CHAPTER 7

PHONON-DRAG MAGNETOTHERMOPOWER

IN

GaAs/GaAlAs HETEROJUNCTIONS

Part of the work presented in this chapter has appeared in

(1993).
7.1 Introduction.

The thermopower, $S$, defined by relation $E = S \cdot \nabla T$ is a tensor in presence of a magnetic field. For an isotropic 2D electron gas, in a magnetic field along $z$-direction, there are only two independent components $S_{xx} = S_{yy}$ and $S_{xy} = -S_{yx}$, the latter being the Nernst-Ettingshausen coefficient.

Under open circuit condition, $J = 0$, and in presence of $\nabla T$ along $x$-axis, the components $S_{xx}$ and $S_{yx}$ of $S$, defined by the second of the equations (6.2.3) are given by

\[
S_{xx} = \rho_{xx}(L_{ET})_{xx} - \rho_{yx}(L_{ET})_{yx}
\]

and

\[
S_{yx} = \rho_{yx}(L_{ET})_{xy} + \rho_{xx}(L_{ET})_{yx}
\]

(7.1.1)

where all the symbols appearing in the above equation have the same meaning as given in section 6.2.1.

In an electric field $E \neq 0$ and $\nabla T = 0$, the heat flux is $U = M \cdot E$, where $M = L_{TE} = \pi$ is the Peltier tensor. Using symmetric conditions and Kelvin relation it is shown that [1,2]

\[
TS_{xx} = \rho_{xx} M_{xx} - \rho_{yx} M_{yx}
\]

and

\[
TS_{yx} = \rho_{yx} M_{xx} - \rho_{xx} M_{yx}
\]

(7.1.2)
For moderate disorders $\rho_{xx} \ll \rho_{yx}$ and $M_{xx} \ll M_{yx}$,

$$TS_{xx} = -\rho_{yx} M_{yx}$$
and

$$TS_{yx} = \rho_{yx} M_{xx}.$$  

In the disorder free limit $\rho_{xx} = 0, M_{xx} = 0$ and hence $S_{yx} = 0$ and $TS_{xx} = \rho_{yx} M_{yx}$. It may be noted that there are again diffusion and phonon-drag contributions to tensors $M$ and hence to $S_{xx}$ and $S_{xy}$.

7.2 Experimental Survey.

Since TP is sensitive to DOS, many low temperature ($T < 10$ K) measurements of magnetothermopower (MTP) of a 2D electron gas were undertaken after the discovery of Quantum Hall effect in order to probe DOS and quantization of TP. Early preliminary measurements on low mobility samples of GaAs/GaAlAs HJs [3-5] show oscillatory behaviour of $S_{xx}$ similar to $\rho_{xx}$ and temperature independent peak value ($\sim \mu V/K$) consistent with diffusion component. However, subsequent measurements [6-9] on samples with large and low mobility showed oscillatory behaviour with large peak value of $S_{xx} \sim mV/K$ which increases with increase of temperature. As a function of temperature MTP exhibits a maximum in the temperature range $1K < T < 10K$. These results are suggested to be due to phonon drag contribution. MTP is also measured in Si - MOSFET [10].
which shows similar behaviour but for $T < 2K$, where MTP is entirely due to diffusion component.

7.3 Theoretical Survey.

Most of the early theoretical investigations [11-14] have dealt with the diffusion magnetothermopower (DMTP). In disorder free limit, the peak amplitudes are given by

$$S_{xx} = \frac{k_B}{e} \ln 2 / \left( N + \frac{1}{2} \right) \approx 60 / \left( N + \frac{1}{2} \right) \mu V/K \quad (7.3.1a)$$

if spin splitting unresolved and

$$S_{xx} = \frac{k_B}{e} \ln 2 / \left( N + 1 \pm \frac{1}{2} \right) \quad (7.3.1b)$$

if the spin splitting is resolved (+ and - signs correspond to upper and lower energy levels, respectively).

In equation (7.3.1), $N$ is the Landau quantum number in which the Fermi level lies and it is derived with the assumption that $\hbar \omega_c \gg k_B T > \Gamma$ (the width of Landau level). Early measurements [3-5] of MTP are consistent with the quantized values given by equation (7.3.1). However, in presence of finite disorder $S_{xx}$ depends on $\Gamma$ where its universal behaviour (equation (7.3.1)) is lost and depends on details of scattering. In this situation $S_{xx}$ is calculated employing the Kubo formula and self consistent Born
Kubakaddi et al [15], for the first time gave the theory of phonon-drag magnetothermopower (PDMTP) modifying the Boltzmann theory of phonon-drag in bulk semiconductors [16]. It is a \( n \)-approach. They considered the weak coupling of 2D electron with 3D phonons via piezoelectric and deformation potentials. Phonon scattering is taken in the boundary scattering regime. Their calculations are in good agreement with the observed large value of \( S_{xx} \) in GaAs/GaAlAs HJs and its temperature dependence in the neighbourhood of liquid helium temperature agrees very well with the experimental data of Fletcher et al [6]. In a similar way Lyo [17] calculated the oscillatory behaviour of \( S_{xx} \) as a function of magnetic field assuming Gaussian broadened Landau levels and including screening. Calculations of Kubakaddi et al [15] and Lyo [17] are also applicable to Si MOSFET. A detailed analysis of temperature and field dependent behaviour of \( S_{xx} \) and \( S_{yx} \) in GaAs/GaAlAs HJs is given by Fremhold et al [2] at liquid helium temperature. However, there are no PDMTP calculations for higher temperatures where phonons are scattered by other phonons and impurities besides the boundary scattering.

In the following we give the calculations of PDMTP incorporating the phonon ‐ phonon and phonon ‐ impurity scattering in phonon relaxation time and account for decrease of \( S_g \) beyond its maximum value around ~ 6K.
7.4 Phonon - Drag Magnetothermopower.

The calculation of $S_g$ is complicated by the need to know the details of the non-equilibrium electron and phonon distribution functions. In the usual procedure Boltzmann theory is developed by calculating the tensor $M$ which determines the phonon heat flux $U = MS$ in a weak electric field $S$. Macroscopic transport equations are then used to express the thermopower tensor $S$ in terms of $M$ and resistivity tensor $\rho$.

In the presence of a small electric field $S$ along the $x$-direction and a magnetic field $B$ along the $z$-direction, the carriers drift along the $y$-direction to produce a net electric current in the $y$-direction which causes a Peltier heat flux to flow in that direction.

With the help of transport equations, employing Kelvin relation $S_{yy} = \frac{n_{yy}}{T}$, one gets for the phonon - drag magneto thermopower in the moderate disorder limit ($\rho_{xx} < \rho_{xy}$), an expression

$$S_{yy}(B) = T^{-1} \left[ \frac{U_y}{g} \right] \rho_{xy}(B) = S_g(B),$$

(7.4.1)

where $U_y$ is the $y$-component of the Peltier heat flux carried by the electrons as a result of electron-phonon interaction.
Kubakaddi et al. [15] obtained formula for \( S_q(B) \) for a 2D electron gas in a HJ by evaluating \( U_y \) due to phonon drag. Their formula is improved by Sankeswar [11] by employing the screened electron-phonon matrix element and by taking into account the proper treatment of Landau level broadening, where they assumed a Gaussian form for spectral density of states with Lorentzian line shape function given by 

\[
\rho(x) = \sqrt{\frac{8}{\pi}} \frac{1}{\Gamma^2} \exp \left( -\frac{2x^2}{\Gamma^2} \right)
\]

characterized by a broadening parameter \( \Gamma \). In the presence of disorder the energy levels are randomized by Landau level broadening. One should take a system average by integrating over 

\[ E = E^0_{\beta} \text{ and } E' = E'^0_{\beta'} \quad (\text{where } \beta = N, k_y, n) \]

with a weighting factor \( \rho(E - E^0_{\beta}) \rho(E' - E'^0_{\beta'}) \).

As discussed in chapter VI, at low temperatures \( T < 5K \) phonon mean free path is limited by boundary scattering alone. As the temperature increases other phonon scattering mechanisms such as phonon-phonon, phonon-impurity scatterings etc. become operative in limiting the phonon mean free path. As a result phonon mean free path drops rapidly for temperatures above 6 K. The maximum in the PDTP data arises from the fact that the heat current is proportional to the product of lattice specific heat (which rises with temperature) and scattering time. The thermal conductivity data support the same conclusion [6]. The rapid decrease of \( S_q \) beyond \( T = 6K \) is attributed to the onset of these additional scattering processes.
We employ the formula as developed by Sankeshawar et al. and incorporate, in addition to the phonon-boundary scattering considered in ref [1], also the other phonon scattering mechanisms described in section (6.2.5) with an intention to see the temperature range over which the drag contribution to the total thermopower is appreciable.

The basic formula derived by Kubakaddi et al. [15] and modified by Sankeshwar [1] is given by

\[
S_{q}^{*}(B) = -\left(\frac{k_{B}}{e}\right) \frac{L}{\theta N_{s}} \times \sum_{s} \int d^{3}q \left(\frac{q}{q^{2}}\right) q_{y} G(\Omega_{qs}) \Gamma^{s}(q) 
\]

(7.4.2)

where \(L\) is the length of the specimen in the \(z\)-direction, \(N_{s}\) is the surface carrier concentration, the summation is over \(s\) phonon mode, \(G(x) = (x/\sinh x)^{2}\), \(\Omega_{qs} = \hbar \omega_{qs}/2k_{B}T\) and

\[
\Gamma^{s}(q) = \frac{\Gamma^{s}(q)}{\Gamma^{s}(q) + \tau^{-1}_{s}(q)} 
\]

(7.4.3)

\(\Gamma^{s}(q)\) is a term due to electron interaction with phonons of energy \(\hbar \omega_{qs} = \hbar \nu_{qs}\). \(\tau^{-1}_{s}(q)\) is the phonon relaxation time. In equation (7.4.3) explicit form of \(\Gamma^{s}(q)\) is
\[ \Gamma_s(q) = \frac{v}{\hbar l^2} \sum |V_{qs}|^2 |J_{N',N}(u)|^2 |F_{n',n}(q_z)|^2 \]

\[ \int dE F_s(E,q) \rho(E-E_{nN}) \times \rho(E + \hbar v_s q - E_{n',n}) \]

(7.4.4)

where \(|V_{qs}|^2\) is the strength of the electron-phonon interaction,

\[ |F_{n',n}(q_z)|^2 = \int dz \xi_n(z) \exp(\imath q z) \xi_n(z), \quad \text{(7.4.4. a)} \]

\[ |J_{N',N}(u)|^2 = \left( \frac{n_2}{n_1} \right) u^{n_1-n_2} \exp(-u) \left[ L_{n_2}^{n_1-n_2}(u) \right]^2, \quad \text{(7.4.4. b)} \]

\[ u = \frac{q^2 l^2}{2}, \quad l \text{ is the cyclotron radius, } n_1 = \max(N',N), \]

\[ n_2 = \min(N',N), \quad L_{n_2}^{n_1-n_2}(u) \text{ is the associated Laguerre polynomial,} \]

\[ \xi_n(z) \text{ is z - part wave function given by equation (2.3.6) for a HJ and} \]

\[ F_s(E,q) = f^0(E) - f^0(E + \hbar v_s q). \quad \text{(7.4.5)} \]

The quantity \(\Gamma_s(q)\) given by equation (7.4.3) corresponds to the fraction of electron-phonon collisions to 'all-phonon' collisions.
collisions. It reflects the efficiency of the drag and hence plays an important role in the determination of phonon-drag thermopower. In the limit $\Gamma_S(q) \ll \tau_S^{-1}(q)$, the quantity $\Gamma_S'(q) \cong \tau_S^{-1}(q) \Gamma_S(q)$ and our expression for $S_q$ reduces to that of Lyo [17].

The strength of the screened electron-phonon interaction is given by equation (6.3.7) but with the screening function, in a magnetic field at low temperatures, given by [18,19]

$$\varepsilon(q_\perp) = 1 + 2\pi e^2 (\kappa S q_\perp)^{-1} f(q_\perp) n(q_\perp)$$  \hspace{1cm} (7.4.6)

with $\kappa S$ representing static dielectric constant and the form factor

$$f(q_\perp) = \int dz \int dz' \xi_n^*,(z) \xi_n(z) \xi_n^*,(z') \xi_n(z') \exp(-q_\perp | z-z' |)$$  \hspace{1cm} (7.4.6 a)

The static polarizability is given by

$$n(q_\perp) = |J_{NN'}|^2 \rho(\xi)$$  \hspace{1cm} (7.4.6 b)

where $N$ is the index of the Landau level closest to the Fermi level and $\rho(\xi)$ is the DOS at the Fermi level.
\[ S_q \text{ in extreme quantum limit.} \]

A case of particular physical interest is that of extreme quantum limit (EQL). In this limit the applied magnetic field is sufficiently strong and the temperature is sufficiently low such that all the electrons are accommodated in the lowest Landau level of the lowest electric subband.

In EQL an expression for PDMP is given by

\[
S_q(B) = \left[ \frac{k_B L}{4\pi^2 e N_s^{1/3}} \right] \sum_{s} \int_{0}^{\infty} x^2 \text{d}x \int_{\mu} \left( 1 - \mu^2 \right) G(\Omega_{qs}) \Gamma'(x,\mu),
\]

(7.4.7)

where \( l \) is the cyclotron radius and

\[
\Gamma'(x,\mu) = \frac{\Gamma_S(x,\mu)}{\Gamma_S(x,\mu) + \tau_S^{-1}(x,\mu)} \quad (7.4.7 \text{a})
\]

In equation (7.4.7 a) electron–phonon interaction term is given by

\[
\Gamma_S(x,\mu) = \frac{C_S(x,\mu)}{\rho v_s^{1/3}} \left( \epsilon(x,\mu) \right)^{-2} \left| F(x,\mu) \right|^2 \exp \left\{ -x^2 (1 - \mu^2) / 2 \right\}
\]

\[ \times \int_{-\infty}^{\infty} \text{d}y \mathcal{F}_S(y, x) (2/\pi \Gamma) \exp \left\{ -2 \left( (y - y_o)^2 + (y - y_o + y_{TS})^2 \right) \right\}, \]

(7.4.8)
with

\[ F_s(y,x) = \frac{\sinh(X_{TS})}{2 \cosh \left( \frac{(\gamma y/k_B T) - \eta}{2} \right) \cosh \left( \frac{(\gamma y/k_B T) - \eta}{2} + X_{TS} \right)} \]  
(7.4.8 a)

\[ Y_{TS} = \frac{\hbar v_s x}{2\Gamma}, \quad X_{TS} = \frac{\hbar v_s x}{2k_B T}, \quad Y_0 = \frac{E_{oo}}{\Gamma}, \]

\[ C_s(x,\mu) = E_d^2 + \frac{(e\hbar \gamma)^2 l^2}{x^2} \left[ \left\{ 8\mu^2(1-\mu^2) + (1-\mu^2)^3 \right\}/4 \right] \]

for longitudinal modes.  
(7.4.8 b)

\[ = \frac{(e\hbar \gamma)^2 l^2}{x^2} \left[ 9\mu^2(1-\mu^2)^2 / 2 \right] \]

for transverse modes,  
(7.4.8 c)

\[ |F(x,\mu)|^2 = \left\{ 1 + (x\mu/lb)^2 \right\}^{-3}, \]  
(7.4.8 d)

\[ \varepsilon(x,\mu) = 1 + \left\{ \frac{2\pi e^2}{k_B Q_s^2} \right\} f(Q) \left\{ \frac{\rho(\zeta - E_{oo})}{\pi l^2} \right\} \exp \left\{ -l^2 Q_s^2/2 \right\}, \]  
(7.4.8 e)

and
\[ Q_s = Q_b \text{ with } Q = \left( \frac{x}{A} \right) (1 - \mu^2)^{1/2}. \]  

(7.4.8 f)

Since phonon relaxation times due to boundary scattering, phonon scattering and impurity scattering are independent of magnetic field, the total phonon - relaxation time is same as given in zero field case. It is given by equation (6.2.6):

\[
\tau_s^{-1}(x,q) = \tau_{\text{Boundary}}^{-1} + \tau_{\text{Impurity}}^{-1} + \tau_{\text{Phonon - phonon}}^{-1}
\]

\[
= \left( \frac{v_s}{A} \right) + A \left( \frac{v_s x}{1} \right)^4 + \left( B_1 + B_2 \right) \left( \frac{v_s x}{1} \right)^2 T^3
\]

for \( x > \left( \frac{k_B T}{h} \right) \left( \frac{1}{v_s} \right) \)

(7.4.9 a)

\[
= \left( \frac{v_s}{A} \right) + \left( \frac{k_B T}{h} \right)^4 + \left( B_1 + B_2 \right) \left( \frac{v_s x}{1} \right)^2 T^3
\]

for \( x < \left( \frac{k_B T}{h} \right) \left( \frac{1}{v_s} \right) \)

(7.4.9 b)

where \( x = q_\perp^1 \).

Simple dependence of \( S_g \) upon the electron surface density \( (N_s) \) or temperature \( (T) \) are not readily apparent from the formulae. Attempts have been made to demonstrate such dependences using simple formulae for zero field \( S_g \) in 2D electron gas [20].

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Though approximate such estimations are expected to provide some insight into the expected behaviour.

7.5 Results and Discussion

We have evaluated numerically the expression (7.4.7) for the phonon-drag magneto thermopower for the specimen 1 (\(N_s = 1.78 \times 10^{11}\), mobility \(\mu = 2.26 \times 10^5\) cm\(^2\)/Vs) of Fletcher et al [6] in a magnetic field of \(B = 15\) T, for a range of temperature 2 - 25K. For the specimen 1 of Fletcher et al [6] the separation between the two lowest subband is 20 meV. For the values of \(N_s\) considered here, Fermi level lies in the first Landau level of the first subband so that the conditions for extreme quantum limit are met and the inter Landau level scattering can be neglected. The material parameters used in the calculations are given in Table III. We use the value of \(\Lambda \approx 0.03\) cm determined experimentally from the temperature dependence of the thermal conductivity, which is close to the sample dimension of 0.036 cm.

We have examined the effect of the broadening parameter \(\Gamma\) on the slope of the \(S_g\) vs \(T\) curve in the low temperature region and on the position of the maximum. It is shown in figure 7.1. It is found that for small values of \(\Gamma\), \(S_g\) increases slowly, reaches a maximum and then decreases with a further increase of temperature. For large \(\Gamma\), the maximum in \(S_g\) occurs at lower temperatures and peak value is larger. This behaviour is attributed to the fact that the maximum energies of the resonant
Fig. 7.1 Variation of phonon - drag magneto thermopower as a function of temperature for different broadening parameters $\Gamma$.

Dashed curve : $\Gamma = 0.5$ meV.
Dashed - dotted curve : $\Gamma = 0.75$ meV.
Continuous curve : $\Gamma = 1$ meV.
phonons are limited by $\Gamma$, so that the classic regime is reached when the thermal energy becomes larger than the width. The value of $\Gamma = 1 \text{ meV}$ is found to best fit the slope of the experimental curve of Fletcher et al for $T < 5K$. For this value of $\Gamma$ the maximum in the PDMTP curve occurs around $T = 6K$. However, it must be noted that in zero field PDTP maximum occurs at temperature around 10K where the upper limit for the wave vector $q$, of the phonons involved is limited only by $k_f$. In figure 7.2 we have shown $S_g(B)$ as a function of temperature ($1 - 25 K$) for $B = 15$ T. Experimental data of Fletcher et al (6) is shown by closed circles. Continuous curve is due to our calculations which includes phonon scattering due to other phonons and impurities in addition to the boundary scattering. The broken curve represents calculated $S_g(B)$ wherein phonons are assumed to be scattered due to boundary alone. We find excellent agreement between the calculated and the experimental values. However, $S_g(B)$ with phonon scattering due to boundary alone cannot account for experimentally observed behaviour at higher temperatures ($T > 6K$). Additional phonon scattering mechanisms cause rapid decrease of $S_g(B)$ after a maximum. The values $A = 3.5 \times 10^{-23} \text{sec}^3$ and $B_1 + B_2 = 6.0 \times 10^{-44} \text{sec/deg}^3$ are used in our calculations. These are reasonable as compared with respective values in bulk GaAs (21) i.e. $A = 2.0 \times 10^{-23} \text{sec}^3$ and $B_1 + B_2 = 3.6 \times 10^{-44} \text{sec/deg}^3$.

Comparison between calculated and experimental values also evinces that at very low temperatures the phonon mean free path
Fig. 7.2 Variation of phonon - drag magneto thermopower as a function of temperature.

Dashed curve : Phonon - boundary scattering alone.

Continuous curve : Phonon - boundary + Phonon - impurity phonon phonon scatterings with

\[ A = 3.5 \times 10^{-44} \text{ sec}^3. \]

\[ B_1 + B_2 = 6.0 \times 10^{-23} \text{ sec/deg}^3. \]

Closed circles : Experimental data of Fletcher et al.
is limited by boundary scattering only. In this regime \( \tau_s(q) \) can be taken out of the integral and combined with \( v_s \) to yield \( A \), the phonon mean free path which is now a sample dimension. Once \( \tau_s(q) \) is constant \( S_g \) falls due to the Bose factors in the integral. With increase in temperature other phonon scattering mechanisms become significant in limiting the phonon mean free path. The net consequence of this is that the phonon momentum is now transferred more to the other sources such as impurities, other phonons etc., than to the electrons. Accordingly a peak is expected to appear in a \( S_g \) versus \( T \) data with a subsequent rapid decrease in its magnitude thereafter. Moreover restrictions imposed by momentum conservation in this region severely limit the number of phonons which can scatter electrons. The net result of all this is to make \( S_g \) to vanish and \( S_d \) to dominate at high temperatures (\( T > 50K \)). As in case of bulk GaAs, the phonon scattering due to point impurities becomes significant for temperatures above liquid helium temperature. At still higher temperatures, phonon - phonon scattering dominates which suppresses the phonon - drag thermopower. To see the effect of addition of phonon scattering mechanisms on \( S_g \) we have shown in figure 7.3 the variation of \( S_g \) with \( T \) for different combinations of \( A \) and \( B_1 + B_2 \). As expected, larger are the values of \( A \) and \( B_1 + B_2 \), faster is the decrease of \( S_g \). The above given set of values of \( A \) and \( B_1 + B_2 \) seem to provide a better compromise for the best fit in the the temperature region for which the experimental data
Fig. 7.3 Variation of phonon - drag magneto thermopower for different combinations of $A$ and $B_1 + B_2$.

Dashed - dotted curve: $A = 3.0 \times 10^{-45} \text{sec}^3$
$B_1 + B_2 = 5.5 \times 10^{-24} \text{sec/deg}^3$.

Dotted curve: $A = 7.5 \times 10^{-44} \text{sec}^3$
$B_1 + B_2 = 8.0 \times 10^{-23} \text{sec/deg}^3$.

Continuous curve: $A = 7.0 \times 10^{-43} \text{sec}^3$
$B_1 + B_2 = 10.0 \times 10^{-22} \text{sec/deg}^3$. 
exists.

In figure 7.4 the contributions of the deformation and piezoelectric modes of the acoustic phonon scattering to the PDMTP as a function of temperature are shown separately. It is found that piezoelectric scattering makes a significant contribution to PDMTP only at temperatures below ~ 2 K and deformation scattering mechanism starts dominating then onwards at higher temperatures. At the peak position piezoelectric scattering mechanism contributes only about 16% to the total PDMTP in contrast to the deformation potential scattering contributing the rest ~ 84%. 
Fig 7.4 Variation of phonon - drag magneto thermopower as a function of temperature.

Curve a: Deformation acoustic phonon scattering.

Curve b: Piezoelectric acoustic phonon scattering.
References.


