CHAPTER – IX
CONCLUSION

This concluding chapter of the thesis presents the unique features, ideas, results, conclusions and proofs given in various chapters in a summarised form. The various results obtained in the form of theorems and related lemmas, truths, remarks, corollaries and examples in a new setting, bring out their novelties.

It is to be noted that the new results and findings in the research work have been mainly based on the published and unpublished research works of the guide and the present researcher of this thesis. Detailed references to the various results are consistently made to maintain a thread of logical continuity in the evaluation of the thesis.

Some of the theorems and corollaries on A-, D- and E- optimality of n-ary partially balanced block designs and A-, D- and E- optimal n-ary block designs under nomoscedastic as well as heroscedastic set-up, that are presented in this thesis. However, they have a natural place here for the simple reason that in that chapter they flow a common base, by which other theorems listed there are proved. Also some theorems and corollaries in the constructions of A, D- and E- optimality of partially balanced n-ary block designs even though published in reputed journals are not covered in this thesis due to shortage of page.

Equivalence theorem, specific optimality criteria like (i) D- optimality, (ii) $D_A$ – optimality, (iii) S- optimality (iv) $D_s$ – optimality (v) A- optimality (vi) G- optimality (vii) E- optimality (viii) V- optimality (ix) Linear optimality (x) M-S- optimality (xi) M-V- optimality (xii) Schur optimality (xiii) Universal optimality and (xiv) T- optimality have been defined clearly relevant to the latest contributions. The interested work on partial optimality for which
literature were collected, are reserved for future works on different optimality of block designs. Concepts and recollection of n-ary block designs and analysis of balanced and partially balanced n-ary block designs which are presented here cannot be found anywhere except from the publications of the guide and the present author of this thesis. They are the latest and up to date recollections.

In chapter II a brief review on the optimum design of experiments based on the literature has been made. The strong reference is made on available theory on optimum experimental designs, with particular emphasis on all up to date incomplete block designs.

It is worthwhile to have a note on latest developments with algorithms for exact designs for off-line quality control associated with Taguchi (1987) designs for non-linear models, the computer aided design of experiments, Bayesian designs, Auto correlated models, stochastic processes and other special type of designs. By taking a comprehensive survey on various types of A-, D- and E- optimality criteria conduced with different designs for construction and analysis of A-, D- and E- optimality of block designs.

Under the first part of the thesis results presented in Chapter III are briefed below, which are accepted for presentation in statistical session of 99th Indian Science Congress Association during 3-8 January 2012. These results, which are further extended, will be published in the Journal of Estadistica (2012) of U.S.A.

Recent developments on optimal block design, A-, D- and E- optimal block designs and latest A- optimal block designs are important research collections in the field of optimal designs. Similarly the review collection of n-ary block designs from Tocher (1952) to recent times, particularly the works of Soundarapandian (1979 a, b, 1980 a, b, c, d, ) and Soundarapandian et.al. (1995 a, b, c, d ) is a landmark in incomplete block design which have been utilized to
find new A-, D- and E- optimal design in our thesis. In this thesis the properties of E- optimal n-ary block designs with various classes of proper n-ary block designs in which treatments are not replicated the same number of times have been investigated. Several sufficient conditions are derived for n-ary block designs to be E- optimal within the class considered and several methods of constructing n-ary designs, which satisfy these sufficient conditions, are presented. The highlight of this chapter is that some results, providing information as how the replications should be assigned to treatments in n-ary E-optimal designs, are presented with illustrations.

In chapter IV the A-, D- and E- optimality of n-ary block designs with various classes having varying replications and unequal block sizes have been analyzed. Several sufficient conditions are obtained for n-ary block designs to be E-optimal within the classes considered. Some important methods of construction of E-optimal n-ary block designs, which satisfy the sufficient conditions, are landmark in the research of optimal n-ary block designs with varying replications and different block sizes. This work has been extended to A- and D- optimality of n-ary block designs.

In Chapter V we have investigated that several families of PBNB designs, with relatively few n-ary blocks are E- optimal over the collection of all n-ary block designs. Various bonds for eigen values of C matrix of n-ary design are given. PBNB designs with $\Lambda_1 = a, \Lambda_2 = 0$ with few blocks; PBNB designs with triangular association scheme of size n, $\Lambda_1 = 0, \Lambda_2 = a$ and block size $K \geq n / 2$ or $\Lambda_1 = a, \Lambda_2 = 0$ and $K \geq (n-1)$; PBNB designs with $L_1$ association schemes based on V treatment with $\Lambda_1 = 0, \Lambda_2 = a$, $K \geq \sqrt{V}$ or $\Lambda_1 = a, \Lambda_2 = 0$ and either $i - 1 \leq \sqrt{V} \leq K$ or $K \leq \sqrt{V} \leq i - 1$; and duals of these n-ary block designs are E- optimal. Here partial geometry may also be utilized. This is a new research finding particularly in partially balanced n-ary block classes of designs. In further PBNB, we have proved that if $D(V, B, K)$ has $d^*$
having non-trivial concurrences equal, then $d^*$ is universally optimal $n$-ary block design over $D(V, B, K)$. In particular they are $A$-, $D$- and $E$- optimal over $D(V, B, K)$. We also proved some theorems, which are associating with a vector $X$ in $k$– dimensional real space. Further it is shown that if $D(V, B, K)$ contains connected two-associated class PBNB design with $\Lambda_2 = \Lambda_1 \pm a$, which has a singular concurrence matrix, then it is $E$- optimal over $D(V, B, K)$ with respect to a large class of criteria including the $A$-, $D$- and $E$- optimal criteria which is also a new finding.

The second part of the thesis we start from Chapter VI we discuss in detail the construction of Balanced Treatment Incomplete Block (BTIB) designs proposed by Bechhofer and Tamhane (1981). Robson (1961) suggested the use of balanced incomplete block design (BIBD) between all of the treatments, including the control. Cox (1958) noted that BIBD may not be very appropriate for the multiple comparisons with the control because of the special role played by the control. For a brief exposition of the history of this problem and the required references the article by Bechhofer and Tamhane (1981) is of seminal importance. Infact this article is basic to the developments in the research. From now on it will be referred as BT.

In many industrial, agricultural and biological experiments, the following problem is encountered frequently: $p$ (new) varieties (or) treatments become available; an efficient, experiment is required to compare the $p$ new treatments (labeled 1, 2, …., $p$) with the one (hereafter, called the control and labeled as 0) and also to compare the new treatments among themselves. But the emphasis will be on the comparison of the $p$-treatments with the control, hence higher precision is desired for these estimates.

In this research the necessary tools for determining a certain set of BTIB designs, which will constitute the MCCGD are given. The minimal complete classes are provided for $p = 4$ (1) 10, $k = 4$ and $p = 5$ (1) 10, $k = 5$ and also for
p = 6, k = 6. The use of the tools is illustrated for p = 4, k = 4 and p = 5, k = 5. Two methods of constructing BTIB designs are also described. The first method is an effective “Ad hoc” method, i.e., it involves a systematic trail and error that allows one to construct MCCGD at least for practical values of (k,p). The second method is a computer algorithm that makes it possible to construct all binary BTIB designs for given (k, p). This method, too, involves a little trail and error.

Now, the important question, “How many units should be assigned to control?” and “How should they be allocated to blocks?” are answered. The guidelines for constructing the A-optimal or nearly A-optimal designs are provided and a computer algorithm that searches for A-optimal designs can be described.

In Chapter VII, we briefly recollect the historical review of optimal binary designs for comparing test treatments with control treatment and then giving A-optimal n-ary designs for comparing V number of test treatments with a control separately for the zero-way, one-way and two-way elimination of heterogeneity.

elimination of heterogeneity, A-optimal n-ary block designs for one-way elimination of heterogeneity and A-optimal n-ary block designs for two-way elimination of heterogeneity are discussed in detail with some theorems, definitions, corollaries and examples for n-ary block designs.

After explaining some optimalities particularly MV- optimality of n-ary block designs, the eighth chapter presents a number of results in the form of definitions, theorems, corollaries and examples, concerning the MV- optimality of n-ary block designs for comparing V number of test treatments with a control for the zero-way, one-way and two-way elimination of heterogeneity. In this chapter other efficient n-ary block designs particularly robust optimal n-ary block designs are discussed and search for an approximately A- or MV-optimal n-ary block designs are taken into consideration. Discussion on A-and MV-optimal n-ary block designs for two or more controls many more theorems and results can be worked out for treatment comparison with one or many control treatments, but due to limit, space and time, results are not included in our present thesis and they will be published separately as research papers in due course of time.