the intersubband relaxation rate for IHZ model

$$\Gamma_{\text{inter}}(E) = \frac{4m^*e^2\omega_{LO}}{eA L} \sum_{n} \pi \theta(E - E_n - \omega_{LO}) \times$$

$$\int_{-L/2}^{L/2} \left[ \left( \frac{2}{L} \int_{-L/2}^{L/2} q^2_n (\phi_n(z))^2 + \left( \frac{d\phi_n(z)}{dz} \right)^2 dz \right)^{-1} N_{LO} |G_{01}^{n}(q_a)|^2 + \right.$$}

$$\left. \left[ (N_{LO} + 1) |G_{01}^{n+1}(q_a)|^2 \right] \right]$$

(6.2.2.5)

where the overlap integral $|G_{01}^{n}(q)|^2$ is given by equation (1.6.5.37).

We obtain the transport relaxation rate for IHZ model by substituting for $|t_{n+1}(q)|^2$ from equation (1.6.5.33) in equation (6.2.2.3)

$$2\Gamma_T(E) = \frac{8m^*e^2\omega_{LO}}{hL} \left( \frac{1}{1 - f(E)} \right) \sum_{n} \pi \theta(E - E_n - \omega_{LO}) \times$$

$$\int_{-L/2}^{L/2} \left[ \left( \frac{2}{L} \int_{-L/2}^{L/2} q^2_n (\phi_n(z))^2 + \left( \frac{d\phi_n(z)}{dz} \right)^2 dz \right)^{-1} x \right.$$}

$$\left. x \left[ (N_{LO} + 1) |G_{00}^{n+1}(q_a)|^2 \right] \right]$$

(6.2.2.6)
and the overlap integral $|G_{00}(q)|^2$ is obtained from equation (1.6.5.35)

**Huang-Zhu model**

In this model even modes contribute for intrasubband transitions and odd modes contribute for intersubband transitions [6.21]. Odd modes start from $n=3$ as $n=1$ corresponds to the interface modes. We use equation (1.6.5.23) for $|t_{n-}(q)|^2$ in equation (6.2.2.1) and obtain the intrasubband scattering rate

$$
\Gamma_{\text{intra}}(E) = \frac{4m^*e^2\omega_{LO}}{e^*\hbar L} \sum_n \left\{ \Theta(E - \hbar \omega_{LO}) \times \right\}
$$

$$
\left[ \left( 3q_n^2 + \frac{\pi^2 n^2}{L^2} \right)^{-1} \left[ |G_{00}^{n=2}|^2 + |G_{11}^{n=2}|^2 - 2|G_{00}^{n=1}||G_{11}^{n=1}| \right] \right] \, d\theta
$$

(6.2.2.7)

where the overlap integrals $|G_{00}^{n=2}|$ and $|G_{11}^{n=2}|$ are given by equations (1.6.5.24) and (1.6.5.25).

We obtain intersubband scattering rate by substituting for $|t_{n+}(q)|^2$ from equation (1.6.5.22) in equation (6.2.2.2) and it is given by

$$
\Gamma_{\text{inter}}(E) = \frac{4m^*e^2\omega_{LO}}{e^*\hbar L} \sum_n \left\{ \Theta(E + E_{10} - \hbar \omega_{LO}) \times \right\}
$$

$$
\left[ \left( 1 + C_n \left( \frac{1}{6} - \mu_n^2 \pi^2 \right) \right) q_n^2 + \left( 3\mu_n^2 - 3 \right) q_n^2 \right] \left( N_{LO} |G_{01}^{n+1}|^2 \right) \, d\theta
$$

(6.2.2.8)

and the overlap integral $|G_{01}^{n+1}|^2$ is given by equation (1.6.5.26).

The transport relaxation rate is obtained by substituting for $|t_{n-}(q)|^2$ from equation (1.6.5.23) in equation (6.2.2.3). It is shown to be
\[ 2\Gamma_{tr}(E) = \frac{8m^*e^2\omega_{1LO}}{\hbar Ll} \sum_n \frac{1}{1 - f(E)} \sum_n \pi \left[ N_{LO} \left\{ 1 + f(E - \hbar \omega_{1LO}) \right\} \left( 3q_e^2 + \frac{\pi^2 n^2}{L^2} \right)^{-1} \right] G_{00}^{-2} \]

where the overlap integral \( |G_{00}^{-2}| \) is given by equation (1.6.5.24).

**Slab modes**

Odd modes in slab model contribute for intrasubband transitions whereas even modes towards intersubband transitions [6.18]. The intrasubband relaxation rate for slab modes is obtained by substituting for \( |t_{xx}(q)|^2 \) from equation (1.6.5.9) in equation (6.2.2.1) and it is shown to be

\[ \Gamma_{\text{intra}}(E) = \frac{4m^*e^2\omega_{1LO}}{\epsilon^{'AL}} \sum_n \left\{ N_{LO} \left( \frac{q_e^2}{L^2} + \frac{\pi^2 n^2}{L^2} \right)^{-1} \left[ G_{00}^{-2} + |G_{11}^{-2}|^2 - 2|G_{01}^{-2}|G_{11}^{-1} \right] + \right\} \]

and the overlap integrals \( |G_{00}^{-2}|^2 \) and \( |G_{11}^{-2}|^2 \) are obtained from equations (1.6.5.10) and (1.6.5.11).

The intersubband relaxation rate is obtained by substituting for \( |t_{xx}(q)|^2 \) from equation (1.6.5.9), in equation (6.2.2.2) and it is given by
The transport relaxation rate for slab model is obtained by substituting for 
\( |t_{\text{tr}}(q)|^2 \) from equation (1.6.5.9) in equation (6.2.2.3). We get

\[
2\Gamma_{\text{tr}}(E) = \frac{8m^*e^2\omega_{\text{LO}}}{\hbar L^2} \int_0^{\pi} \frac{d\theta}{(1-f(E))} \left\{ \left( \frac{q^2 + \pi^2 n^2}{L^2} \right)^{-1} \Theta(E + E_{\text{i}0} - \hbar\omega_{\text{LO}}) \right\}
\]

where the overlap integral \(|G_{01}^{n-1}|^2\) is given by equation (1.6.5.10).

The linewidth and average linewidth are given by the equations (6.2.1) and (6.2.1.9).

### 6.3 Linewidth due to Interface modes

The electron scattering by interface modes is discussed in section (1.6.6) [6.22]. Symmetric modes contribute for intrasubband (0-0) and (1-1) transitions and antisymmetric modes contribute for intersubband (0-1) transitions.

The scattering vectors for emission and absorption in case of symmetric modes are
Intrasubband scattering rate for interface modes is obtained from equation (6.2.2.1) as

\[
q_{s_{su}}^{2} = \left[ \frac{2k^2 - \frac{2m^* \omega_{LO}}{\hbar} - 2k \sqrt{k^2 + \frac{2m^* \omega_{LO}}{\hbar} \cos \theta}}{2} \right]
\]  

\[
q_{as_{su}}^{2} = \left[ \frac{2k^2 + \frac{2m^* \omega_{LO}}{\hbar} - 2k \sqrt{k^2 - \frac{2m^* \omega_{LO}}{\hbar} \cos \theta}}{2} \right]
\]  

Intersubband scattering rate is obtained from equation (6.2.3), and it is found to be

\[
\text{Intersubband scattering rate for interface modes is obtained from equation (6.2.2.1) as}
\]

\[
\frac{r_{\text{int}}^L}{r_{\text{int}}^L} = \frac{2m^* c^2}{\hbar^2} \left[ N_{as_{su}} \left| \psi_{\text{em}} \right| \psi_{\text{abs}} \right] \left[ \psi_{\text{em}} \right] \left[ \psi_{\text{abs}} \right] \left[ \frac{e^{-\frac{q_{s_{su}}}{\hbar}}}{1 - e^{-\frac{q_{s_{su}}}{\hbar}}} \right]
\]

\[
\text{where } \left| G_{\text{abs}}(q_{s_{su}}) \right|^2 \text{ and } \left| G_{\text{em}}(q_{s_{su}}) \right|^2 \text{ are obtained from equations (1.6.6.9) and (1.6.6.10)}
\]

and \( q_{as_{su}} \) and \( q_{s_{su}} \) are given by equations (6.3.1) and (6.3.2).

The scattering vectors for emission and absorption in case of antisymmetric modes are

\[
q_{as_{su}}^{2} = \left[ \frac{2k^2 + \frac{2m^* E_1}{\hbar^2} - \frac{2m^* \omega_{LO}}{\hbar} - 2k \sqrt{k^2 + \frac{2m^* E_1}{\hbar^2} - \frac{2m^* \omega_{LO}}{\hbar} \cos \theta}}{2} \right]
\]

\[
q_{s_{su}}^{2} = \left[ \frac{2k^2 + \frac{2m^* E_1}{\hbar^2} - \frac{2m^* \omega_{LO}}{\hbar} - 2k \sqrt{k^2 + \frac{2m^* E_1}{\hbar^2} - \frac{2m^* \omega_{LO}}{\hbar} \cos \theta}}{2} \right]
\]  

Intersubband scattering rate is obtained from equation (6.2.3), and it is found to be

\[
\]
where \( |G_{01}(\bar{q}_{\text{as}u})|^2 \) and \( |G_{01}(\bar{q}_{\text{equ}})|^2 \) are obtained from equations (1.6.6.11).

We obtain transport relaxation rate by substituting equation (1.6.6.9) in equation (6.2.4) as

\[
2\Gamma_{tr}(E) = \frac{4m^* e^2}{\hbar^3} \frac{1}{(1 - f(E))} \int_0^\pi \Theta(E - \hbar\omega_{\text{LO}}) \left[ \left( N_{\text{as}} + 1 \right) \frac{f_{\text{as}u}}{\omega_{\text{as}u}} \frac{1}{\bar{q}_{\text{as}u}} \left[ G_{00}(\bar{q}_{\text{as}u}) \right]^2 \frac{e^{-\bar{q}_{\text{as}u}L}}{1 - e^{-\bar{q}_{\text{as}u}L}} \right] d\theta \]

(6.3.7)

where \( |G_{00}(\bar{q}_{\text{equ}})|^2 \) is given by equation (1.6.6.9).

### 6.4 Results and discussion

The parameters used in our numerical calculations are for GaAs/AlGaAs QW. The GaAs effective conduction band mass \( m^* = 0.06m_0 \), the energy of a bulk GaAs LO phonon \( \hbar\omega_{\text{LO}} = 36.5\text{meV} \), the static and high frequency dielectric constants for GaAs \( \varepsilon_s = 12.91 \) and \( \varepsilon_c = 10.92 \), \( \hbar\omega_{\text{LO}1} = 36.25\text{meV} \), \( \hbar\omega_{\text{LO}2} = 34.45\text{meV} \), \( \hbar\omega_{\text{TO}1} = 33.29\text{meV} \) and \( \hbar\omega_{\text{TO}2} = 32.99\text{meV} \). We have used the value for the quantum well barrier height as 1eV [6.10].
Figure 6.1 shows $2\Gamma_{\text{opt}}(E)$, $\Gamma_{\text{intra}}(E)$, $\Gamma_{\text{inter}}(E)$ and $2\Gamma_{\text{tr}}(E)$ due to LO phonon scattering as function of in-plane kinetic energy $E$ for $n_e=5\times10^{11}$ cm$^{-2}$, $L=80\text{Å}$ and $T=300\text{K}$. Figures 6.1a-6.1d are, respectively, for bulk LO phonons, LO phonons in HZ model, LO phonons in IHZ model and LO phonons in slab mode model. In all the cases we observe that the intrasubband contribution $\Gamma_{\text{intra}}(E)$ depends strongly on the energy and becomes large for the lower subbands while the intersubband contribution $\Gamma_{\text{inter}}(E)$ is nearly independent of energy. Similar behavior is observed for bulk LO phonons [6.12]. When the kinetic energy $E$ is larger than the LO phonon energy $\hbar\omega_{\text{LO}}=36.5\text{meV}$ intrasubband LO phonon emission is allowed, which makes $2\Gamma_{\text{tr}}(E)$ and $\Gamma_{\text{intra}}(E)$ larger. For all the LO confinement models we observe that $2\Gamma_{\text{tr}}(E)$ is much larger than $\Gamma_{\text{inter}}(E)$ which is attributed to the small form factor. The magnitude of the difference is different for different confinement models. For e.g. at $E=40\text{ meV}$, $2\Gamma_{\text{tr}}(E)/\Gamma_{\text{inter}}(E)$ is about 12 for bulk LO phonons, 8 for HZ model, 5 for IHZ model and 7 for slab model. It is also found that $2\Gamma_{\text{tr}}(E)$ is always greater than $\Gamma_{\text{intra}}(E)$ for all the models. But the differences are smaller compared to the differences of $2\Gamma_{\text{tr}}(E)$ and $\Gamma_{\text{inter}}(E)$ for IHZ and HZ models and larger for slab modes and bulk phonons.

Effect of LO phonon confinement can be seen in all the models by comparing $2\Gamma_{\text{tr}}(E)$ and $2\Gamma_{\text{opt}}(E)$ of Figure 6.1a with those in Figures 6.1b-d. $2\Gamma_{\text{tr}}(E)$ due to bulk LO phonons is 2-3 times greater than those due to confined LO phonons. This is consistent with the scattering rate calculations. However, $2\Gamma_{\text{opt}}(E)$ due to bulk phonons is much smaller than its value due to confined LO phonons in HZ and IHZ models. $2\Gamma_{\text{opt}}(E)$ due to bulk LO phonons is found to be nearly same as $2\Gamma_{\text{opt}}(E)$ due to slab modes. These observations indicate that the same scattering mechanisms have different influences on linewidth and scattering rates leading to apparent lack of correlation between the two.
Figure 6.1: Energy broadening as a function of in-plane kinetic energy of electron
(a) due to bulk LO phonons, (b) due to confined LO phonons in H-Z model,
(c) due to confined LO phonons in IHZ model and (d) due to confined LO phonons in
slab mode model.
The absorption linewidth $2\Gamma_{op}$ and transport energy broadening $2\Gamma_{tr}$ are also calculated for GaAs/GaAlAs QW as a function of temperature and well width using the equation (6.2.1.9).

A. Temperature dependence

We performed the calculations of linewidth $2\Gamma_{op}$ and transport broadening $2\Gamma_{tr}$ in a GaAs/AlGaAs QW with a well width of $L=80\,\text{Å}$ and a sheet electron concentration of $n_s=9.8 \times 10^{11}\,\text{cm}^{-2}$ [6.10] for temperatures ranging from 50K to 300K considering scattering by confined LO phonons described by IHZ model, Huang-Zhu model and slab model. The linewidth due to interface phonons (IF) and bulk phonons are also calculated.

The calculated results for $2\Gamma_{op}$ and $2\Gamma_{tr}$ versus temperature are shown in Figures 6.2 and 6.3, respectively. The contribution from IF phonons to $2\Gamma_{op}$ is very small contrary to the energy loss rate calculations. The effect of LO phonon confinement on linewidth is more at higher temperatures than at lower temperatures. $2\Gamma_{op}$ is observed to have a higher value for confined phonons described by IHZ and HZ model model as compared to bulk phonons. But $2\Gamma_{op}$ due to slab model is nearly same as that due to bulk phonons.

The calculations of $2\Gamma_{op}$ show an increase with increasing temperature for all the models and also for bulk phonons. The same behavior is observed in ref [6.11] for bulk LO phonons. The increase is more rapid and $2\Gamma_{op}$ is highest for H-Z model as compared to other models. With the increasing temperature the absorption peak becomes broader and its maximum shifts towards lower energy [6.8]. Phonon scattering process becomes more active as temperature rises. The contribution of $2\Gamma_{op}$ at 300K is in the range of 1 to 3meV for all the models.
The calculations of $2\Gamma_{tr}$ as a function of temperature also show an increase with increasing temperature for all the models and also for bulk phonons. It is noticed that the contribution of IF phonons to $2\Gamma_{tr}$ is significant.

**Figure 6.2:** Temperature dependence of the intersubband absorption linewidth $2\Gamma_{op}$. Curve 1 denotes the energy broadening due to bulk phonons, curve 2 denotes energy broadening due to confined phonons described by IHZ model. Curve 3 denotes energy broadening due to confined modes described by HZ model and curve 4 denotes energy broadening due to confined modes described by slab model. Curve 5 denotes energy broadening due to IF phonons.
Figure 6.3: Temperature dependence of the transport broadening $2\Gamma_{tr}$. Curve 1 denotes the transport broadening due to bulk phonons, curve 2 denotes transport broadening due to confined phonons described by IHZ model. Curve 3 denotes transport broadening due to confined modes described by HZ model and curve 4 denotes transport broadening due to confined modes described by slab model. Curve 5 denotes transport broadening due to IF phonons.

B. Well-width dependence

Linewidth and transport broadening are calculated in a GaAs/AlGaAs QW with a barrier height of 1eV and $n_s=6\times10^{11}\text{cm}^{-2}$ [6.11] for various well widths in the range $L=75\text{Å}-110\text{Å}$ at $T=300K$. The calculated results for linewidth and transport broadening as function of well width, respectively are shown in Figures (6.4) and (6.5). The contribution of LO phonon scattering slowly increases as QWs become wider. Our results show a definite enhancement of the linewidth with increasing well width. This is consistent with the scattering rate calculations [6.19]. The absorption linewidth due to...
LO phonons in HZ and IHZ models are larger than those due to slab mode model and bulk phonons for all the well widths considered here. The transport broadening is also found to increase with the increasing well width for confined phonons where as it decreases for larger well width for bulk LO phonons.

**Figure 6.4:** Well width dependence of the intersubband absorption linewidth $2\Gamma_{op}$.

Curve 1 denotes the energy broadening due to bulk phonons, curve 2 denotes energy broadening due to confined phonons described by IHZ model. Curve 3 denotes energy broadening due to confined modes described by HZ model, curve 4 denotes energy broadening due to confined modes described by slab model. Curve 5 denotes energy broadening due to IF phonons.
6.5 Conclusions

We have formulated the energy dependent intersubband linewidth $2\Gamma_{\text{op}}(E)$ and transport broadening $2\Gamma_{\text{tr}}(E)$ due to scattering by confined LO phonons and interface phonons. The LO phonon confinement is considered in the HZ model, IHZ model and slab model. We have numerically calculated the absorption linewidth $2\Gamma_{\text{op}}$ and transport energy broadening $2\Gamma_{\text{tr}}$ for GaAs/AlGaAs QW. The broadening is determined by intrasubband and intersubband processes. The value of $2\Gamma_{\text{tr}}(E)$ is found to be greater than $2\Gamma_{\text{op}}(E)$ for all energies. The confinement of LO phonons enhances the values of $2\Gamma_{\text{tr}}(E)$ and $2\Gamma_{\text{op}}(E)$ over the contribution due to bulk LO phonons. The contribution of interface phonons is found to be very small. The $2\Gamma_{\text{op}}$ and $2\Gamma_{\text{tr}}$ are also studied as a

![Figure 6.5: Well width dependence of the transport broadening $2\Gamma_{\text{tr}}$. Curve 1 denotes the energy broadening due to bulk phonons, curve 2 denotes energy broadening due to confined phonons described by IHZ model. Curve 3 denotes energy broadening due to confined modes described by HZ model, curve 4 denotes energy broadening due to confined modes described by slab model. Curve 5 denotes energy broadening due to IF phonons.](image)
function of temperature and well width and are found to increase with increasing temperature and well width.

The results on the line narrowing are important in a device context, as a broad response would be undesirable for detectors and lasers based on the intersubband transition.

References


