CHAPTER 2

BASIC ELECTRONIC PROPERTIES

OF

LOW - DIMENSIONAL

SEMICONDUCTOR STRUCTURES
2.1 Introduction

In the last few years there has been a dramatic increase of interest in the physics and applications of LDSS. Low dimensionality refers to the system in which the mobile charge carriers are constrained by the potential barriers so that they loose one or more degrees of freedom for motion. The system becomes two, one or even zero dimensional depending on whether the potential barriers confine the electrons in one, two or three directions, respectively. Consequently, the system of electronic states associated with each structure differs from the usual Bloch states from which they derive. The resulting optical and electronic properties are thus different from those in the bulk and are of interest from the point of view of fundamental research and technological applications. Heterojunction (HJ) devices are attracting increased attention due to their superior flexibility. This flexibility originates from the use of two different semiconductors with two different sets of parameters. In principle the control of these parameters enables the designers to tailor the HJ characteristics to fit the desired functions.

To study any physical property of these low dimensional structures it is essential to know their electronic structure. In this chapter, after a brief introduction to the electronic structure of bulk semiconductors in section 2.2, we will give the
electronic structure of various systems, used in our work to study 2D physics, in section (2.3.1). In section 2.3.2 we give the electronic structure of SLs.

The electronic properties of LDSS are strongly modified by the application of a magnetic field. The energy spectrum consists of Landau levels leading to an oscillatory behaviour of all physical properties as a function of the magnetic field. Such experiments have been used to explore the 2D nature of the carriers in QWS and SLs. We give a brief discussion of the electronic structure of 2D systems and SLs in a quantizing magnetic field in sections 2.4.1 and 2.4.2., respectively.

2.2 Bulk Semiconductors

The band structure of semiconductors is of central importance in understanding the charge carrier transport and optical properties. In the ‘one electron approximation’ [11], an electron sees the average potential \( V(R) \) due to all other charges in the medium which is assumed to be periodic with lattice period. Then the solution of the one electron Schrödinger equation

\[
\nabla^2 \Psi_K(R) + \frac{2m^*}{\hbar^2} (E_K - V(R)) \Psi_K(R) = 0 \quad (2.2.1)
\]

is of the form
\[ \Psi_K(R) = u_K(R) e^{iK \cdot R} \quad (2.2.2) \]

In the above equations \( m^* \) is the effective mass of the electron which is generally a tensorial quantity, \( K \) and \( R \) are the 3D wave vector and position vector, respectively, and \( u_K(R) \) is the cell periodic part.

Making use of the band theoretical approach \( [2] \) one can determine \( E_K \) versus \( K \) over all \( K \) points or at least at some special high symmetry points in the Brillouin zone (BZ). In dealing with transport problems we are concerned usually with the lowest minima of conduction band (CB) and the highest maxima of the valence band (VB) as they are populated, respectively, by the electrons and the holes under normal conditions. In these regions, irrespective of the complexity of the band structure, the quadratic relation between \( E_K \) and \( K \),

\[ E_K = \frac{\hbar^2 K^2}{2m^*} \quad (2.2.3) \]

holds.

Figure 2.1 displays the band structure of bulk GaAs. The CB is characterized by two minima, one at the \( \Gamma \) point and the other at a distance of 0.75\( \pi /a \) (where \( a \) refers to the lattice constant of GaAs) from the centre of the BZ in [100] direction.

In bulk semiconductors the \( K \)-states are quite close in the
Fig. 2.1  Band diagram of a bulk semiconductor.

$E_g$ - Energy gap.
BZ and they form a quasi continuum. To study the electrical transport and optical properties of any system one should investigate the density of states (DOS) carefully, because the change in the DOS affects directly these properties. The DOS per spin and per unit volume of a three dimensional electron gas for the dispersion relation (2.2.3) is given by

$$\rho_{3D}(E) = \left(\frac{1}{2\pi}\right)^2 \left(\frac{2m^*}{\hbar^2}\right)^{3/2} E^{1/2}$$

(2.2.4)

Making use of the above model and incorporating the appropriate scattering mechanisms, one can understand the general features of electron transport and optical phenomena. The excess carriers of either type can only be created by doping the material with appropriate impurities but the mobility of the charge carriers are limited owing to the increased scattering due to impurities. One can overcome this limitation to a reasonable extent in two dimensional structures like heterojunctions, quantum wells etc.

2.3 Low Dimensional Systems

In this section we briefly give the band structure of some of the LDSS, namely, heterojunctions, quantum wells and superlattices which are relevant to our study. Former two are quasi two-dimensional (Q2D) systems while the latter is a quasi
three-dimensional (Q3D) system.

2.3.1 Q2D Systems

The first device in which the carrier confinement was realized is Si-MOSFET (Metal Oxide Semiconductor Field Effect Transistor). In Si-MOSFET the 2D electron gas is formed, at the Si-SiO$_2$ interface, in the potential well arising from the creation of an inversion layer by the application of a positive bias to the gate. Subsequently higher quality 2D systems such as GaAs/GaAlAs HJs and QWS were fabricated and most of our work is related to these systems.

2.3.1 (a) Heterojunction (HJ)

Q2D electron systems can also be realized in semiconductor heterostructures fabricated by growing an interface of two lattice matched semiconductors of different band gap. A schematic diagram of GaAs/GaAlAs HJ is shown in figure 2.2 a) with its band diagram in figure 2.2 b). The Q2D electron gas is held close to the interface by the attractive potential of the donor impurities in the barrier. Spacer layer (undoped GaAlAs) is used between the impurities and the interface to reduce impurity scattering and to achieve very high mobilities. The effective potential well is nearly triangular in shape at the interface.
Fig. 2.2 a) Modulation doped n-type GaAs/GaAlAs heterostructure.

Fig. 2.2 b) Band diagram of a modulation doped heterojunction between AlGaAs (with undoped spacer and Si doped regions) and undoped GaAs.
The spatial extent of the Q2D electron gas is typically 30-100 Å and the surface density of the electrons, \( N_s \), is of the order of \( 10^{12} \text{ cm}^{-2} \). In HJ, unlike MOSFET, \( N_s \) is governed by Si donor concentration in GaAlAs.

For low carrier densities a good analytical approximation can be obtained using a triangular potential well for \( V(z) \) [3]

\[
V(z) = \begin{cases} 
\infty & z \leq 0 \\
e Fz & z > 0 
\end{cases} \quad (2.3.1)
\]

where

\[
F = 4\pi \left[ N_{\text{depl}} + \frac{1}{2} N_s \right] \frac{e}{\epsilon_s} \quad (2.3.2)
\]

is the effective field felt by the electrons. In equation (2.3.2) \( \epsilon_s \) is the static dielectric constant and \( N_{\text{depl}} \) is the charge concentration. The factor \( \frac{1}{2} \) gives the average field in the inversion layer.

Since the potential energy is taken to be a function of \( z \) only, the electron wave function \( \Psi_{nk}(r,z) \) can be written as the product of the Bloch function \( u_K(r,z) \), a plane wave vector representing the free motion in the xy-plane and a \( z \)-dependent factor \( \xi_n(z) \).
Here \( r \) \(=(x,y)\) and \( k \) \(=(k_x,k_y)\) are the position vector and the wave vector, respectively, of the electron in the plane of the layer. \( \xi_n(z) \) is the solution of the Schrödinger equation

\[
\frac{\hbar^2}{2m_z} \frac{d^2 \xi_n(z)}{dz^2} + \left( E_n - V(z) \right) \xi_n(z) = 0
\tag{2.3.4}
\]

where \( m_z \) is the electron effective mass perpendicular to the surface, \( E_n \) is the confinement energy of the carriers.

With the approximation (2.3.1), the solution of Schrödinger equation (2.3.4) are the Airy functions \([3]\) and the energy levels are given to a good approximation by

\[
E_n = \left( \frac{\hbar^2}{2m_z} \right)^{1/3} \left[ \frac{3\pi eF}{2} \left( n + \frac{3}{4} \right) \right]^{2/3}
\tag{2.3.5}
\]

with \( n = 0, 1, 2, \ldots \).

For low temperatures and concentrations only lowest subband \( E_0 \) is occupied and a better approximate solution is Fang-Howard variational wave function \([4]\). It is given by

\[
\xi_0(z) = \left( \frac{b^3}{2} \right)^{1/2} z \exp\left( -\frac{bz}{2} \right)
\tag{2.3.6}
\]
The optimum value of the variational parameter $b$ is given by

$$b = \left[ \frac{48\pi mz^2}{\hbar^2} \left( N_{\text{depl}} + \frac{11}{32} N_s \right) \right]^{1/3}$$  \hspace{1cm} (2.3.7)

For the wave function (2.3.6) the corresponding eigen value is given by

$$E_o = \left[ \frac{36e^2}{2\hbar x_s} \right] \left( N_{\text{depl}} + \frac{55}{96} N_s \right).$$  \hspace{1cm} (2.3.8)

The motion parallel to the interface (i.e. in $xy$-plane) is free and $E_0$ acts as a bottom of 2D subband. The energy eigen values associated with the wave function of equation (2.3.3) are given by

$$E_{nk} = E_n + \frac{\hbar^2 k^2}{2m^*} = E_n + E_k', \hspace{1cm} n = 0,1,2,....$$  \hspace{1cm} (2.3.9)

where

$$E_k = \frac{\hbar^2 k^2}{2m^*}$$  \hspace{1cm} (2.3.10)

and $m^*$ is the effective mass in the $x$-$y$ plane.
The DOS function per unit area per spin for a pure 2D system may be expressed as

\[ \rho_{2D}(E) = \frac{m^*}{2\pi\hbar^2} \]  

(2.3.11)

The DOS of pure 2D system is therefore independent of energy and layer thickness.

The DOS of a Q2D electron system, per unit area per spin for the dispersion relation (2.3.9) can be written as [5]

\[ \rho = \sum_{n,k} \delta (E - E_{nk}) = \rho_{2D} \sum_{n} \Theta (E - E_n), \]  

(2.3.12)

where \( \Theta(x) \) is the unit step function defined by

\[ \Theta(x) = \begin{cases} 
0, & x < 0 \\
1, & x \geq 0.
\end{cases} \]  

(2.3.13)

The DOS given by equation (2.3.12) is evidently step like. Thus Q2D DOS shows discontinuities for each \( n \). The Q2D system is truly 2D when one deals with only one electric subband. If more subbands come into play the system loses gradually its 2D features.
Another layered structure that contains a 2D electron system is a quantum well (QW). A semiconductor QW is fabricated by sandwiching a layer of a smaller band gap material between two layers of a larger band gap material of thickness much greater than the penetration length of the confined wave function. Schematic band diagram of QW formed by GaAs and GaAlAs is shown in figure 2.3. The CB edge in the sandwiched layer is rectangular well with the bottom line curved upward at the center. The curving depends on the thickness of the layer and for very thin layers becomes negligible. The band discontinuities are such that both types of carriers are confined in the smaller band gap material. The number and energetic positions of confined states depend on the size of the CB and VB offsets, $\Delta E_C$ and $\Delta E_V$, and on the well width $L$ (i.e., on the thickness of the GaAs layer). In GaAs/GaAlAs QW $\Delta E_C \approx x \, eV$ and $\Delta E_V \approx 0.3 \, x \, eV$ where $x$ is the Al mole fraction.

QW is usually grown using MBE technique. A buffer layer of undoped GaAs (about 1µm thickness) is first grown on semi-insulating (usually Cr doped) GaAs substrate. Layers of GaAlAs and GaAs of different thicknesses are then grown alternately. In modulation doped samples the dopants are placed in the GaAlAs layers. The width of the GaAs QW is typically 50-100 Å and heights of the potential barriers (i.e., band discontinuities) at the interfaces are of the order of $\Delta E_C \approx 300$.
Fig. 2.3  Schematic diagram of a quantum well with rectangular well approximation.

C - Conduction band edge.
FL - Fermi level.
R - Rectangular approximation.

Fig. 2.4  Q2D density of states (DOS) along with 2D DOS (dotted curve) and 3D DOS (broken curve).
meV and $\Delta E_v \approx 100$ meV, for $x = 0.3$.

The wave functions and the eigen values for a QW of thickness $L$, in the effective mass approximation can be written, respectively, as

$$\psi_{nk}(r,z) = \Omega^{-1/2} \exp(i k \cdot r) \xi_n(z)$$  \hspace{1cm} (2.3.14)

and

$$E_{nk} = E_n + E_k,$$  \hspace{1cm} (2.3.15)

where $\Omega$ is the area of the layer, $\xi_n(z)$ is the envelope wave function, $E_k$ is given by (2.3.10) and $E_n$ is the nth subband energy. $\xi_n(z)$ is determined to a good approximation by a Schrödinger wave equation (2.3.4).

For a QW of width $L$ and infinite height the solution of equation (2.3.4) is given by

$$\xi_n(z) = \left[ \frac{2}{L} \right]^{1/2} \sin \left( \frac{n \pi z}{L} \right)$$  \hspace{1cm} (2.3.16)

The corresponding eigen values are given by

$$E_n = n^2 E_0,$$  \hspace{1cm} (2.3.17 a)

where
\[ E_0 = \frac{n^2 \hbar^2}{2mL^2}, \quad n = 1, 2, 3, \ldots \quad (2.3.17\ b) \]

\( n^2 E_0 \) are the quantum energy levels arising from the confinement of the carriers in the narrow potential well. Each value of \( n^2 E_0 \) marks the beginning of a 2D continuum subband. Thus the spectrum shows increased level spacing with increasing quantum number \( n \) which is opposite to the behaviour in a triangular well. For very large \( L \) the above equation yields a continuum of states and the system no longer exhibits quantum effects.

For a QW of finite depth \([5]\), the effective potential in the growth direction as seen by the electron can be written as

\[
V(z) = \begin{cases} 
V_0, & z < L/2 \\
0, & -L/2 < z < L/2 \\
V_0, & z > L/2.
\end{cases} \quad (2.3.18)
\]

The QW, with an origin of the coordinate system chosen at the centre of the well has an inversion symmetry around the centre of the well and hence the solutions for wave functions can only be even or odd. In such a case the solution of equation (2.3.4), incorporating equation (2.3.18) can be written as
\[ \xi_n(z) = \begin{cases} \exp(az) & z < -L/2 \\ B \cos(k_z z) & -L/2 < z < L/2 \\ A \exp(-az) & z > L/2 \end{cases} \] (2.3.19)

where

\[ \alpha = \sqrt{\frac{2m_z(V_o - E_n)}{\hbar^2}}, \] (2.3.20 a)

\[ \tan\left(\frac{k_z L}{2}\right) = \left[ \frac{2m_z V_o}{\hbar^2 k_z^2} - 1 \right]^{-1/2} \] (2.3.20 b)

A and B are normalization constants given by

\[ A = B \cos\left(\frac{k_z L}{2}\right) \exp(\alpha L/2) \] (2.3.20 c)

\[ B^2 = \left[ \frac{L}{2} + \frac{\sin(k_z L)}{2k_z} + \frac{\cos^2(k_z L/2)}{\alpha} \right]^{-1} \] (2.3.20 d)

The equation (2.3.20 b) can be solved numerically or graphically for \( k_z \). Then the energy eigenvalues are obtained from the equation

\[ E_n = \frac{\hbar^2 k_z^2}{2m_z} \] (2.3.21)

Turning to the hole quantization problem, it is shown [6]
that the VB degeneracy of bulk GaAs is lifted due to confinement and separate heavy and light hole bands are formed. In each of these bands the energy spectrum of holes is given by equations similar to (2.3.15) with $m^*$ replaced by $m_+$ and $m_-$, the heavy and light hole masses, respectively. They are given by

$$m_+ = \frac{m_0}{(\gamma_1 - 2\gamma_2)} \quad (2.3.22\ a)$$

$$m_- = \frac{m_0}{(\gamma_1 + 2\gamma_2)} \quad (2.3.22\ b)$$

where $\gamma_1$ and $\gamma_2$ are the Kohn - Luttinger parameters and $m_0$ is the free electron mass.

The DOS $\rho_{Q2D}(E)$ for a Q2D electron gas in a QW, in view of equation (2.3.15), is the same as that already given in equation (2.3.12) for a HJ and is displayed in figure 2.4.

2.3.1 (c) Multiple Quantum Well (MQW)

The remarks about a single QW apply to the case of MQW. A number of single QW structures are grown successively on the same substrate with barrier thickness being chosen such that there is no interaction between successive layers as shown in figure 2.5 a). Hence MQW problem reduces to that of a single QW especially when the thickness of potential barrier is much larger than the electron tunnelling length. Band diagram for a modulation doped MQW is shown in figure 2.5 b).
Fig. 2.5 a) Schematic band diagram of a multiple quantum well (MQW) structure.

\[ \Delta E_c : \text{conduction band discontinuity.} \]
\[ \Delta E_v : \text{valence band discontinuity.} \]

Fig. 2.5 b) Electron band states in the modulation doped MQW structure.

FL : Fermi Level.
2.3.2 Superlattice (SL)

Superlattice (SL) is a periodic structure consisting of alternating ultrathin layers of two semiconductors of different band gaps. Such SL structure was first proposed by Esaki and Tsu [7] and realized by Chang et al [8]. In a SL the barrier layers are so thin that (~ 50 Å) the wave functions of the carriers in the neighbouring wells overlap significantly resulting in broadening of the otherwise discrete isolated QW energy levels into 'minibands'. Artificial periodicity of this spatial modulation is much larger than the lattice periodicity. This results in a reduced BZ scheme of 'minibands' separated by 'miniband gap'. The width of miniband and the miniband gap depend on the barrier height, width of the wells and the barriers. This tunability of miniband structure in SLs can be exploited to enhance the nonlinear response of the material. These novel features of SL make it possess very interesting transport properties which are quite different from bulk materials. Usually these properties are studied in the direction parallel (Bloch transport) and perpendicular to the SL growth axis. In our work we will consider mainly type - I SLs where the minimum of the conduction band and the maximum of the valence band of one semiconductor lie inside the energy gap of the other semiconductor. We restrict ourselves to the study of type - I SLs of GaAs/GaAlAs type, which is shown in figure 2.6 a).
1 - GaAs   2 - AlGaAs

Fig. 2.6 a) Schematic band diagram of a Type-I superlattice (SL).

Fig. 2.6 b) Kronig-Penny potential of a SL.
For a lattice matched GaAs / GaAlAs SL the effective potential in the growth direction as seen by the electrons can well be described by the Kröning-Penny potential

\[ V_{SL}(z) = \begin{cases} 
V_0 & a < z < a + 2b \\
0 & -a < z < a \\
V_0 & -(a + 2b) < z < -a 
\end{cases} \]  
(2.3.23)

as shown in figure 2.6 b). In equation (2.3.23) \(2a\) and \(2b\) are the widths of the well and the barrier layers, respectively. An exact solution of the Schrödinger equation can be obtained using Kröning-Penny model. However, this leads to a complicated expression for eigen values and wave functions. The band structure of a SL can be obtained most simply in the tight binding model using the effective mass theory and the envelope function approximation [5].

Assuming the effective mass change in the alternate layers to be small, as in a GaAs / GaAlAs SL, the envelope function \(\xi_{nk_z}(z)\), in the tight binding approximation can be expressed as,

\[ \xi_{nk_z}(z) = N_z^{-1/2} \sum_m \exp(ik_z md) \Phi_n(z-md), \]  
(2.3.24)

where \(\Phi_n(z-md)\) describes the solution of the mth GW centered at \(z = md\) and is obtained using the boundary conditions
\[ \phi^A = \phi^B \quad \text{and} \quad \frac{d\phi^A}{dz} = \frac{d\phi^B}{dz} \quad \text{at the interface A:B.} \]  
\[ \text{(2.3.25)} \]

In equation (2.3.24) \( d = 2a + 2b \) is the SL period and \( N_z \) is the number of SL periods.

The dispersion along the \( z \)-direction, for the overlap of only the nearest neighbour QW wave functions, can be written as

\[ E_{nk_z} = E_k + E_{nk_z} \]  
\[ \text{(2.3.26)} \]

where \( E_k \) is same as in equation (2.3.10),

\[ E_{nk_z} = t_n [1 - \cos(k_z d)] \]  
\[ \text{(2.3.27)} \]

and \( 2t_n \) is the band width of miniband which comes from the interaction of one well with its two neighbours in the chain. Thus, in a SL the dispersion of the electrons perpendicular to the layers i.e. along the SL axis to a good approximation is of tight binding type, while parallel to the layers can be assumed to be of the free electron type.

The SL effect introduces a profound change in the 2D DOS. The steppness of DOS of 2D system is removed. DOS, \( \rho_{SL}(E) \), for a SL, is given by
\[ \rho_{SL}(E) = \sum_n \rho_n(E), \quad (2.3.28) \]

where

\[ \rho_n(E) = \frac{2N_d}{\pi} \rho_{2D}(E) \int dk_z \Theta(E - E_{nk_z}). \quad (2.3.29) \]

The limiting values of \( \pm k_z \) occur for \( k = 0 \) and are given by

\[ k_z = \left\{ \begin{array}{ll}
\pm \cos^{-1}(1 - E/t_n) & 0 < E \leq 2t_n \\
\pm \pi & E > 2t_n
\end{array} \right. \]

Making use of the dispersion relation for \( E_{nk_z} \), \( \rho_n(E) \) can be evaluated and is given by

\[ \rho_n(E) = \left\{ \begin{array}{ll}
0 & E < 0 \\
\rho_o/\pi \cos^{-1}(1 - E/t_n) & 0 < E \leq 2t_n \\
\rho_o & E > 2t_n
\end{array} \right. \quad (2.3.30) \]

where \( \rho_o = 2N_z \rho_{2D} \). In figure 2.7 we have shown \( \rho_{SL}(E) \) for electrons in a GaAs/GaAlAs SL.

Thus, for energies above the miniband, the carrier motion is entirely 2D i.e. \( \rho_{SL} \cdot d = \rho_{2D} \).
Fig. 2.7 Density of states of a superlattice with $2t_n$, representing the width of nth miniband.
2.4 Low-Dimensional Systems In a Quantizing Magnetic Field

2.4.1 Q2D Systems.

The properties of Q2D electron system in an external magnetic field are very much interesting and they are extensively studied. Application of a strong magnetic field normal to the plane of Q2D electron system leads to Landau quantization of the electron energy in the plane. The energy spectrum of Q2D electron gas, therefore, becomes completely discrete. The constant DOS of Q2D electrons is converted into series of Dirac-delta functions. Such Q2D electron system provides an ideal tool for study of quantum transport phenomena. The observation of oscillatory behaviour of almost all physical quantities of Q2D systems as a function of the magnetic field is a direct evidence of the discrete DOS. QHE (91) is one of the important phenomena which is due to the complete quantization of the electron gas. In real systems Landau levels are broadened due to electron interaction with various systems present in the structure. The measurement of the width, $\Gamma$, of Landau levels helps to determine some of the physical properties of the Q2D electron system.

In a magnetic field, $B$, along $z$-direction, the 2D electron system is confined to move in the $xy$-plane. Using the gauge $A = (0, Bx, 0)$, the wave functions and eigen values are, respectively, given by
\[ \psi_{N,n,k_z} = \phi_N(x - x_0(k_y)) L_y^{-1/2} \exp(\text{i} k_y y) \xi_n(z) \] (2.4.1)

and

\[ E_{Nn} = \left( N + \frac{1}{2} \right) \hbar \omega C + E_n = E_N + E_n \] (2.4.2)

where the Landau quantum number \( N = 0, 1, 2, \ldots \), \( E_n \) is the energy of the \( n \)th subband, \( L_y \) is the sample dimension and \( k_y \) is the electron wave vector in the \( y \)-direction and \( \omega_C = eB/m^* \) is the cyclotron frequency, \( l = (\hbar/eB)^{1/2} \) is the cyclotron radius, \( \phi_N \) is the normalized simple harmonic oscillator wave function

\[ \phi_N(x) = \left[ \frac{2^N N!}{\sqrt{\pi}} \right]^{-1/2} \exp(-x^2/2l^2) H_N(x/l). \] (2.4.3)

and \( H_N \) is the Hermite polynomial. In equation (2.4.1), \( \xi_n(z) \) is the normalized wave function due to the confining potential \( V(z) \).

In the gauge chosen the electron states are plane waves in the \( y \)-direction and are localized in the \( x \)-direction. The coordinate \( x_0 \), related to the momentum \( k_y \) through

\[ x_0(k_y) = \frac{l^2}{2} k_y \] (2.4.4)

has the meaning of a center of the oscillator.

If the electron spin is considered then the magnetic energy
term need to be added to equation (2.4.2).

The constraint that the center of motion $x_0$ must lie within the sample of dimensions $L_x$ and $L_y$ shows that the degeneracy of each Landau level is

$$\frac{L_x L_y}{2\pi l^2} = l_x l_y \left[ \frac{eB}{\hbar} \right]. \quad (2.4.5)$$

Each Landau Level contains

$$N_L = \left( \frac{eB}{\hbar} \right) \quad (2.4.6)$$

states per unit area per spin. An additional factor of 2 can be added if spin splitting is not resolved.

Using the result (2.4.6) for the degeneracy one can specify the electron density $N_S$ in terms of the Landau level filling factor $\gamma$ defined by

$$\gamma = \frac{N_S}{N_L} = \frac{N_S}{(eB/h)} = 2\pi l^2 N_S. \quad (2.4.7)$$

This dimensionless number indicates how many of the successive Landau levels are filled. If we assume that the carrier density $N_S$ does not change with the magnetic field then the Fermi level $\xi$
will lie (at $T = 0$) in the $N$th Landau level, where

\[ N = \text{INT} \left( \frac{N}{2N_L} \right). \quad (2.4.8) \]

As a function of magnetic field $\zeta$, will exhibit a saw tooth oscillation, making a discontinuous jump each time an integer number of Landau levels are filled.

Equations (2.4.7) and (2.4.8) imply that for even integral value of $\gamma$, $\zeta$ lies in the orbital gap between the Landau levels whereas for odd $\gamma$, $\zeta$ coincides with a given Landau level.

The DOS function $\rho(E)$ for the dispersion relation (2.4.2) can be written as

\[ \rho(E) = \frac{1}{2\pi^2} \sum_{N,n} \delta(E - E_{Nn}). \quad (2.4.9) \]

Due to complete quantization of $E_{Nn}$, $\rho(E)$ shows a series of equidistant $\delta$-function peaks at each $E = E_{Nn}$ (see figure 2.8).

2.4.2 **Superlattice**

With the application of the magnetic field normal to the SL layers, the in-plane motion of the electron is quantized into Landau levels whereas the energy spectrum along the direction of the SL axis remains unaffected.
Fig. 2.8 Schematic diagram of density of states of a 2DEG in a magnetic field perpendicular to the plane of the layer.

a) without Landau level broadening.

b) with Landau level broadening.
In the effective mass model the one electron eigen functions and eigen values, for magnetic field along the SL axis, are, respectively, given by

$$\Psi_{N,n,k_y,k_z} = \phi_N[x - x_0(k_y)] L_y^{-1/2} \exp(ik_y y) \zeta_{nk_z}(z)$$

(2.4.10)

and

$$E_{Nnk_z} = \left( N + \frac{1}{2} \right) n\omega \omega + E_{nk_z}$$

(2.4.11)

where $\phi_N[x - x_0(k_y)]$, $\zeta_{nk_z}(z)$ and $E_{nk_z}$ are, respectively, given by equations (2.4.3), (2.3.24) and (2.3.27).

The DOS per unit area per spin for the dispersion of equation (2.4.11) is given by

$$\rho_{\text{SL}}(E) = 2 \sum_{N,n,k_y,k_z} \delta \left( E - E_{Nnk_z} \right)$$

(2.4.12)

Since $E_{Nnk_z}$ is independent of $k_y$ the summation in equation (2.4.12) over $k_y$ can be carried out which yields the degeneracy factor $\Omega N_L$, where $\Omega$ is the interface area of the SL. Thus,

$$\rho_{\text{SL}}(E) = 2\Omega N_L \sum_{N,n,k_z} \delta \left( E - E_{Nnk_z} \right).$$

(2.4.13)
Using the transformation

\[
\sum_{k_z} \rightarrow \frac{L_z}{2\pi} \int dk_z
\]

with the limits of integration given by

\[
(k_z)_{\text{max}} = \left\{ \begin{array}{ll} \pm \frac{1}{d} \cos^{-1}(1-\delta_{n}) & 0 < \delta_{n} < 2 \\ \pm \pi/d & \delta_{n} \geq 2 \end{array} \right.
\]

(2.4.14)

with \( \delta_{n} = (E - E_{N})/t_{n} \). Summation over \( k_z \) can be carried out. Exploiting the evenness of the integrand and restricting ourselves to the lowest miniband i.e. \( n = 1 \), we ultimately get,

\[
\rho_{SL}(E) = \rho_{0} \sum_{N} \left[ \sin \left( \cos^{-1}(\mu) \right) \right]^{-1}
\]

(2.4.15)

where

\[
\rho_{0} = \left( \frac{V}{\pi^{2}2^{2}t_{1}d} \right) \text{ and } \mu = 1 - \frac{E - E_{N}}{t_{1}} .
\]

(2.4.16)
References


