CHAPTER V

FLOW BETWEEN TWO ROTATING DISCS
(WITH INJECTION OR SUCTION)
5.1 Introduction

In the previous chapter we have studied the problem of viscous flow between a rotating and a stationary disc using computer extended perturbation series. In this chapter we attempt for the series solution of more involved problem viz the flow between two rotating discs with injection (or suction).

As explained in chapter 4, the flows driven by rotating discs have constituted a major field of study in Fluid Mechanics for the better part of this century. These flows have technical applications in many areas, such as rotating machinery, lubrication, viscometry, computer storage devices and crystal growth processes. However, they are of special theoretical interest, because they represent one of the few examples for which there is an exact solution to the Navier-Stokes equations. This problem was first discussed by Batchelor [31] who generalised the solution of von Karman [40] and Bodewadt [41] for the flow over a single infinite rotating disc. The problem was further discussed by Stewartson [32] who obtained approximate perturbation solution when the Reynolds number was small. Later, Hoffman [33] has studied this problem using computer extended series. The numerical solutions for this problem have been obtained by Lance and Rogers [34], Mellor, Chapple and Stokes [36] and Brady and Durlofsky [39]. The problem of injection (suction) of a viscous incompressible fluid through a rotating porous disc onto a
rotating co-axial disc was studied by Narayana and Rudraiah [42] and Wang and Watson [43]. Through this span of a period of half a century, since the Batchelor - Stewartson contributions, the interaction between physically based conjectures, numerical calculations, formal asymptotic expansions and rigorous mathematical treatment has been quite intensive.

The present analysis is primarily concerned with the possible extension of Wang's [43] low Reynolds numbers perturbation series by computer and its analysis. The forms of the few manually calculated functions in low Reynolds numbers perturbation solution of two point boundary value problem allows to propose the generation of universal functions in compact form which are solutions of infinite sequence of linear problems. Using these universal coefficient functions we obtain series solution and calculate various physical parameters of interest. The present series, which is expected to be limited in convergence by the presence of a singularity may be extended to moderately high Reynolds numbers by analytic continuation.

The aims of the present work are twofolds. First, to calculate enough terms of the low-Reynolds numbers perturbation series by computer so that the nature and location of the nearest singularity (which limits the convergence) can be determined accurately second, to show that the analytic continuation can be
used effectively to extend the perturbation series to moderately high Reynolds numbers.

5.2 Formulation

As shown in Fig.5.1 we denote the spacing between the discs by \( d \), the angular velocity of bottom disc by \( \Omega_1 \), and that of the upper disc by \( \Omega_2 \). Let the injection (or suction) at the lower disc be \( W \) and let \( u, v, w \) be the velocity components in the direction \( r, \theta, z \) respectively (Fig.5.1). The governing equations of the problem are (4.2.1) to (4.2.4).

The boundary conditions are

at \( z = 0 \) \( u = 0, v = r \Omega_1, w = \pm W \) \hspace{1cm} (5.2.1)

\( z = d \) \( u = 0, v = r \Omega_2, w = 0 \). \hspace{1cm} (5.2.2)

For similarity solution, the boundary conditions and the continuity equation suggest the transformations (Wang [43])

\[ u = r f'() \frac{W}{d}, \quad v = r g() \frac{W}{d}, \quad w = -2 f() W \] \hspace{1cm} (5.2.3)

\[ p = \rho r^2 A \frac{W^2}{(2d)^2} + P() \] \hspace{1cm} (5.2.4)

where \( \eta = \frac{z}{d} \) and \( A \) is a constant to be determined. With these transformations the equations of motion reduce to

\[ (f')^2 - 2 f f'' - g^2 = A + \frac{1}{R} f''' \] \hspace{1cm} (5.2.5)

or after differentiation, we have
Fig. 5.1: Schematic diagram of the problem.
-2 R (f f'' + g g') = f''''  \quad (5.2.6)

2 R (f' g - f g') = g''  \quad (5.2.7)

P(\eta) = -\frac{\Omega d}{W} (-2 \frac{f^2 - W^2}{d} - 2 \nu W \frac{f'}{d}) + p_0  \quad (5.2.8)

Here R = \frac{Wd}{\nu} is the cross flow Reynolds number. The constant p_0 is determined by the pressure at the edge of the discs. The boundary conditions take the forms

f'(0) = 0, g(0) = \frac{\Omega d}{W} = \alpha, f(0) = -\frac{1}{2}  \quad (5.2.9)

f'(1) = 0, g(1) = \frac{\Omega d}{W} = \beta, f(1) = 0  \quad (5.2.10)

In order to investigate the mutual interaction of rotation and injection (suction), we shall assume \alpha and \beta to be of order of unity. This includes many interesting cases where both rotation and injection (suction) are not minor perturbations.

5.3 Method of solution

We seek the solution of equations (5.2.6) and (5.2.7) in power series of R in the forms

\[ f(\eta) = f_0(\eta) + \sum_{n=1}^{\infty} R^n f_n(\eta) \]  \quad (5.3.1)

\[ g(\eta) = g_0(\eta) + \sum_{n=1}^{\infty} R^n g_n(\eta). \]  \quad (5.3.2)

Substituting (5.3.1),(5.3.2) into (5.2.6),(5.2.7) and comparing like powers of R on both sides, we get
\[ f''''(n) = -2 \sum_{r=0}^{n-1} (f_r f''_{n-1-r} + g_r g'_{n-1-r}) \]  
(5.3.3)

\[ g''''(n) = 2 \sum_{r=0}^{n-1} (f'_r g_{n-1-r} - f_r g'_{n-1-r}) \]  
(5.3.4)

The relevant boundary conditions are

\[ f'(0) = 0, f(1) = 0, f(0) = 0, f(1) = 0 \]  
(5.3.5)

\[ g_o(0) = \alpha, g_o(1) = \beta \]  
(5.3.6)

The solutions of above equations up to the o(R) are

\[ f_0 = \frac{1}{2} - \frac{3}{2} \eta^2 + \eta^3 \]  

\[ f_1 = -\frac{11}{70} \eta^2 + \frac{13}{35} \eta^3 - \frac{1}{4} \eta^4 + \frac{1}{20} \eta^5 - \frac{1}{70} \eta^7 - \frac{\alpha}{12} (\beta - \alpha) \eta^2 - 2 \eta^3 + \eta^4 - \frac{(\beta - \alpha)^2}{60} (2 \eta^2 - 3 \eta^3 + \eta^5) \]  
(5.3.7)

\[ g_0 = (\beta - \alpha) \eta + \alpha \]  

\[ g_1 = \frac{(\beta - \alpha)}{20} (4 \eta^5 - 5 \eta^4 - 10 \eta^2 + 11 \eta) + \frac{\alpha}{2} (\eta^4 - 2 \eta^3 + \eta) \]  

The slow convergence of the series ((5.3.1),(5.3.2)) require large number of terms for obtaining the approximate sum. As we proceed for higher approximations, the algebra becomes cumbersome and it is difficult to calculate the terms manually. As in earlier
chapters we propose a systematic series expansion scheme with polynomial coefficients so that whole process can be made automatic using computer. For this purpose, we consider \( f_n \) and \( g_n \) to be of the forms

\[
f_n(\eta) = (1 - \eta)^2 \sum_{k=2}^{4n+1} A_{n(k)} \eta^k \quad (5.3.8)
\]

\[
g_n(\eta) = (1 - \eta) \sum_{k=1}^{4n} B_{n(k)} \eta^k \quad (5.3.9)
\]

in (5.3.1) and (5.3.2) respectively. This expression yields exactly the above calculated terms \( f_1 \) and \( g_1 \) besides this it enables us to find \( f_i \) and \( g_i \) for \( i \geq 2 \) using computer. We substitute equations (5.3.8), (5.3.9) into (5.3.3), (5.3.4) and equate various powers of \( \eta \) on both sides and obtain two recurrence relations for unknowns \( A_{n(k)} \) and \( B_{n(k)} \) in the forms

\[
A_{n(k)} = 2A_{n(k+1)} - A_{n(k+2)} + \frac{1}{(k+2)(k+1)K(k-1)} \left\{ \sum_{i=1}^{6} A_{(n-i)(k+2-i)} P_i(k+2-i) 
\right.
\]

\[
+ \sum_{i=1}^{9} B_{(n-i)(k-i)} Q_i(k-i) + \sum_{r=1}^{n-2} \sum_{i=0}^{4} A_{r(l+1-nk-i)} A_{(m-1)(t-2)} P_{7+i}(t-2) 
\]

\[
+ \sum_{i=0}^{2} \sum_{j=1}^{nk+i} B_{r(l-nk-i)} B_{(m-l)(t-9)} Q_{4+i}(t-3) \right\} \quad (5.3.10)
\]

\[ K = 2, 3, \ldots \ldots , (4n+1) \]
\[
B_{n(k)} = B_{n(k+1)} - \frac{1}{k(k+1)} \left[ \sum_{i=1}^{5} B_{(n-\ell)(k+1-i)} T_i (k+1-i) \right. \\
+ \sum_{i=1}^{4} A_{(n-\ell)(k+1-i)} T'_i (k+1-i) \\
+ \sum_{r=1}^{n-2} \sum_{i=0}^{nk+i} A_{r(l+1-nk-i)} B_{(m-\ell)(l-3)} T_i (l+1-nk-i, l-3) \right]
\]

(5.3.11)

where

\[
m = n-r, \quad l = 4r+j, \quad t = 4m-j, \quad nk = 4n-2-k,
\]

\[
\begin{align*}
P_1(k) &= \pm k(k-1)(k-2), \\
P_2(k) &= \mp 2(k+1)k(k-1), \\
P_3(k) &= \mp (3k(k-1)(k-2) - (k+2)(k+1)k), \\
P_4(k) &= \pm (2k(k-1)(k-2) + 6(k+1)k(k-1)+12), \\
P_5(k) &= \pm (4(k+1)k(k-1) + 3(k+2)(k+1)(k-1)+24), \\
P_6(k) &= \pm (2(k+2)(k+1)k+12), \\
P_7(k) &= -2k(k-1)(k-2), \\
P_8(k) &= 4(k+1)k(k-1) + 4k(k-1)(k-2), \\
P_9(k) &= -2(k+2)(k+1)k - 8(k+1)k(k-1) - 2k(k+1)(k-1), \\
P_{10}(k) &= 4(k+2)(k+1)k + 4(k+1)k(k-1), \\
P_{11}(k) &= -2(k+2)(k+1)k, \\
Q_1(k) &= -2k\alpha
\end{align*}
\]
\[ Q_2(k) = -2k(\beta-\alpha) + 2(k+1)\alpha - 2(\beta-\alpha), \]
\[ Q_3(k) = 2(k+1)(\beta-\alpha) + 2(\beta-\alpha), \]
\[ Q_4(k_1) = -2k_1, \]
\[ Q_5(k_1) = 2(2k_1 + 1), \]
\[ Q_6(k_1) = -2(k_1 + 1), \]
\[ T_1(k) = \pm k, T_2(k) = + (k+1), T_3(k) = + (3k-6), T_4(k) = \pm (5k-9), \]
\[ T_5(k) = + (2k-4), T_6(k,k_1) = 2k - 2k_1, T_7(k,k_1) = 2(3k_1 - 3k-1), \]
\[ T_8(k,k_1) = 2(3k_1 - 3k + 2), T_9(k,k_1) = 2( -k + k_1 -1), \]
\[ T'_1(k) = 2k\alpha, T'_2(k) = 2(\beta-\alpha)k - 4(k+1)\alpha - 2(\beta-\alpha), \]
\[ T'_3(k) = -4(\beta-\alpha)(k+1) + 2(k+2)\alpha + 4(\beta-\alpha), \]
\[ T'_4(k) = 2(k+2)(\beta-\alpha) - 2(\beta-\alpha), \]
\[ A_{12} = - \frac{11}{70} - \frac{\alpha}{12} (\beta-\alpha) - \frac{(\beta-\alpha)^2}{30}, \]
\[ A_{13} = \frac{4}{70} - \frac{(\beta-\alpha)^2}{60}, A_{14} = \frac{3}{140}, A_{15} = - \frac{1}{70}, \]
\[ B_{11} = \pm \left( \frac{\alpha}{2} + \frac{11(\beta-\alpha)}{20} \right), B_{12} = \pm \left( \frac{\beta+9\alpha}{20} \right), \]
\[ B_{13} = \pm \left( \frac{\alpha}{2} - \frac{(\beta-\alpha)}{20} \right), B_{14} = \pm \frac{(\beta-\alpha)}{5}. \]

For the radial velocity profile \( f'(\eta) \), we have
\[ f'(\eta) = \pm (3\eta^2 - 3\eta) + \sum_{n=1}^{\infty} R_n \sum_{k=2}^{n+1} A_{n,k}(k\eta^{k-1} - 2(k+1)\eta^k + (k+2)\eta^{k+1}) \]
\[ (5.3.12) \]
The constant $A$ ((5.2.5)) which is proportional to the lift is given by

$$A = -\frac{1}{R} f'''(1) - \beta^2$$

$$= -\frac{1}{R} \left[ -6 + \sum_{n=1}^{\infty} R^n \sum_{k=2}^{4n+1} 6k A_{n(k)} \right] - \beta^2$$

(5.3.13)

Case(1): $\alpha = 0, \beta = 0$ which corresponds to the case when both discs are stationary and the flow is due to injection only. In this case the coefficients of the series for $f'''(1)$, which is used to calculate $A$, has terms which are all positive (Table 5.1). Using the computed coefficients we draw Domb-Sykes plot (Fig.5.2) for $f'''(1)$ (series (5.3.13)) to find the nature and location of the nearest singularity which restricts the convergence of the series. In this case singularity is found to be a square root singularity at $R = 17.9826$. This singularity on the positive real axis is not a real singularity, but an indication of double valuedness of the function. This artificial restriction on convergence can be eliminated by reverting the series. This type of reversion is also successfully employed in chapter 3 (3.3(a)). Towards this goal the reversion of the series (5.3.13) for $f'''(1)$ is performed as follows. Consider
\[ f''''(1) = -6 + \sum_{n=1}^{\infty} a_n R^n. \] (5.3.14)

Let

\[ Y = f''''(1) + 6 = \sum_{n=1}^{\infty} a_n R^n. \]

Reverting the above series, we have

\[ R(Y) = \sum_{n=1}^{\infty} B_n Y^n \] (5.3.15)

where

\[ B_1 = \frac{1}{e(1,1)} \]

\[ B_m = \frac{1}{e(1,m)} \sum_{i=0}^{m-2} B_{i+1} e(m-i,i+1) \]

\[ e(1,\chi) = (a_1)^\chi \]

\[ e(k+1,\chi) = \frac{1}{K_{a_1}} \sum_{i=0}^{k-1} \left\{(k-i)\chi - i\right\} e(i+1,\chi) a_{k-i+1} \]

\[ K = 1, 2, \ldots, n; \ \chi = 1, 2, \ldots, n. \]

Besides reversion we use Padé approximants for summing the reverted series (5.3.15) which yields analytic continuation. These results are shown in Fig. 5.3.

**Case 2:** \( \alpha = 4, \beta = 0 \) ie lower disc is rotating and upper disc is stationary. Also, there is an injection at the lower disc. The coefficients \( a_n \) of the series (5.3.13) for \( f''''(1) \) are listed in
Table 5.2. They are decreasing in magnitude and have no regular pattern of sign. We invoke Pade' approximants to achieve analytic continuation of the series (5.3.13) (Van Dyke [16]) and the corresponding results are shown in Fig. 5.4.

Case (3): $\alpha = 0, \beta = 0.5$ in this case upper disc is rotating and the lower one is stationary with injection. The coefficients $(a_n)$ of the series (5.3.13) for $f''(1)$ are listed in Table 5.3. They are decreasing in magnitude but have no regular pattern of sign. So, as in the previous case we use Pade' approximants to sum the series. The results obtained are shown in Fig. 5.5.

Lastly, we have considered the case of suction at the stationary lower disc and the upper one rotating ($\alpha = 0, \beta = 1$). The radial velocity profiles $f'(\eta)$ are shown in Fig. 5.8.

5.4 Discussion of results

Here the problem of injection (suction) of a viscous incompressible fluid through a rotating porous disc onto a rotating co-axial disc is studied using computer extended series analysis. The motion of the fluid is governed by a pair of coupled nonlinear ordinary differential equations (5.2.5) and (5.2.6) together with the boundary conditions (5.2.9) and (5.2.10). The series expansion scheme with polynomial coefficients ((5.3.8), (5.3.9)) proposed enables in obtaining recurrence
relations (5.3.10) and (5.3.11). Using these interactive relations we generate large number \((n = 25)\) of universal coefficients \(\left(\begin{array}{c} A_{nk} \\
 \end{array}\right), k = 2,3,\ldots,4n+1, n = 1,2,\ldots,25\) and \(\left(\begin{array}{c} B_{nk} \\
 \end{array}\right), k = 1,2,\ldots,4n, n = 1,2,\ldots,25\). To this order there are 1300 coefficients \(A_{nk}\) and 1300 of the \(B_{nk}\). A careful FORTRAN program consisting of number of DO loops makes it possible in performing complex algebra involved. The series (5.3.12) representing radial velocity profiles in various cases \(\alpha = 0, \beta = 0 \) (with injection); \(\alpha = 4, \beta = 0 \) (with injection); \(\alpha = 0, \beta = 0.5 \) (with injection) and \(\alpha = 0, \beta = 1 \) (with suction)) are analyzed using Pade' approximants and these results are shown in Figs.5.6-5.8. The velocity profiles have trend similar to that obtained by Wang ([43],[27]). Using the universal coefficients of the series (5.3.8),(5.3.9) we obtain series expansion for \(A\) which is directly proportional to the lift. The coefficients \(a_n\) of the series (5.3.13) for \(A\) in the case of \(\alpha = 0, \beta = 0\) are listed in Table 5.1. They decrease in magnitude and have same sign. Fig.5.2 shows the Domb-Sykes plot for series (5.3.13) in the case of \(\alpha = 0, \beta = 0\). The slope of the curve indicates square root singularity corresponding to double valuedness of the solution (by using rational extrapolation exact position of the singularity is found to be at \(R = 17.9826\) with an error of order \(10^{-5}\)). So the region of validity of the series (5.3.13) for \(A\) in the case of \(\alpha =\)
$\beta = 0$ will be increased by reverting the series (by changing the role of dependent and independent variables). We use Padé approximants for summing the reverted series (5.3.15) which accelerates the convergence and yields its analytic continuation. The results agree most favourably with results of Wang [27] (numerical), Bujurke and Naduvinamani [28] (semi-numerical) and Phan-Thien and Bush [30] (power series). It is of interest to note that $[2/2]$ and $[2/3]$ Padé approximants bracket the numerical results of Wang [27] (Fig.5.3). Double precision arithmetic used guarantees the accuracy of Padé approximants. Also, the round off errors will be of negligible order as the Padé approximants bracketing the numerical results are of the form where denominators are polynomials of degree $\leq 4$ (Graves-Morris [44]). Table 5.2 contains the list of coefficients $a_n$ of the series (5.3.13) for the case of $\alpha = 4, \beta = 0$. These coefficients decrease in magnitude but have no regular sign pattern. We invoke Padé approximants to achieve analytic continuation of the series (5.3.13). The results agree favourably with earlier numerical findings (Wang [43]). Also, we observe that $[2/2]$ and $[2/3]$ Padé approximants bracket the numerical results which are given in Fig.5.4. The coefficients $a_n$ for the case of $\alpha = 0, \beta = 0.5$ are listed in the Table 5.3. In this case also coefficients are decreasing in magnitude and have no regular sign pattern. As in
the previous case analytic continuation of the series (5.3.13) is achieved by using Pade' approximants. The [3/4] Pade' approximant is found to be very near to the numerical results (Wang [43]) which are shown in Fig.5.5.

The method proposed here is quite flexible and efficient in implementing on computer compared with the pure numerical methods. Once the universal coefficients are generated rest of the analyses can be done at a stretch requiring hardly any computer time and storage. Whereas other methods ([43], [27]) require more computer time and large storage and are silent about analytic structure of the solution function.
Table 5.1: The coefficients of the series (5.3.13) for \( f'''(1) \) in the case of \( \alpha = 0, \beta = 0 \).

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Table 5.2: The coefficients $a_n$ of the series (5.3.13) for $f^{'''}(1)$ in the case of $\alpha = 4$, $\beta = 0$.

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Table 5.3: The coefficients $a_n$ of the series (5.3.13) for $f'''(1)$ in the case of $\alpha = 0, \beta = 0.5$.

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Fig. 5.2: Domb-Sykes plot for coefficients of series (5.3.13) in the case of \( \alpha = 0, \beta = 0 \).
Fig. 5.3: Values of $A$ as a function of $R$ ($\alpha = 0$, $\beta = 0$).
Fig. 5.4: Values of $A$ as a function of $R$ ($\alpha = 4, \beta = 0$).
Fig. 5.5: Values of $A$ as a function of $R$ ($\alpha = 0$, $\beta = 0.5$).
Fig. 5.6: Radial velocity distribution \( f'(\eta) \) at \( R = 10 \).
Fig.5.7: Radial velocity distribution $f'(\eta)$ at $R = 20$. 
Fig. 5.8: Radial velocity distribution $f'(\eta)$ for different $R$
(suction).