CHAPTER VII

LUBRICATION OF POROUS JOURNAL BEARING WITH ROUGH SURFACES
7.1 INTRODUCTION:

In the preceding chapters, we have studied the roughness effects in squeeze film bearing with transversely oriented roughness. The present chapter is concerned with the study of short porous rough journal bearing, with transverse or longitudinal roughness structure.

Recently Sinha and Singh [74] have studied the lubrication of rough surfaces, in line with Christensen [36], including the cases of longitudinal and transverse roughness. The dynamic characteristics of journal bearing for the same cases of roughness has been studied by Sinha and Raj [75], but they did not consider porous effects. Prakash and Vij [76] have studied the performance of narrow porous journal bearing using Beavers-Joseph criterion of velocity slip but they neglected the roughness effect. In the present chapter, we attempt to consider both roughness and porosity effects. We find that the load carrying capacity increases for transverse roughness and decreases for longitudinal roughness, further we show the porosity and roughness effects on the performance of short journal bearings.

7.2 FORMULATION OF THE PROBLEM AND THE METHOD OF SOLUTION:

A schematic diagram of the system under study is presented in fig.7.1. It shows a porous journal of radius R rotating with a uniform tangential velocity U. The flow is assumed to be laminar and the fluid is incompressible. The governing
fig. 7.1  Geometry of the rough journal bearing.
equation for the oil film is given by a modified form of the Reynolds equation (5.2.1).

The equation for pressure in the porous region is

\[ \nabla^2 p_A = 0. \]  

(7.2.1)

The boundary conditions for the porous matrix are

\[ p_A(y, \pm l/2) = 0 \]  

(7.2.2)

\[ \frac{\partial p_A}{\partial y} \bigg|_{y=-H} = 0 \]  

(7.2.3)

\[ p_A(0, z) = p(z) \]  

(7.2.4)

7.3 SURFACE ROUGHNESS STRUCTURE:

The study of the surface roughness is based on the assumption of the stochastic film thickness defined by the equation (4.2.10).

7.4 TRANSVERSE ONE-DIMENSIONAL ROUGHNESS:

The film thickness can be described by

\[ h = h(x, z) + h(z, \xi) . \]  

(7.4.1)

The three-dimensional Reynolds-type equation derived using Christensen theory is given by equation (6.2.11).

The boundary conditions for the oil film are

\[ p(\pm l/2) = 0. \]  

(7.4.2)

Using the short-bearing approximation, equation (6.2.11) becomes
\[
\frac{\partial}{\partial z} \left[ \frac{1}{E(1/h^3)} \frac{\partial}{\partial z} E(p) \right] = 6U \mu \frac{\partial E(h) }{\partial x} + 12k \sum_{i=1}^{\infty} \frac{\partial p_i}{\partial y} \bigg|_{y=0}.
\] (7.4.3)

In accordance with the short-bearing approximation, the effect of the circumferential pressure gradient on the flow is neglected and equation (7.4.1) reduces to

\[
\frac{\partial^2 p_1}{\partial y^2} + \frac{\partial^2 p_1}{\partial z^2} = 0.
\] (7.4.4)

The solution of equation (7.4.4) is obtained by separation of variables in the form

\[
p_1(y, z) = \sum_{n=1}^{\infty} a_n \sin(\beta_n(z+L/2)) \cosh(\beta_n(y+H))
\] (7.4.5)

where

\[
\beta_n = \frac{n\pi}{L}.
\]

Therefore,

\[
p_1(0, z) = \sum_{n=1}^{\infty} a_n \sin(\beta_n(z+L/2)) \cosh(\beta_n H).
\] (7.4.6)

Also,

\[
\frac{\partial p_1(0, z)}{\partial y} = \sum_{n=1}^{\infty} a_n \beta_n \sinh(\beta_n H) \sin(\beta_n(z+L/2)).
\] (7.4.7)

Substituting the value of \(\frac{\partial p_1(0, z)}{\partial y}\) into equation (7.4.3), we get

\[
\frac{\partial}{\partial z} \left[ \frac{1}{E(1/h^3)} \frac{\partial}{\partial z} E(p) \right] = 6U \mu \frac{\partial E(h) }{\partial x} + \\
+ 12k \sum_{i=1}^{\infty} \sum_{n=1}^{\infty} a_n \beta_n \sin(\beta_n(z+L/2)) \sinh(\beta_n H).
\] (7.4.8)
The particular value of probability density function and expected values are defined \((4.2.22), (4.2.23), (4.2.24)\) in chapter iv. Integrating equation \((7.4.3)\) and using equation \((7.4.2)\), we get

\[
E(p) = \frac{3\mu}{E(1/h^3)} \frac{dh/dx}{(z^2 - \frac{L^2}{2})} - \frac{12k}{E(1/h^3)} \sum_{n=1}^{\infty} \frac{a_n}{\beta_n} \sinh[\beta_n H] \sin[\beta_n (z + \frac{L}{2})].
\]

\((7.4.9)\)

From the continuity of pressure at the interface and using orthogonality property of eigenfunctions, we get

\[
a_n \cos[\beta_n H] = \frac{12\mu}{E(1/h^3)} \frac{dh/dx}{(1 - (-1)^{n+1})} \frac{12k}{\beta_n^3} \left[ \frac{1}{E(1/h^3)} + \frac{12k}{\beta_n^3} \tanh[\beta_n H] \right] \sin[\beta_n (z + \frac{L}{2})].
\]

Substituting this into equation \((7.4.6)\), we have

\[
p_1(0, z) = \sum_{n=1}^{\infty} \frac{12\mu c \varepsilon}{\beta_n^3 R L} \sin[\beta_n (z + \frac{L}{2})].
\]

\((7.4.10)\)

Letting \(h = (1 + \varepsilon \cos \theta)\), then \(dh/dx = - \frac{1}{R} \varepsilon \varepsilon \sin \theta\) and

\[
p_1(0, z) = \sum_{n=1}^{\infty} \frac{12\mu c \varepsilon}{\beta_n^3 R L} \left[ \frac{1}{E(1/h^3)} + \frac{12k}{\beta_n^3} \tanh[\beta_n H] \right] \sin[\beta_n (z + \frac{L}{2})].
\]

\((7.4.11)\)

The load capacity of the bearing can be obtained by the integration of the pressure profile. The load components along and normal to the line of centers are, respectively
\[ W_0 = -2R \int_{0}^{L/2} \int_{0}^{\pi} p_1 \cos \theta \, d\theta \, dz \] (7.4.12)

\[ W_{\pi/2} = 2R \int_{0}^{L/2} \int_{0}^{\pi} p_1 \sin \theta \, d\theta \, dz \] (7.4.13)

Substituting (7.4.11) into (7.4.12), we have

\[ W = \frac{W_0}{\mu UL} (C/R)^2 (D/L)^2 = 96\varepsilon \sum_{n=1}^{\infty} \frac{[1 - (-1)^n]}{n^4} M_n \]

Substituting (7.4.11) into (7.4.13), we get

\[ W = W_{\pi/2} (C/R)^2 (D/L)^2 = 96\varepsilon \sum_{n=1}^{\infty} \frac{[1 - (-1)^n]}{n^4} M_n \]

where

\[ L_n = \int_{0}^{\pi} \frac{\sin \theta \cos \theta \, d\theta}{f(h, \psi)} \]

\[ M_n = \int_{0}^{\pi} \frac{\sin^2 \theta \, d\theta}{f(h, \psi)} \]

\[ f(h, \psi) = E(1/h^3) + \frac{12\psi}{nnH} \tanh(nnH) \]

The load capacity in terms of load number is given by (Cusano[71])

\[ S(C/L/D)^2 = \frac{1}{\pi \bar{W}} \] (7.4.14)

where

\[ \bar{W} = \left( W_1^2 + W_2^2 \right)^{1/2} \]

The coefficient of friction can be obtained from [71] in the form

\[ f(R/C) = \frac{2n^2 S}{(1 + \varepsilon^2)^{1/2}} \]

where \( S \) is the Sommerfeld number.
In this case the film thickness and three-dimensional Reynolds equation take the forms

\[ h = h_1(\theta, z, t) + h_2(\theta, \xi) \]

and

\[
\begin{align*}
\frac{\partial}{\partial x} \left[ \frac{1}{E(1/h^3)} \frac{\partial E(p)}{\partial x} \right] + \frac{\partial}{\partial z} \left[ E(h^3) \frac{\partial E(p)}{\partial z} \right] &= 6U \mu \frac{\partial E(h)}{\partial x} \\
&+ 12k_1 \frac{\partial p}{\partial y} |_y = 0 \tag{7.5.1}
\end{align*}
\]

respectively.

In a way similar to that of the previous section, the pressure distribution and the load components perpendicular to and along the line of centers can be obtained in the forms

\[
p_1 = \sum_{n=1}^{\infty} \frac{12U \mu \varepsilon \sin \theta \left[ 1 - (-1)^{n+1} \right]}{\beta_n^3 RL \left[ 3E(h^3) + \frac{12k_1}{\beta_n^3} \tanh(\beta_n H) \right]} \sin(\beta_n [z + L / 2])
\]

and

\[
w_1 = 96 \varepsilon \sum_{n=1}^{\infty} \frac{[1 - (-1)^{n+1}]}{n \pi^4} L_N
\]

and

\[
w_2 = 96 \varepsilon \sum_{n=1}^{\infty} \frac{[1 - (-1)^{n+1}]}{n \pi^4} M_N
\]

where

\[
L_N = \int_0^\pi \frac{\sin \theta \cos \theta \, d\theta}{g(h, \psi)} , \quad M_N = \int_0^\pi \frac{\sin^2 \theta \, d\theta}{g(h, \psi)}
\]

\[ g(h, \psi) = E(h^3) + \frac{12\psi}{nH} \tanh(nH) \]
7.6 DISCUSSION OF RESULTS:

In the present study, the major influencing dimensionless parameters are, roughness parameter $C_r$, permeability parameter $\psi$, and the eccentricity ratio $\varepsilon$. The parameter $C_r$ is a characteristic of the bearing geometry and arises due to the surface roughness. It is obvious that the roughness effects would be accentuated when $C_r$ is comparable to the bearing clearance. A detailed discussion and limitation of these parameters are given by Christensen and Tonder [77] and Christensen [36]. We find that $C_r \to 0$ gives the case of smooth surfaces.

The graphs in fig.7.2 show the characteristics of roughness structures with respect to eccentricity ratio. The load carrying capacity increases for transverse roughness and decreases for longitudinal roughness compared with smooth case. This is primarily caused by the directional patterns of the ridges. In longitudinal roughness these are in the direction of fluid motion whereas, in transverse roughness case they are in the transverse direction of fluid motion. The increase of 3.1% for transverse roughness and a decrease of 1.45% in load capacity for longitudinal roughness over the smooth case is observed.

The graphs in fig.7.3, show the variation of load against $\log_{10}(\psi)$ for different values of $\varepsilon$ for the constant value of roughness parameter $C_r=0.8$. It is observed that the load decreases for decreasing values of $\varepsilon$ and its variation is
small for higher values of $\log_{10}(\psi)$. The profiles in fig. 7.4
show the variation of load number $\Lambda$, roughness parameter $C_r$,
for different values of eccentricity ratio. The decrease of
3.51% in load number for the transverse roughness and an
increase of 1.30% in load for longitudinal roughness over
smooth case is found.

The coefficient of friction $f(C/\Sigma)$ has been plotted as a
function of eccentricity ratio for different roughness
parameters in fig. 7.5. It is seen that, the longitudinal
roughness leads to a significant increase in coefficient of
friction compared to smooth and transverse roughness surfaces.
Fig. 7.2 Variation of load with $\varepsilon$ for different roughnesses.
fig. 7.3 Variation of load with log(\(\psi\)) with \(\varepsilon\) for different \(\varepsilon\).
fig. 7.4 Variation of load number with $\bar{c}_r$ for different $\varepsilon$ and roughnesses.
fig. 7.5 Variation of load number with ϵ for different $C_r$ and roughnesses.
fig. 7.6 Variation of $f(R/C)$ with $\varepsilon$ for different $\bar{C}_f$ and roughnesses.