CHAPTER VIII

THE EFFECT OF VARIABLE PERMEABILITY AND ROTATION ON THE PERFORMANCE CHARACTERISTICS OF POROUS BEARINGS*

* Part of this chapter is to appear in WEAR (1991)
8.1. INTRODUCTION

The characteristics of squeeze film with variable permeability and random roughness is studied in chapter vi. In the present chapter, we consider the rotation effect together with variable permeability. The lubrication aspect of short journal bearing with variable permeability is chosen for the purpose of illustration.

The present chapter is an attempt to study the effect of rotation on the lubrication mechanism by considering porous nature of the short journal bearing. The permeability in the axial direction is assumed to be \( k(z) = k_0 \left[ 1 + a \cos \frac{nz}{L} \right] \). This representation approximates the curves given by Margon [78] for the permeability variation along the length of the porous bearing. The permeability in the radial direction is assumed to be constant and it is designated as \( k_y \).

8.2. FORMULATION OF THE PROBLEM AND METHOD OF SOLUTION:

Fig.8.1 shows a short porous journal bearing under steady load. Considering the oil to be incompressible, the viscosity to be constant, and the radius of the bearing to be large compared to the wall thickness, the governing equation for the pressure distribution in the porous matrix can be derived in the form [71]

\[
\frac{\partial}{\partial y} \left[ k_y \frac{\partial p_1}{\partial y} \right] + \frac{\partial}{\partial z} \left[ k(z) \frac{\partial p_1}{\partial z} \right] = 0. \tag{8.2.1}
\]

Considering

\[
K_z = k_z / k_y, \quad \bar{p}_1 = p_1 \frac{c^2}{R_d U}, \quad \bar{y} = y/(L/2), \quad \bar{z} = z/(L/2),
\]
fig. 8.1 The physical model of the problem.
equation (8.2.1) reduces to

\[ \frac{\partial^2 p_1}{\partial y^2} + K \frac{\partial}{\partial z} \left( \frac{1}{2} \cos \frac{\pi z}{L} \right) \frac{\partial p_1}{\partial z} = 0. \]

The boundary conditions for the porous matrix are

\[ p_1(\bar{y}, \pm 1) = 0, \quad (8.2.3) \]

\[ \frac{\partial p_1}{\partial \bar{y}} (-H, \bar{z}) = 0, \quad (8.2.4) \]

\[ p_1(0, \bar{z}) = \bar{p}(\bar{z}). \quad (8.2.5) \]

Condition (8.2.3) shows that the ends of the bearing are exposed to the atmosphere. Condition (8.2.4) indicates that the porous bearing is fitted into an impermeable bushing and condition (8.2.5) gives the continuity of pressure at the interface between the bearing and the oil film.

The governing equation for oil film is given by a modified form of the Reynolds equation [79], viz;

\[ \frac{d^2 p}{dz^2} - Mh \frac{d}{dx} \frac{d}{dz} \frac{d^2 p}{dz^2} = 6 \frac{\phi}{h^3} \left( \frac{\partial p_1}{\partial y} \right)_{y=0} - 6(\frac{CL}{D}) \frac{dh}{dx} \]

where \( M = \frac{\Omega p c^2}{2\mu} \), \( \psi = \frac{k_y H}{c^3} \).

Assuming a separation solution of the form

\[ p_1(\bar{y}, \bar{z}) = F_1(\bar{y}) F_2(\bar{z}) \]

and substituting into equation (8.2.2) the following equations

\[ k(z) = k(1 + a \cos \frac{\pi z}{L}) \]
\[
\frac{d^2 F_1}{d y^2} - \chi^2 F_1 = 0 \quad (8.2.7)
\]

\[
\frac{d}{dz} \left[ (1 + \cos \frac{n z}{L}) \frac{d F_2}{dz} \right] + \frac{\chi^2}{k_2} F_2 = 0 \quad (8.2.8)
\]

for \( F_1 \) and \( F_2 \) are obtained, where \( \chi \) is a separation constant.

The value of \( \chi \) can be found by using Galerkin's method \([72]\).

The solution of the equation \((8.2.8)\) is assumed to be of the form

\[
F_2(z) = \sum_{i=1}^{n} c_i \phi_i(z) \quad (8.2.9)
\]

A system of functions \( \phi_i(z) \) which is complete and satisfies the boundary conditions given by \((8.2.3)\) is

\[
\phi_i(z) = \cos \beta_i z
\]

where

\[
\beta_i = \frac{\pi}{L} (2i-1).
\]

Considering \( L \) to be a differential operator and setting

\[
L(F_2) = -\frac{d}{dz} \left[ (1 + \cos \frac{n z}{L}) \frac{d F_2}{dz} \right] + \frac{\chi^2}{4 k_2} F_2 = 0, \quad (8.2.10)
\]

the \( \chi \)'s are found by requiring the functions \( \phi_1, \phi_2, \phi_3, \ldots, \phi_n \) to be orthogonal to \( L(F_2) \), i.e.

\[
\int_{-1}^{1} L(F_2) \phi_1(z) \, dz = 0.
\]

Substituting for \( L(F_2) \) and \( \phi_1(z) \), the above integral gives

\[
\sum_{i=1}^{n} \left( a_{ij} + \chi^2 b_{ij} \right) c_i = 0 \quad j = 1, 2, 3, \ldots, n
\]

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\[ b_{ij} = \frac{\delta_{ij}}{4K_2} \]
\[ a_{ij} = -\delta_{ij} \beta_i^2 - \frac{a\beta_i}{\pi} \left( -1 \right)^{i+j} \left[ \frac{1}{2i+2j-9} - \frac{1}{2i+2j-1} + \frac{1}{2i-2j+1} \right] \]
\[ - \frac{1}{2i+2j-1} \right] + \frac{a\beta_i}{\pi} \left[ \frac{1}{2i+2j-9} + \frac{1}{2i+2j-1} - \frac{1}{2i-2j+1} \right]. \]

The \( \chi \)'s are obtained by setting the determinant of the system \( a_{ij} + \chi b_{ij} \) equal to zero. Using the boundary conditions given by equation (8.2.3) and noting that the solution of equation (8.2.7) is
\[ F_1 = a e^\chi y + b e^{-\chi y}, \]

it can be shown that the pressure distribution in the porous bearing is given by
\[ p_1(y,z) = \sum_{n=1}^{\infty} B_n \left( e^{\chi_n y} + e^{-\chi_n y} \right) \cos \beta_n z. \quad (8.2.11) \]

At \( \overline{y} = 0 \), equation (8.2.11) can be written as
\[ p_1(0,z) = \sum_{n=1}^{\infty} \chi_n A_n \cosh \chi_n \cos \beta_n z, \quad (8.2.12) \]

it can also be shown that
\[ \frac{\partial \overline{p}_1}{\partial \overline{y}} \bigg|_{\overline{y}=0} = \sum_{n=1}^{\infty} \chi_n A_n \sinh \chi_n \cos \beta_n z. \quad (8.2.13) \]

Solution of equation (8.2.6) satisfying boundary conditions \( p = 0 \) at \( \overline{z} = \pm 1 \) is
\[ \overline{p} = \frac{\overline{\lambda}_2}{M \overline{\lambda}_1} \left[ 1 + \overline{z} - \frac{\overline{M} \overline{\lambda}_2}{\sinh M \overline{\lambda}_1} \left( e^{\overline{\lambda}_1 \overline{z}} - e^{-\overline{\lambda}_1 \overline{z}} \right) \right] \quad (8.2.14) \]

where
Equation (8.2.14) may be written as

\[ \bar{p} = \sum_{n=0}^{\infty} f_n \bar{n} \]

where

\[ f_0 = \frac{\bar{\lambda}^2}{\bar{\eta}} (\bar{z}^2 - 1) \]

\[ f_1 = -\frac{\bar{\lambda} \bar{\lambda}^2}{\bar{\eta}} (\bar{z}^{3+1}) \]

etc.

Using the condition given by (8.2.5), ie, equating equations (8.2.12) and (8.2.14) at \( \bar{y}=0 \), multiplying both sides by \( \cos \beta \bar{z} \), and integrating it from \( \bar{z} = 0 \) to \( \bar{z} = 1 \), we get

\[ A = -\frac{3L/D}{(1+M\bar{A}_4/3)} \]

\[ \beta_n \cosh \beta_n \left[ h^3 + 3\chi_n (L/H) (1+M\bar{A}_4/3) \tanh \beta_n \right] \]

(8.2.12)

Substituting the value of \( A \) into \( \bar{p} \), we get

\[ \bar{p}_i = -\frac{3(1+M\bar{A}_4/3)}{(D/L)} \sum_{n=1}^{\infty} \frac{(dh/d\bar{x}) \cos \beta_n \bar{z} (-1)^{n+1}}{\beta_n \left[ h^3 + 3\chi_n (L/H) (1+M\bar{A}_4/3) \tanh \beta_n \right]} \]

(8.2.16)

For journal bearing the film thickness is defined as \( h=1+\varepsilon \cos \theta \), \( \bar{x} = R\theta \) and \( dh/d\bar{x} = -\frac{\varepsilon \sin \theta}{R} \). Therefore, equation (8.2.16) becomes

\[ \bar{p}_i = \frac{1}{RC/D/L} \sum_{n=1}^{\infty} D_n \]

(8.2.17)
where

\[ D = \frac{3\varepsilon \sin \theta (1 - \varepsilon M \sin \theta (1 + \varepsilon \cos \theta) / 3) \ (-1)^{n+1} \cos \beta \ z}{\beta_n \left( (1 + \varepsilon \cos \theta)^3 + 3\psi L / H \right) \chi_n \left( 1 - (1 + \varepsilon \cos \theta) \varepsilon \sin \theta / 3 \right) \tanh \chi_n} \]

The load capacity of the bearing can be obtained by integrating the pressure profile. The dimensionless load components along and normal to the line of centers are respectively

\[
W = \int_{-1}^{1} \int_{0}^{\pi} \bar{p}_1 \cos \theta \, d\theta \, dz
\]

\[
= \sum_{n=1}^{\infty} \int_{0}^{\pi} \frac{3\varepsilon \sin \theta \cos \theta (1 - \varepsilon \sin \theta (1 + \varepsilon \cos \theta) / 3) \, d\theta}{\beta_n^2 \left( (1 + \varepsilon \cos \theta)^3 + 3\psi \chi_n \left( L / H \right) \left( 1 - (1 + \varepsilon \cos \theta) \varepsilon \sin \theta / 3 \right) \tanh \chi_n \right)}
\]

\[
W_2 = \int_{-1}^{1} \int_{0}^{\pi} \bar{p}_2 \sin \theta \, d\theta \, dz
\]

\[
= \sum_{n=1}^{\infty} \int_{0}^{\pi} \frac{3\varepsilon \sin^2 \theta (1 - \varepsilon \sin \theta (1 + \varepsilon \cos \theta) / 3) \, d\theta}{\beta_n^2 \left( (1 + \varepsilon \cos \theta)^3 + 3\psi \chi_n \left( L / H \right) \left( 1 - (1 + \varepsilon \cos \theta) \varepsilon \sin \theta / 3 \right) \tanh \chi_n \right)}
\]

The load capacity of the bearing is given by

\[
\bar{W} = \pi \left( W_1^2 + W_2^2 \right)^{1/2}
\]

The coefficient of friction can be obtained in the form [7]

\[
f = \frac{2 \pi^2 S}{(1 + \varepsilon^2)^{1/2}}
\]

where \( S \) is the Sommerfeld number and for load number, we have

\[
1 / S (L / D)^2 = \pi (W_1^2 + W_2^2)^{1/2}
\]

The altitude angle is given by the relation [9]

\[
\Phi = \tan^{-1} \left( -\frac{W_1}{W_2} \right)
\]

\[ \text{126} \]
8.3. DISCUSSION OF RESULTS:

In porous bearings the usual method for finding the permeability is to close its ends and measure the flow through the wall caused by pressurizing a fluid in the bore. In calculating the load capacity of porous bearing it is usually assumed that the permeability is constant throughout the structure of the bearing and that its magnitude is equal to the average radial permeability obtained from experiments. Because of this fact, it is of interest to compare the performance characteristics of a porous bearing having an axial and radial permeability of $k \left[ 1 + \cos \frac{\pi z}{L} \right]$ and $k \left[ 1 + \frac{z}{\pi} \right]$ respectively. Only 13 terms of the series ((8.2.18), (8.2.19)) are required to meet the sufficient accuracy for the evaluation of load, coefficient of friction and altitude angle. To minimize the roundoff error we have used double precision arithmetic.

The values in the Table 8.1, give the variation of load capacity ($W$), coefficient of friction ($F$) and altitude angle ($\psi$) with respect to eccentricity ratio $\varepsilon$. It is observed that the values of these physical parameters are depending upon the orientation of rotation. These have large values for $M=-0.2$ and small values for $M=0.2$ compared to that of non-rotational case ($M=0.0$). We observe the same type of variations in [3]. Also from the results mentioned in Table 8.1 and 8.2, we find that $W$, $F$ and $\psi$ have large values in anisotropic case compared with isotropic case.
Table 8.3 shows the variation of $W$, $F$ and $\phi$ with the design variable $\psi$, for fixed value of rotational parameter (MD) and eccentricity ratio ($e$). As $\psi$ increases both $W$ and $F$ decrease whereas $\phi$ increases for both isotropic and anisotropic case.
Table 8.1. Variation of $W$, $F$ and $\psi$ with $\varepsilon$ for anisotropic case.

$k_c(z) = k_c(y) = k_c \left( 1 + \frac{z}{\pi} \right)$, $\psi = 0.0001$

<table>
<thead>
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<th>$M$</th>
<th>$\varepsilon$</th>
<th>Load capacity</th>
<th>Coefficient of friction</th>
<th>Altitude angle</th>
</tr>
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Table 8.2. Variation of $W$, $F$ and $\psi$ with $\varepsilon$ for isotropic case.

\[
k(z) = k(y) = k \left( \varepsilon + \varepsilon_0 \right)
\]

<table>
<thead>
<tr>
<th>$M$</th>
<th>$\varepsilon$</th>
<th>Load capacity</th>
<th>Coefficient of friction</th>
<th>Altitude angle</th>
</tr>
</thead>
<tbody>
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Table 8.3. Variation of $\bar{W}$, $F$ and $\psi$ with $\psi$.

\[ M = -0.2, \quad \epsilon = 0.6 \]

<table>
<thead>
<tr>
<th>$\psi$</th>
<th>Load capacity</th>
<th>Coefficient of friction</th>
<th>Altitude angle</th>
</tr>
</thead>
</table>
|       | \begin{align*}
0.0001 & : 381.37590+01 \\
0.001 & : 379.26050+01 \\
0.01 & : 360.16890+01 \\
0.1 & : 264.3130+01 \\
1.0 & : 996.3951+00 \\
\end{align*} | \begin{align*}
0.0001 & : 645.52540+02 \\
0.001 & : 641.94480+02 \\
0.01 & : 609.62640+02 \\
0.1 & : 447.75160+02 \\
1.0 & : 168.65210+02 \\
\end{align*} | \begin{align*}
0.0001 & : 81.493820 \\
0.001 & : 81.670420 \\
0.01 & : 83.596600 \\
0.1 & : 92.397800 \\
1.0 & : 1.1898890 \\
\end{align*} |

Anisotropic case

<table>
<thead>
<tr>
<th>$\psi$</th>
<th>Load capacity</th>
<th>Coefficient of friction</th>
<th>Altitude angle</th>
</tr>
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</table>
|       | \begin{align*}
0.0001 & : 381.37590+01 \\
0.001 & : 377.75390+01 \\
0.01 & : 349.91400+01 \\
0.1 & : 233.34920+01 \\
1.0 & : 763.28750+00 \\
\end{align*} | \begin{align*}
0.0001 & : 645.52540+02 \\
0.001 & : 639.39480+02 \\
0.01 & : 592.27230+02 \\
0.1 & : 394.97220+02 \\
1.0 & : 1.291.9240+02 \\
\end{align*} | \begin{align*}
0.0001 & : 81.493820 \\
0.001 & : 81.795140 \\
0.01 & : 84.104490 \\
0.1 & : 95.883880 \\
1.0 & : 1.12490430 \\
\end{align*} |

Isotropic case