CHAPTER - I

INTRODUCTION

1.1 Place of Mathematics in Secondary School Curriculum

'Mathematics is the gate and key of the Sciences .... Neglect of Mathematics works injury to all knowledge, since he who is ignorant of it cannot know the other Sciences or the things of the World. And, what is worse, men who are thus ignorant are unable to perceive their own ignorance and so do not seek a remedy.'

- Roger Bacon

The importance of mathematics was realised even in the earlier days also. Then the aim of education was teaching 3 R's, i.e., Reading, Writing and Arithmatic. Thus it had an important position even in the earlier school curriculum.

Kothari Commission (1966) of Indian Education has wisely remarked that 'One of the outstanding characteristics of Scientific culture is quantification. Mathematics there-
fore, assumes a prominent position in modern education. Apart from its role in the growth of the physical sciences, it is now playing an increasingly important part in the development of the biological sciences. The advent of automation and cybernetics in this country marks the beginning of the new scientific-industrial revolution and makes it all the more imperative to devote special attention to the study of mathematics. Proper foundations in the knowledge of the subject should be laid at school' (21:199).

Study of mathematics is a constituent of full life. It is quite necessary for the allround development of the child. The modern science and technology stress the importance of mathematics in the school curriculum. Mathematics is the mother of all sciences. In the absence of mathematics, science cannot progress. It is due to the contribution of mathematics man has moved even to the Moon. Mathematics does not mean merely a sum total of addition, subtraction, division and multiplication. It is a mother discipline which forms a major part of human life. It is quite essential even for a layman and shopkeeper. Hence it occupies strategic position in school curriculum.

Everybody needs some knowledge of mathematics in one way or the other. But it is felt that for an ordinary man, the knowledge acquired during the primary and middle stage will suffice ......
The place of mathematics in secondary schools cannot be undermined. It has to play its own part in shaping the future generation of the country. India after independence is striving hard to improve herself into all her commitments whatever may be the sphere. Only individual and technological developments will extend its helping hand in making India an advanced country. To achieve this development the knowledge of mathematics is essential. The proficiency in mathematics complements to the proficiency in science knowledge. The future of India should be shaped only in the Indian classrooms. In order to make Indians better citizens, secondary school curriculum should be so formulated that proper importance be given to mathematics.

Mathematics is very useful subject for most vocations and higher specialized courses of learning. But everybody who is studying it in the school is not going to be an accountant, engineer or statistician. But at such an early stage of education it is difficult to know who is going to be an engineer or banker. Psychologists, however, have shown that it is possible at this stage to determine what student could be in later life. Therefore the duty of the school is to give to the high school student a broad view of what he is capable of achieving in future. He should get a broad course to be able to choose a suitable line out of that. Regarding this point S.D. Bajpai opines that 'secondary education is the backbone of university education. At the
primary and secondary stage, the greatest impact of the present changes will be felt. The future development of the country depends on mathematical knowledge that is gained in the primary and secondary education. If the individual understands each and every step in this stage he feels happy and gets success. In view of the above facts it is not merely necessary but it is urgent to discuss what developments are desirable and means by which they can be achieved at the secondary level' (3:33-40).

In 1932, Executive Board of the Progressive Education Association (America) established the Commission of Secondary School Curriculum. This Commission subsequently established several committees to explore the respective contributions of various subject fields to general education at the secondary level; among them was the committee on the function of mathematics in general education. This committee finally submitted its report in four parts. The first of these presents the educational philosophy which guided the committee in the formulation of the report, central in this philosophy being the premise that mathematics in order to justify its place in the secondary school curriculum must contribute to the satisfaction of the needs of the students. These needs are enumerated in terms of the following four basic aspects of living:
i. Personal living,
ii. Immediate personal-social relationships,
iii. Social civic relationships, and
iv. Economic relationships.

This part of the report closes with a discussion of the role of mathematics in satisfying the needs of people with respect to these four aspects of living.

Mathematics does help in training and disciplining the mind. It develops the power of thinking and reasoning and gives mental exercises best fitted for strengthening the faculties of the mind. Although the opportunities of such exercises may also be given in the studying of other subjects, but mathematics enjoys a unique position in this respect. Young remarks 'Mathematics is the only subject that encourages and develops logical thinking. It enables the student to determine between essentials and non-essentials'. Bread and butter value of the subject mathematics can hardly be denied. Directly or indirectly mathematics does not only help everybody in earning but it helps in wise spending.

In the present day democracy, number and order are playing a very important role in the field of man and a nation. Number and order make or mar the career of many politicians and scholars. In the present complex society
mathematics is playing a vital role affecting physical, intellectual and cultural life of men and nation.

'Mathematics is the unshaken foundation of all sciences, fruitful parent of all arts, fountain head of advantages to human affairs.'

- B. Isaac

Due recognition should therefore be given to this vital science and best advantage of this all powerful science should be taken by every individual.

On seeing its role in life, mathematics has become an indispensible factor for progress of our present day World. It is a pivot of all civilization. This is the subject which indisputably forms the very basis of entire world's commercial system. It is a contributory factor in prosperity of human race. No other subject can be a substitute for mathematics. Therefore, Pettern Lincoln Spencer opines that "Every phase of the daily experience offers an opportunity to develop some aspect of mathematical competence because Mathematics is a 'social study', a 'language art', a 'science', an 'esthetic art' and 'recreational activity':

Mathematics is a social study because it has been developed to serve the personal and social needs of human
kind. Mathematics is also a language in which carefully defined language and concise symbol representation are used. Mathematics is an integral part of aesthetic art because it is concerned with symmetry of form and order. Mathematics is a recreational study because it is used in many recreational pursuits and because people find pleasure and relaxation in delving into its content" (19:2).

This importance of mathematics in the secondary school curriculum, at present has been considered and widely accepted as a subject that enables the child to develop rational and logical thinking, understanding of the percepts and concepts. The value and importance of mathematical ability has greatly increased so much that every man and woman of liberal education is expected to understand the use of quantitative data in the solution of social problems and use of mathematics in daily life. If every man and woman is to manage his or her own affairs and to participate in the management of community life, he or she needs some medium of mathematical knowledge and competence. Our personal and family budgeting, our shopkeeping, sale-tax, business, our insurance, income-tax and provident fund figures and the like requires quick division and multiplication skills.

Keeping all these points in mind, the Kothari Commission of Indian Education, 1966 remarked that 'Science and Mathematics should be taught on a compulsory basis to all
pupils as a part of general education during the first ten years of schooling. In addition, there should be provision of special courses in these subjects at the secondary stage, for the students of more than average ability' (21:197-8).

National Policy on Education 1986 remarked that Mathematics should be visualised as the vehicle to train a child to think, reason, analyse and to articulate logically. Apart from being a specific subject, it should be treated as a concomitant to any subject involving analysis and reasoning (20:23).

With the recent introduction of computers in schools, educational competency and the emergence of learning through the understanding of cause-effect relationships and the interplay of variables, the teaching of mathematics will be suitably redesigned to bring it in line with modern technological devices' (20:23).

1.2 Aims and Objectives of Teaching Secondary School Mathematics

'Mathematics is the most marvellous instrument created by the genius of man for the discovery of truth.'

- Laisant

Knowledge of educational values helps the teacher to avoid aimlessness in teaching. Value is the spring-
board of aim and vice versa. There is nothing controversial about the two. When we teach mathematics in the light of its aims we shall realise its values. Aimless teaching will realise no values.

Broadly speaking there are three main considerations for which a child is sent to school. Education must contribute towards the acquirement of these values.

i. Knowledge and skill,

ii. Intellectual habits and power, and

iii. Desirable attitudes and ideals.

These three values can be called;

i. Utilitarian or practical,

ii. Disciplinary, and

iii. Cultural values of education respectively.

1.2.1 The Practical Value of Mathematics

In order to determine with precision, the methods of teaching any subject, it is necessary to arrive at a clear understanding of the reasons for teaching it. This inquiry is particularly important in mathematics, since the aims and pedagogical advantages of this study differ widely from those of most other subjects.
The practical value of mathematics is very great. It is indeed no exaggeration to assert that our whole modern civilization owes its peculiar stamp indirectly to mathematics. Modern thought and modern life owe their character to the great progress of the exact sciences, and to the wonderful development of the technical arts. These two in turn are closely connected with and based upon mathematics.

A science becomes exact, when it advances from the formation of mere qualitative relations to quantitative laws, and thereby becomes accessible to mathematical investigations. Hence Kant said 'A science is exact only in so far as it employs mathematics'. The knowledge deflection of a ray of light entering from one medium into another of different density was of small value until the quantitative law of refraction was discovered. Thereby all problems of dioptrics became mathematical problems, and the entire art of making optical instruments was put upon an exact scientific basis.

The production and distribution of every necessity or luxury partly depend upon the technical sciences which owe their perfection to their exact mathematical basis. 'Our entire present civilization', says Professor Voss 'as far as it depends upon intellectual penetration and utilization of nature has, its real foundation in the mathematical sciences'. Engineering, architecture, navigation,
rail road, building and surveying are more or less based upon mathematical foundations.

It would be an error to infer, from great usefulness of mathematics to our civilization, an equal practical usefulness to every individual. The percentage of students who are likely to have practical use for mathematics after leaving school or college, is certainly small. The majority of business or professional callings requires no algebra, geometry or trigonometry and even the professions which use these subjects do so to a much smaller extent than is generally supposed. There are navigators, surveyors, coolies, carpenters, tailors, engineers, etc., who make their calculations in an almost mechanical manner, without having perfectly clear notions of the underlying mathematical principles.

Any person ignorant of mathematics will be at the mercy of others and will be easily cheated. A person may belong to the lowest or the highest class of society, but he utilises knowledge of mathematics in one form or another.

Even Napoleon said 'The progress and the improvement of mathematics are linked to the prosperity of the state'. Thus it occupies all walks of life. In order to create system in life, we have to fix timings, prices, wages, rates, rations, fares, percentages, targets, exchanges, commissions discounts, lengths, breadths, areas, volumes, etc.
In absence of these fixations, life in the present complex society will revert back into confusion and chaos. The number imparts system to our life. In many types of our practical difficulties, number or measurements come to our help and solve the difficulties for us.

1.2.2 The Disciplinary Value of Mathematics

The principle value of mathematical study arises from the fact that it exercises the reasoning power more, and claims from the memory less, than any other secondary school subject. The study of mathematics should result in the development of power, rather than the acquisition of facts. Not he who knows a great many mathematical facts is a good mathematician, but he who can apply these facts intelligently, who can discover facts that are new to him, and who can construct those which have been forgotten.

It is power and not knowledge that furnishes the true test of mathematical ability, and if the power is acquired, then and only then - will the knowledge follows as a natural consequence. Mathematical instruction in a secondary school is - or rather should be mainly a systematic training in reasoning and not on imparting of information.

The reasoning in mathematical work is of a peculiar kind, possessing characteristics that make it especially fitted for training the minds of the students. According to
Locke 'Mathematics is a way to settle in the mind a habit of reasoning'. It trains or disciplines the mind. Some of the peculiar characteristics are the following:

i. Simplicity,
ii. Accuracy,
iii. Certainty of results,
iv. Originality,
v. Similarity to the reasoning of life, and
vi. Amount of reasoning.

(i) Simplicity: Mathematics allows an almost perfect grading, commencing with exceedingly simple work, and leading the student by degrees to harder and harder problems. It teaches that definite facts are always expressed in a simple language and definite facts are always easily understandable.

(ii) Accuracy: In this subject, the mere repetition of words or phrases will not hide the ignorance of the pupil. The student must think accurately, he has to speak accurately, to master mathematics. Accuracy, exactness and precision compose the beauty of mathematics. It is in the nature of this subject that it cannot be learnt through vagueness of thought and argument.
(iii) **Certainty of results**: Any piece of mathematical work is either right or wrong and it is usually a very simple matter to find out whether or not it is right. Certainly there can be no difference of opinion between student and teacher as to the final result. A student who has discovered a geometric original, or has solved an algebraic problem, and verified his answer, knows that he is right, and therefore is conscious of having accomplished something. He inculcates the habit being certain about his achievement. So it develops faith in self effort.

(iv) **Originality**: Most work in mathematics demands original thinking. Reproduction and cramming of ideas of others is not very much appreciated. In other subjects, of course, ideas of others occupies a prominent place and have to be grasped by student. Therefore he can safely depend on memory in other subjects; but without original thinking and intelligent reasoning, there cannot be satisfactory progress in mathematics. When he has new or a different mathematical reasoning or problem it is only his originality which keeps him going. The discovery or establishment of a new formula is also his original work. The practice in originality enables the child to face new problems and situations with confidence in his career.
(v) **Similarity to the reasoning of daily life:**

While nobody questions the value of mathematical training for scientific work and rigid logical deductions, it is often asserted that mathematical thinking is of an entirely different order from the kind used in human affairs and that consequently mathematical training has no practical value.

However it is undoubtedly true that the mental qualities cultivated by mathematical study alone are not sufficient to insure ability for solving practical problems, but on the other hand, it is evident that without these qualities one can hardly hope for success in the affairs of life. Clearness and exactness of thinking are just as necessary in daily life as in mathematical study.

The person who undertakes an industrial or commercial venture must possess a clear idea of the existing conditions and of his aims - in other words, he must have firm grasp on the situation; just as a student in mathematics has to recognise the hypothesis and the conclusion. In all steps he must have a clear notion of the situation, of the means to be adopted, of the end to be reached. More than one business, man has testified that he owes his success in life to the habits of exact thinking which he formed when studying mathematics.
(vi) **Amount of reasoning:** Mathematics appeals more to the reasoning power and less to the memory than any other high school subject. This is particularly true of geometry. Here if the subject is properly taught - nearly everything is reasoning. The few facts to be known are so palpable as to require no special memorizing.

After proper training in exercise work, the regular propositions soon become natural consequences of general methods - not be memorized, but to be discovered and to be reconstructed when forgotten. They acquire the power to think effectively. They learn to think and do not merely acquire mechanical habits. The intellectual power of the learner is strengthened. It generates the otherwise latent powers of thinking, reasoning, discovering and judgement of the individual.

1.2.3 **Cultural Value**

The understanding of the world in which man lives, of the civilization to which he belongs and the culture of which he is very proud, requires the understanding of scientific and social principles, the development of which depends, in turn, upon mathematical principles. It has been truely said that 'Mathematics is the mirror of civilization'.

The cultural value of mathematics is steadily increasing day by day. J.W.A. Young is justified in saying 'Whenever we turn in these days of iron, steam and electricity, we find that mathematics has been the pioneer. Were its backbone removed, our material civilization would inevitably collapse. Modern thought and belief would have been all together different, had mathematics not made the various sciences exact. Different laws of science and the scientific instruments are based on exact mathematical concepts. Astronomy and physics are the most exact sciences and the exactness is the result of the usefulness of mathematics'.

Modern civilization owes its advancement to the progress of various occupations, such as agriculture, engineering, surveying, medicine, industry, navigation, rail road, building, etc. These occupations buildup culture and they are its backbone. But one should not forget that mathematics contributes and has contributed extensively to the advancement of these occupations. Therefore mathematics shapes culture as a play back pioneer.

Mathematics is also a pivot for cultural arts, such as music, poetry, and painting. It might not be altogether a matter of chance that the Greeks, the greatest geometers, were also very successful in fine arts.
1.2.4 Some Other Advantages of Mathematical Study

(i) Development of the power of concentration:
Very few young people seem to be able to concentrate their minds for even a few minutes upon an idea. This is a faculty which can be acquired and mathematical study is admirably adapted to develop it.

(ii) Development of the constructive imagination or the intuitive faculty:
Far from being a dry science requiring pedantic accuracy and little imagination, true mathematical work consists in inventing, in finding something that is unknown to the worker and in this success is impossible without the use of creative power of mind.

To the student, the solving of a difficult problem is a discovery; and constant training in such work develops those faculties that lead to discovery and invention.

(iii) Growth of mental self-reliance:
Young students, as a rule, rely too much upon facts taken from books or some other authority, and too little upon their own faculties, a trait which shows that they have no confidence in their own mental powers. Their former training has led them to believe utterly in authority, and especially to think that all knowledge depends upon authority. Mathematics however trains students to develop self-confidence.
(iv) **Development of character**: Mathematics study trains the students in systematic and orderly habits and the pleasure connected with the successful conquering of a difficulty stimulates the will power. It has also been claimed that dealing with a subject, that is absolutely true, that rejects and shows up any error, is bound to increase respect for truthfulness and honesty.

(v) **Power of expression**: The habit of expressing oneself clearly and accurately is a great asset in practical life. In mathematics, one has always to be very careful while using appropriate words and terms. The habit of clearness, brevity, accuracy, precision, and certainty in expression are formed and strengthened by the study of this subject. Its concepts and symbolism provide a means of concise expression which is elegant in its simplicity and exactness.

(vi) **Increase in general culture**: An acquaintance with the fundamental facts and methods of mathematics seems to be necessary for general culture. A science that is closely interwoven with most mental achievements of the race, that is found in all civilizations, that represents the most finished types of exact thinking cannot be ignored by the man of culture.

A person unfamiliar with elements of mathematics cannot fully comprehend the simplest facts of astronomy,
he is not able to read to grasp the accounts of the wonderful discoveries and inventions of our time.

In conclusion, mathematics is primarily taught on account of the mental training it affords, and only secondarily on account of the knowledge of facts it imparts. The true end of mathematical teaching is, power and not knowledge.

1.2.5 Aims of Teaching Mathematics

Universally recognised aims or large purposes of teaching mathematics in secondary schools are as follows:

i. To give the individual an understanding of ideas and operations in number and in quantity needed in daily life by the citizens of our country as individuals,

ii. To develop in the individual an awareness of the mathematical principles and operations which will enable the individual to understand and participate in the general, social, and economic life of his community,

iii. To provide through mathematical ideas, aesthetic and intellectual enjoyment and satisfaction and to give an opportunity for creative expression,

iv. To provide the basis of mathematical skills and processes which will be needed for vocational purpose, and
v. To help the child develop mathematical skills and attitudes to meet the demands of (a) daily life, and (b) future mathematical work in the related field of knowledge.

1.2.6 Objectives of Teaching Mathematics

The following objectives are to be achieved by the teaching of mathematics at the secondary stage, as presented in the guidelines and syllabi for secondary stage, 1988 by the NCERT, New Delhi.

The teaching learning of mathematics should enable the child to:

i. consolidate the mathematical knowledge and skills acquired at the upper primary stage,

ii. acquire knowledge and understanding of the terms, symbols, concepts, principles, processes, proofs, etc.,

iii. develop mastery of basic algebraic skills,

iv. develop drawing skills,

v. apply mathematical knowledge and skills to solve real mathematical problems by developing abilities to analyse, to see inter-relationship involved, to think and reason,
vi. develop the ability to articulate logically,

vii. develop skill in the use of mathematical tables for problem solving,

viii. develop ability to write, interpret algorithms for problem solving,

ix. develop necessary skills to work with modern technological devices such as calculators, computers, etc., where available and develop understanding of the cause effect relationships and the interplay of variables,

x. develop interest in mathematics and participate in mathematical competitions and other mathematics club activities in the school,

xi. develop appreciation for mathematics as problem solving tool in various fields for its beautiful structures and patterns etc., and,

xii. develop reverence and respect towards great mathematicians particularly towards the Indian mathematicians for their contributions to the field of mathematics, astronomy, etc.

1.2.7 Specifications of Instructional Objectives of Mathematics

The broad objectives presented above have been translated into specific instructional objectives in behavioural terms as follows:
(i) **Objective - 1**: The pupil acquires **knowledge** of terms, symbols, formulae, definitions, concepts, facts, processes and principles in mathematics.

Specifications - The pupil

1.1 recalls terms, symbols, formulae, definitions, concepts, processes and principles,
1.2 recognises symbols, formulae, concepts, etc., and
1.3 carries out proofs successfully.

(ii) **Objective - 2**: The pupil develops **understanding** of terms, symbols, formulae, definitions, concepts, processes, principles and facts in mathematics.

Specifications - The pupil

2.1 explains mathematical concepts in his own words,
2.2 illustrates mathematical terms, formulae, definitions, etc.,
2.3 translates verbal statement into symbols and vice-versa,
2.4 detects errors in mathematical definitions, processes, formulae, etc.,
2.5 corrects errors in mathematical definitions, processes, formulae, etc.,
2.6 identifies relationship among the given data,
2.7 compares and contrasts various concepts,
2.8 discriminates between closely related concepts,
2.9 classifies the data,
2.10 estimates the results, and
2.11 verifies mathematical results.

(iii) **Objective - 3**: The pupil applies his knowledge and understanding of mathematics to unfamiliar situations.

**Specifications** - The pupil

3.1 analyses the data into parts,
3.2 judges the sufficiency or insufficiency, consistency or inconsistency and the relevancy of data,
3.3 establishes relationship among the data,
3.4 suggests different methods for solving a problem,
3.5 selects the most appropriate method or line of attack,
3.6 draws conclusions or inferences (or reasons deductively), and
3.7 generalises (reasons inductively).
(iv) **Objective - 4**: The pupil acquires **skill** in -

a. computation,

b. handling mathematical instruments,

c. drawing mathematical figures, and

d. reading charts, graphs, and tables.

(a) Computation skills - Specifications - The pupil -

4.1.1 does oral calculations correctly and quickly.

4.1.2 does written calculations correctly quickly and legibly.

(b) Skill in handling mathematical instruments - Specifications - The pupil -

4.2.1 handles mathematical instruments with ease, speed and accuracy.

4.2.2 works with computers.

(c) Drawing skills - Specifications - The pupil -

4.3.1 draws fairly, accurately, freehand figures,

4.3.2 draws figures to given specifications,
4.3.3 draws figures neatly and speedily, and

4.3.4 labels the figures or graphs appropriately.

(d) Skill in reading graphs, charts or tables -
Specifications - The pupil -

4.4.1 reads and interprets graphs, charts or tables correctly for problem solving.

(v) Objective - 5 : The pupil develops interest in mathematics.
Specifications - The pupil -

5.1 engages himself in mathematical activities during his leisure time,

5.2 collects graphs, charts, puzzles, etc., and interprets,

5.3 frames puzzles or new problems in mathematics,

5.4 suggests new methods of solving a problem,

5.5 makes mathematical designs or models,

5.6 approaches the teacher with problems outside the class,
5.7 contributes articles on mathematics in the school magazines, and

5.8 becomes a member of the mathematics club.

(vi) **Objective - 6**: The pupil acquires **positive attitude** towards mathematics.

Specifications - The pupil -

6.1 develops reverence and respect towards great mathematicians,

6.2 likes teachers of mathematics and mathematical literature,

6.3 feels at home in the company of students of mathematics,

6.4 helps students who are weak in mathematics, and

6.5 likes to take tests in mathematics.

(vii) **Objective - 7**: The pupil develops **scientific attitude** through the study of mathematics.

Specifications - The pupil -

7.1 does not jump to conclusions,

7.2 does not accept an argument unless logically convinced,

7.3 owns his mistakes,
7.4 points out errors of others boldly and tries to correct them, and
7.5 is not disappointed by his failures.

1.3 Achievement of Indian Secondary School Pupils in Mathematics

A National level survey report (NCERT) - The broad objective of this national survey was to find out the achievement of pupils in mathematics at three levels of education - primary, middle, high school, all over India except Bihar and Tamilnadu. For this purpose 15,000 schools at each level were given the tests. The results revealed that the national performance was quite below the desired level. On the objective dimension items involving applications of concepts proved difficult. Similarly was the case with items that involved more reasoning rather than rote knowledge. In most of the states, items on objectives - application and logical reasoning were comparatively difficult than those on the objective knowledge and skill. Comparing the average difficulty values of the items according to objective seemed to be application followed by logical reasoning, knowledge and skill. This trend appeared in most of the states. The students had failed badly on application of knowledge to new situations, logical reasoning, evaluation, synthesis and so on (13:79).
Patel (1984) among other things discovered that the pupils possessing high reasoning ability were found to be better in mathematical ability than those with low reasoning ability (17:704-705).

1.4 Need for the Study

Results of the national survey report and the Ph.D. study clearly reveal that (i) the performance of our students is far from satisfactory with reference to application questions, and (ii) higher cognitive abilities contribute more in mathematics performance. Further it is matter of common experience of mathematics teachers that students by and large do not perform well on application and reasoning questions.

An attempt therefore is made in the present study to try out a model of teaching (Developmental model) and investigate its effectiveness in developing higher cognitive abilities in addition to general achievement in a branch of mathematics - geometry.
REFERENCES


17. Patel, N.R. 'An Investigation into the Mathematical Ability of Pupils of Class IX and X in the Content of Some Cognitive and Affective Variables'.


