CHAPTER 5

ELECTRON - LOCALIZED PHONON SCATTERING RATES IN QUANTUM WELLS IN QUANTIZING MAGNETIC FIELD

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5.1 INTRODUCTION

An understanding of transport and optical properties of Q2D electron system in a QW or a SL can be achieved by the knowledge of electron-phonon interaction in these structures. In the absence of magnetic field the problem of scattering of electrons by polar optical phonons in semiconductor QWs has been treated by number of workers /13,14,59,88,89/. In these investigations the usual Fröhlich interaction which is based on bulk description of LO phonons is used. Experiments like time resolved Raman scattering /42,90/ and infrared spectroscopy /43/ show the dominance of electron interaction with LO phonons and reveal important information about the vibrational modes in the layers forming multiple QWs and SLs. Based on infrared bleaching experiments Sielemeier et al /43/ have studied intersubband hot-electron relaxation in GaAs QWs and found an order of magnitude discrepancy between theory (based on bulk description of LO phonons) and experimental results. Rudin and Reinecke /27/ have calculated electron-confined LO phonon scattering rates in GaAs quantum wells for three models describing phonon modes: the slab modes based on the dielectric continuum theory with electrostatic boundary conditions as discussed by Licari and Evrard /37/, the confined or guided modes based on the continuum model with mechanical boundary conditions as discussed by Ridley /38/...
and the modes formulated as analytical approximations to the microscopic model proposed by Huang and Zhu /39/. These calculations have been extended for quantum wells with finite depth by Weber et al./41/. Recently, da Cunha Lima and Reinecke /28/ have calculated, based on HZ model, infrared absorption coefficient in GaAs/AlAs QWs due to electron interaction with optical phonons. They found that effects of confinement on phonon spectrum reduce the absorption. Also, there have been theoretical investigations /29/ of electron scattering rates in QWs due to confined LO phonons, described by above three models, in the presence of an applied electric field.

Calculations of electron scattering rate /27/ and infrared absorption/28/ due to interface modes in GaAs/AlAs QWs, following the method of Lassnig /45/ [§ 1.5.2], indicate that interface modes are as important as confined modes. Very recent investigations of electron scattering rates due to interface optical phonons in GaAs/AlAs QWs /30,31/ and SLs /32/ in the presence of electric field have indicated the strong coupling between electrons and interface modes. In these investigations contribution to electron scattering rates due to interface modes is found to increase with decreasing quantum well layer thickness. These theoretical attempts to obtain an understanding of the physical situation point out that the effects of electron interaction with confined and interface optical phonon modes
should be taken into account to obtain realistic estimate for the electron-phonon scattering rates in QW structures in quantizing magnetic field as well.

Electron scattering rates in the presence of quantizing magnetic field, in GaAs QWs /84,85/ and SLs /91/, have been studied, theoretically, when electrons are scattered by bulk acoustic, nonpolar and polar optical phonons. Recent experimental /33,34/ and theoretical /35/ investigations of I-V characteristics of GaAs/AlAs double barrier resonant tunneling structures, in the presence of high magnetic field applied perpendicular to the layers, have revealed electron relaxation through the emission of confined and interface phonons. In view of the importance of confined and interface phonon modes it is interesting and important to investigate electron scattering rates due to confined and interface optical phonon modes in a QW structure in the presence of quantizing magnetic field. In § 5.2 we give the theory of scattering rates due to confined modes described by HZ model, Fuchs-Kliewer slab modes and Ridley's guided mode models, which we have described in chapter 1. For the scattering rates due to interface modes we employ the model due to Lassnig. We compare our results with those obtained with bulk description of phonons. In § 5.3 we present the results and discussion.
5.2 THEORY

In this section we present calculations of electron scattering rate in QWs due to confined and interface optical phonons in the presence of quantizing magnetic field.

5.2.1 BASIC FORMULA, ELECTRON WAVEFUNCTION AND EIGENVALUES

The scattering rates, $W$, for the absorption or emission of a phonon are obtained from the usual Fermi golden rule and is given by

$$W = \frac{2\pi}{\hbar} \sum_{f} |<f|H_{el-ph}|i>|^2 \delta(E_f - E_i \pm \hbar \omega_p), \quad (5.1)$$

where $E_i$ and $E_f$ represent the initial and final state energies of the electron and $\hbar \omega_p$ is the confined or interface phonon energy. The - and + signs refer to the scattering with the absorption and emission of phonon, respectively. The summation is over all final states 'f' of the system.

As described in chapter 1 the electron wave function and energy eigen values for Q2D electron gas in a quantizing magnetic field are given by

$$\Psi_{Nl\k_y} = \left(\frac{1}{L_y}\right)^{1/2} \phi_N(x - x_o) \exp(ik_y y) \chi_1(z) \quad (1.18)$$
and

\[ E_{\text{NL}} = (N + 1/2)\hbar \omega_c + \frac{1}{2} E_0, \quad (1.19) \]

where the envelope wavefunction \( \chi_1(z) \) is given by Eqn.(1.9) for quantum well of infinite depth centered at \( z = 0 \). Other quantities in the above equations are as described in § 1.5.

5.2.2. **ELECTRON-CONFINED LO PHONON SCATTERING**

In this subsection we calculate the electron scattering rates due to confined LO phonons described by Huang and Zhu model, Fuchs-Kliewer slab modes and Ridley’s guided mode models. Using the matrix elements given by Eqn.(4.1) for the electron confined phonon interaction Hamiltonian [Eqn.(1.26)] in Eqn.(5.1) we obtain for electron scattering rate due to confined phonons

\[
W_{\text{cp}} = \frac{4\pi e^2 \omega_{\text{LO1}}}{\varepsilon_\text{L}} \left( N_{\text{LO1}} + 1/2 \mp 1/2 \right)
\]

\[ \times \sum_{N', l', n, \alpha} \sum_{n, \alpha} \int_0^\infty dq q |J_{NN'}(u)|^2 |Q_{11'}^n\alpha| |t_{n\alpha}(q)|^2 \]

\[ \times \delta \left( (l'^2 - 1^2)E_0 + (N' - N)\hbar \omega_c \mp \hbar \omega_{\text{LO1}} \right), \quad (5.2) \]
where the subscript 'cp' refers to 'confined phonon', \( N_{LO1} \) denotes the phonon distribution function and function \( J_{NN'}(u) \) is as defined in chapter 3 [Eqn.(3.8)]. \( G_{11'}^{\alpha \alpha} \) is the overlap integral defined in Eqn.(2.8)

\[
\frac{L}{2} \int_{-L/2}^{L/2} \chi_{1'}(z) u_{n\alpha}(z) \chi_1(z) \, dz . \tag{2.8}
\]

Assuming broadening to be same for all Landau levels, we replace the Dirac delta function in Eqn.(5.2) by a Lorentzian of width \( \Gamma \). In the extreme quantum limit i.e., for \( l = l' = 1 \) and \( N = 0 \), expression for the scattering rate becomes

\[
W_{cp} = \frac{4e^2 \omega_{LO1}}{\varepsilon' L \hbar \omega_c} \left( 2N_{LO1} + 1 \right) \sum_{N'} \sum_{n, \alpha = \pm} |G_{11'}^{\alpha \alpha}|^2 \times \frac{1}{N' - 1} \int_0^\infty \frac{d t}{t} \left( bt^2 + n^2 \pi^2 \right)^{-1} \exp\left[ - \frac{\lambda t^2}{2L^2} \right] \exp\left[ - \frac{\lambda' t^2}{2L^2} \right] \times \frac{\Gamma'}{(N' - C)^2 + \Gamma' \cdot 2} , \tag{5.3}
\]

where \( b = 3 \) for HZ model and \( b = 1 \) for the other two models, \( t = qL \), \( C = \omega_{LO1}/\omega_c \) and \( \Gamma' = \Gamma/\hbar \omega_c \). We have evaluated the overlap integral for intrasubband transitions in chapter 2 for HZ model [Eqns.(2.9) and (2.10)], slab modes

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[Eqns.(2.13) and (2.14)] and guided modes [Eqns.(2.17) and (2.18)]. Using the appropriate $G_{11}^{n\alpha}$ in Eqn.(5.3) scattering rates can be calculated for the three models. It may be noted that for intrasubband transitions only even modes contribute in HZ model whereas in slab modes only odd modes contribute, and in the case of guided modes contribution is from only $n = 2$ mode.

To study the effect of finite height for barriers on intrasubband scattering rate we use $G_{11}^{n-}$ given by Eqn.(4.6) for HZ model. Using Eqn.(4.6) in (5.3) the expression for scattering rate becomes

$$W_{HZ} = \frac{4e^2 \omega_{LO1}}{\varepsilon^* L \hbar \omega_c} (2N \omega_{LO} + 1) \frac{k_{A1L}}{k_{A1L} + \sin(k_{A1L})}$$

$$\times \sum_{N'} \sum_n \left\{ \frac{k_{A1L}}{(k_{A1L})^2 - (0.5n\pi)^2} \sin(k_{A1L}) \cos \left( \frac{n\pi}{2} \right) \right\}$$

$$- (-1)^{n/2} \left[ 1 + \frac{\sin(k_{A1L})}{k_{A1L}} \right] \frac{1}{N'!} \int_0^\infty dt \ t \left( 3t^2 + n^2 \pi^2 \right)^{-1}$$

$$\times \left\{ \frac{\lambda_{t^2}}{2L^2} \right\}^{N'} \exp\left( - \frac{\lambda_{t^2}}{2L^2} \right) \frac{\Gamma'}{(N' - C)^2 + \Gamma'^2}, \quad (5.4)$$

where $k_{A1}$ is the $z$-component of the electron wave vector and subscript 'HZ' refers to 'Huang and Zhu model'.

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5.2.3 ELECTRON-INTERFACE OPTICAL PHONON SCATTERING

In this subsection we derive the expression for electron scattering rate due to interface phonons. Using the matrix elements given by Eqn.(4.10) for the electron interface optical phonon interaction Hamiltonian [Eqn.(1.39)] in Eqn.(5.1) we obtain for electron scattering rate:

\[ W_{ip} = \frac{2\pi e^2}{\hbar^2} \sum_{N',1'} \sum_{\mu} \int_0^\infty dq \frac{\gamma}{1 \pm \gamma} |J_{NN'}(u)|^2 |G_{11'}^\beta|^2 \times (N_{\beta\mu} + 1/2 \mp 1/2) \left( \frac{\omega_{\beta\mu}}{\omega_{\beta\mu}} \right) \times \delta \left( (1' - 1'^2)E_0 + (N' - N)\hbar\omega_c \mp \hbar\omega_{\beta\mu} \right), \]  

(5.5)

where \( \hbar\omega_{\beta\mu} \) represents the energy of \( \beta \)th interface phonon mode with parity \( \mu \), \( N_{\beta\mu} \) is the phonon distribution function and \( \gamma = \exp(-qL) \). Function \( J_{NN'} \) is as defined in chapter 3. \( G_{11'}^\beta \) is given by (§ 2.2.4)

\[ G_{11'}^\beta = \int_{-L/2}^{L/2} dz \chi^*_1(z) \left[ \exp(qz) \pm \exp(-qz) \right] \chi_1(z). \]  

(2.30)

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For intrasubband transitions i.e., for $l = l' = 1$, overlap integral reduces to

$$G^s_{11} = \frac{8\pi^2}{qL} \exp\left(-\frac{qL}{2}\right)(1 - \gamma) \left(qL^2 + 4\pi^2\right)^{-1} \tag{2.33}$$

for symmetric modes and

$$G^a_{11} = 0 \tag{2.34}$$

for antisymmetric modes. Thus, only symmetric modes contribute for intrasubband scattering rates.

Replacing Dirac delta function in Eqn.(5.5) by Lorentzian of width $\Gamma$, the expression for scattering rates, in the extreme quantum limit i.e. for $l = l' = 1$ and $N = 0$, becomes

$$W_s = \frac{8e^2}{h^3 \omega_c L} \sum_{N',\mu} \int_0^\infty dt \left(\frac{f_{s\mu}}{\omega_{s\mu}}\right) \frac{(1 - e^{-t})^2}{t^2(1 + e^{-t})(1 + t^2/4\pi^2)^2}$$

$$\times \exp\left(\frac{\lambda^2}{2L^2}\right) \left(\frac{\lambda^2}{2L^2}\right)^{N'} \frac{(2N_{s\mu} + 1) \Gamma'}{(N' - F_{s\mu})^2 + \Gamma'^2}, \tag{5.6}$$

where $F_{s\mu} = (\omega_{s\mu}/\omega_c)$ and subscript 's' refers to 'symmetric' interface modes.
5.2.3 ELECTRON-BULK LO PHONON SCATTERING

The matrix elements for electron polar optical phonon interaction Hamiltonian [Eqn.(1.21)] can be written as

\[ |<k_y',N',l'|H_{e-ph}|k_y,N,l>|^2 = \frac{2\pi e^2 \hbar \omega_{LO1}}{Q^2} |G_{11'}(\pm q_z)|^2 \]

\[ \times \left(N_Q + 1/2 \mp 1/2\right) |J_{NN'}(u)|^2 \delta_{k_y \pm q_y', k_y'} \]  \hspace{1cm} (5.7)

where the functions \( J_{NN'} \) and \( G_{11'} \) are as defined in chapter 3. Using Eqn.(5.7) in Eqn.(5.1) and replacing the Dirac delta function by Lorentzian of width \( \Gamma \), for extreme quantum limit, we obtain the following expression for electron scattering rate

\[ W_{bp} = \frac{2e^2 \hbar \omega_{LO1}}{\epsilon' \hbar c} (2N_{LO1} + 1) \sum_{N'} N'! \int_0^\infty dt \ t \ \exp \left[ -\frac{\lambda^2 t^2}{2L^2} \right] \]

\[ \times \left( \frac{\lambda^2 t^2}{2L^2} \right)^N \left\{ t^{-2} \left[ 1 - F(t) \right] + 0.5 + \left[ 2 - \frac{t^2}{t^2 + 4\pi^2} \right] F(t) \right\} \]

\[ \times \frac{1}{t^2 + 4\pi^2} \frac{\Gamma'}{(N' - C)^2 + \Gamma'^2}, \]  \hspace{1cm} (5.8)
with $F(t) = \frac{1 - \exp(-t)}{t}$. It may be noted that while obtaining this expression we have evaluated the integration over $z$-component of phonon wave vector ($q_z$), explicitly, following Leburton [see Eqn.(3.25)]/59/. Expressions for scattering rates found in literature/84,85/ employ approximations in the evaluation of $q_z$ integral.

5.3 RESULTS AND DISCUSSION

We have evaluated numerically the above expressions for scattering rates obtained for electron interaction with confined, interface and bulk phonons in GaAs/AlAs QW system. The material parameters used are given in Table 1. The scattering rate can be estimated from the broadening parameter, $\Gamma$, according to $W = \frac{\Gamma}{\hbar}$. The width parameter is assumed to same for all Landau levels and value of $W$ occurring on left hand side of the Eqns.(5.3), (5.4), (5.6) and (5.8) were made self-consistent with $\frac{\Gamma}{\hbar} = W$.

Figure 5.1 shows the magnetic field dependence of scattering rates calculated for a QW of width $L = 100 \AA$ at a temperature $T = 200$ K. Curve A is for bulk phonons, B for HZ model and C for slab modes. We have not shown the results for guided modes as the scattering rate is found to be much smaller. Each maximum in the curves correspond to the resonance condition $m\omega_C = \omega_{LO1}$, where $m$ is an integer. In HZ model only even modes contribute and maximum contribution is
Fig. 5.1 Magnetic field dependence of scattering rate in GaAs/AlAs QW for bulk phonons (curve A) and confined modes described by HZ model (curve B) and slab modes (curve C) calculated for \( L = 100 \) A and \( T = 200 \) K.
from $n = 2$, whereas in slab modes only odd modes contribute and maximum contribution is from $n = 1$ mode. In the case of guided modes only $n = 2$ mode contributes and the scattering rate is much smaller compared to that obtained from HZ or slab mode models. In the case of scattering by bulk phonons all modes contribute and hence the scattering rate is greater than those with confined modes. The increase in sharpness and the amplitude of oscillation of scattering rate at higher magnetic fields is similar to that seen in the recent magneto-tunneling experiments/33,34/ and calculations /35/ in which the peaks in the tunneling current were found to be sharper at higher fields than those at lower fields [see Fig.5.2]. It may be observed that oscillations are more distinct in the case of HZ and slab mode models compared to the bulk phonons case. This is due to the decrease in the broadening parameter $\Gamma$ with the decrease of scattering rate. In Fig. 5.3 we show the well width dependence of scattering rate calculated at $B = 2.62$ T ($m = 8$) and $T = 200$ K. Curve 1 is for bulk phonons, 2 for HZ model, 3 for slab modes and 4 for guided modes. We find that scattering rate due to confined modes increases with increase of well width in contrast to that seen with the bulk phonons. In the figure we have also shown the results of the calculations, for HZ model, with the finite value of $1$ eV for the height of the QW barrier (curve 5). The effect of finite barrier height is to reduce the scattering rate.
Fig. 5.2  Total phonon-assisted-tunneling current due to confined modes and symmetric interface modes at various magnetic fields: (a) 15 T, (b) 6 T, and (c) 4 T. [after Turley and Teitsworth /35/]
Fig. 5.3 Scattering rate is shown as function of QW width calculated for $B = 2.62$ T and $T = 200$ K due to bulk phonons (curve 1) and confined modes described by HZ model (curve 2), slab modes (curve 3) and guided mode (curve 4) models. Curve 5 is for HZ model with finite QW barrier height.
For finite QW the electronic wave functions lie partially outside the QW which reduces the electron-phonon coupling strength and consequently the scattering rate. This is similar to that seen in the zero field calculations /27,41/. The interface phonons are slightly dispersive with their energy [see figure 2.2] varying between bulk $\omega_{LO}$ and $\omega_{TO}$ as a function of the wave vector. We have taken this into consideration in our calculations. Figure 5.4 shows the magnetic field dependence of the scattering rate calculated for $d = 100$ Å at $T = 200$ K. Full curve is for bulk phonons, broken curve is for sum of the scattering rates due to two interface phonon modes $\omega_{3+}$ and $\omega_{3-}$, dashed curve is for symmetric AlAs like ($\omega_{3+}$) mode and dotted curve is for symmetric GaAs like ($\omega_{3-}$) mode. Each maximum in both curves corresponds to the resonant transition between the Landau levels. It may be seen that maxima in the curves occur at different magnetic fields. This arises due to the dispersive nature of the interface phonon modes. In figure 5.5 we show the well width dependence of the scattering rates calculated at $T = 200$ K and $B = 2.62$ T. Curve A is for interface $\omega_{3+}$ modes, B is for interface $\omega_{3-}$ modes and C is for confined modes described by HZ model. The scattering rate due to both the interface modes decreases with increasing of well width in contrast to that seen with the confined modes. The electron-interface phonon scattering rates dominates over that due to confined modes for well
Fig. 5.4 Magnetic field dependence of scattering rates due to interface modes calculated for a QW of width L = 100 Å at T = 200 K.
Broken curve: Sum of interface modes $\omega_{3+}$ and $\omega_{3-}$.
Dashed curve: Interface $\omega_{5+}$ mode.
dotted curve: Interface $\omega_{5-}$ mode.
Full curve: Bulk LO phonons.
Fig. 5.5 Well width dependence of scattering rates due to interface modes calculated at $T = 200$ K and $B = 2.48$ T.

Curve 1: Interface $\omega_{s+}$ mode,
Curve 2: Interface $\omega_{s-}$ mode,
Curve 3: Confined phonon - HZ model.
widths less than $\approx 80$ Å. Similar behavior is seen for scattering rates in the absence of magnetic field /27/.

In conclusion, we have presented in this chapter the calculations for intrasubband electron scattering rates due to confined and interface optical phonons in a GaAs QW. The scattering rate exhibits oscillatory behavior with magnetic field. We find that scattering rate due to confined and interface modes is lower than that due to bulk phonons. Further, scattering rate due to confined modes, for all the models, increases with the increasing well width in contrast to that seen with the bulk phonons. However, in the case of interface phonons scattering rate increases with the decreasing well width.