CHAPTER IV

COUNTABILITY AND PARALINDELOF PROPERTY

IN FUZZY TOPOLOGY

1. INTRODUCTION:

The study of the first axiom of countability, the second axiom of countability and separability in fuzzy topology was carried out by C. K. Wong [52 & 53], S. R. Malghan and S. S. Benchalli [33 & 5] and many others.

In this chapter, the concept of $\alpha$-local base and $\alpha$-base have been defined. These concepts have been used to introduce and study the first axiom of countability ($\alpha$-C$_1$), the second axiom of countability ($\alpha$-C$_{11}$) and separability ($\alpha$-separable) in fuzzy topology. Among other results, it is proved that $\alpha$-C$_1$ property is invariant under $F$-continuous and $F$-open surjections and that $\alpha$-separability is invariant under $F$-continuous surjections. Further it is also proved that an open crisp subspace of a $\alpha$-separable fts is $\alpha$-separable and that every crisp subspace of a $\alpha$-C$_{11}$ fts is $\alpha$-C$_{11}$. It is also proved that every $\alpha$-C$_{11}$ fts is $\alpha$-Lindelof.

In the third section of this chapter, the new class of $\alpha$-paralindelof fts has been introduced and studied. In this section, it is proved that $\alpha$-paralindelof property is a generalization of $\alpha$-Lindelof property, $\alpha$-paracompactness and $\alpha$-$C_{11}$ property. It is also proved that every closed crisp subspace of a $\alpha$-paralindelof fts is $\alpha$-paralindelof and that the property of a fts being $\alpha$-paralindelof is invariant under $F$-continuous, $F$-open surjections.

In the fourth section of this chapter, a weaker form of $\alpha$-paralindelof property namely weakly $\alpha$-paralindelof property in fuzzy topology is introduced and studied. This new concept is based on the concept of $\alpha$-point countable family of fuzzy sets introduced in this section. It is proved that a closed crisp subspace of weakly $\alpha$-paralindelof fts is weakly $\alpha$-paralindelof and that weakly $\alpha$-paralindelof property is invariant under $F$-continuous, $F$-open surjections.

The fifth section of the present chapter is devoted to the study of a stronger form of $\alpha$-paralindelof property. In this section, the class of strongly $\alpha$-paralindelof fts is introduced and studied, using the concept of $\alpha$-star countable collection of fuzzy sets. It is proved that a closed crisp subspace of a strongly $\alpha$-paralindelof fts is strongly $\alpha$-paralindelof and that the class of strongly $\alpha$-paralindelof fts is invariant under $F$-continuous, $F$-open surjections.

In the sixth section of this chapter, the concept of nearly $\alpha$-paralindelof property in fuzzy topology is introduced and studied. It is proved that this concept
generalizes $\alpha$-compactness, nearly $\alpha$-compactness, $\alpha$-paracompactness, nearly $\alpha$-paracompactness and $\alpha$-paralindelof property. It is also proved that a fts $X$ is nearly $\alpha$-paralindelof if and only if every fuzzy regular open $\alpha$-shading of $X$ has a $\alpha$-locally countable, fuzzy regular open refinement. Invariance under maps, of such class of fts is also obtained.

The next section of this chapter contains the study of nearly weakly $\alpha$-paralindelof fts. In this section, the new concept of nearly weakly $\alpha$-paralindelof property in fuzzy topology is introduced and studied. It is shown that this concept is a weaker form of $\alpha$-compactness, nearly $\alpha$-compactness, $\alpha$-paracompactness, nearly $\alpha$-paracompactness, weakly $\alpha$-paracompactness and nearly weakly $\alpha$-paracompactness. A characterization and an invariance under maps of such a class of fts are also obtained.

Finally in the eighth section of this chapter, the concept of nearly strongly $\alpha$-paralindelof property in fuzzy topology is introduced and studied. It is shown that this concept generalizes strong $\alpha$-paracompactness, nearly strongly $\alpha$-paracompactness and strongly $\alpha$-paralindelof property. An invariance of such a class under maps is obtained.

Most of the ideas in this chapter are motivated by the work of Gantner et.al [16] and the corresponding concepts in general topology.
2. α - COUNTABILITY AND α - SEPARABILITY IN FTS

The study of separability and countability axioms in fuzzy topology was carried out by C. K. Wong [52 & 53], S. R. Malghan and S. S. Benchalli [33 & 5] and many others.

In this section the concepts of α - local base, α - C₁, α - separability and α - C₁₁ in fuzzy topology have been introduced and studied. It is proved among other results, that α - C₁ is invariant under F - continuous, F - open surjections, α - separability is invariant under F - continuous surjections, α - separability is open hereditary and that α - C₁₁ is a hereditary property.

The concept of α - local base is introduced in the following

2.1 Definition: Let α ∈ [0, 1) ( resp. α ∈ (0, 1]). Let (X, T) be a fts and x ∈ X. A subfamily ℋ of T is called a α - local base ( resp. α* - local base ) at x iff B(x) ≥ α ( resp. B(x) ≥ α ) for each B ∈ ℋ_x and for every A ∈ T with A(x) > α ( resp. A(x) ≥ α ), there exists a member B₀ ∈ ℋ_x such that B₀ ≤ A.

The class of α - C₁ fts is introduced in the following.

2.2 Definition: Let α ∈ [0, 1) ( resp. α ∈ (0, 1]). A fts (X, T) is said to be α - C₁ ( resp.α* - C₁ ) iff every x ∈ X has a countable α - local base ( resp. countable α* - local base ).

Separability in fuzzy topology is defined in the following.
2.3 Definition: Let $\alpha \in [0, 1)$ (resp. $\alpha \in (0, 1]$). A fts $(X, T)$ is said to be $\alpha$-separable (resp. $\alpha^*$-separable) iff there exists a sequence of points \( \{x_i : i = 1, 2, 3, \ldots \} \) such that for every member $A$ of $T$ and $A \neq 0$ there exists a $x_i$ such that $A(x_i) > \alpha$ (resp. $A(x_i) \geq \alpha$).

The following result contains a property of $\alpha$-C1 fts.

2.4 Theorem: If $(X, T)$ is a $\alpha$-C1 fts then for each $x \in X$ there exists a countable $\alpha$-local base of $x$ say $\mathcal{V} = \{ A_i : i = 1, 2, 3, \ldots \}$ such that $A_1 \geq A_2 \geq A_3 \geq \ldots$.

Proof: Let $(X, T)$ be a $\alpha$-C1 fts. Let $x \in X$. Then there exists a countable $\alpha$-local base say $\mathcal{B} = \{ B_i : i = 1, 2, 3, \ldots \}$ of $x$. Now define $A_i = B_i,$ $A_2 = B_1 \land B_2,$ $A_3 = B_1 \land B_2 \land B_3,$ \ldots. Let $\mathcal{V} = \{ A_i : i = 1, 2, 3, \ldots \}$. $\mathcal{V}$ is a $\alpha$-local base at $x$; Since $\mathcal{B}$ is a $\alpha$-local base at $x$, for each $B_i \in \mathcal{B}$, $B_i(x) > \alpha$.

Therefore $A_i(x) > \alpha$ for each $i$. Let $G \in T$ with $G(x) > \alpha$. Again, since $\mathcal{B}$ is a $\alpha$-local base, there exists a $B_{i_0} \in \mathcal{B}$ such that $B_{i_0} \leq G$. Also $B_{i_0}(x) > \alpha$ for each $i = 1, 2, 3, \ldots\ i_0$, as each $B_i \in \mathcal{B}$ and $\mathcal{B}$ is a $\alpha$-local base at $x$. Therefore $\bigwedge_{i = 1}^{i_0} B_i(x) > \alpha$. That is $(A_{i_0})(x) > \alpha$, as $A_{i_0} = B_1 \land B_2 \land B_3 \land \ldots \land B_{i_0}$. Also $A_{i_0} \leq B_{i_0}$. But $B_{i_0} \leq G$. Therefore $A_{i_0} \leq G$. Thus $\mathcal{V}$ is a countable $\alpha$-local base at $x$ such that $A_1 \geq A_2 \geq A_3 \geq \ldots$.

The next result shows that the class of $\alpha$-C1 fts is invariant under $F$-continuous, $F$-open surjections.
2.5 Theorem: Let \( f : (X, T) \to (Y, S) \) be an \( F \)-continuous, \( F \)-open surjection. If \((X, T)\) is \( \alpha - C_1 \) (resp. \( \alpha^* - C_1 \)) fts then \((Y, S)\) is also \( \alpha - C_1 \) (resp. \( \alpha^* - C_1 \)) fts.

Proof: Let \( f : (X, T) \to (Y, S) \) be a \( F \)-continuous, \( F \)-open surjection and \((X, T)\) is \( \alpha - C_1 \) fts. To prove that \((Y, S)\) is \( \alpha - C_1 \) fts. Let \( y \in Y \). Then there exists \( x \in X \) such that \( f(x) = y \). Since \((X, T)\) is \( \alpha - C_1 \), \( x \) has a countable \( \alpha \)-local base for \( T \), say \( \mathcal{B}_x \). Then the family \( \mathcal{U}_y = \{ f(A) : A \in \mathcal{B}_x \} \) forms a countable \( \alpha \)-local base at \( y \) in \( S \); For each \( A \in \mathcal{B}_x \), \( A(x) > \alpha \) for each \( A \). Let \( \mathcal{V}_y \) be a subfamily of \( S \) which is countable, as \( \mathcal{B}_x \) is countable. Let \( \mathcal{V}_y \) be countable and \( \mathcal{V}_x \) is a subfamily of \( S \) which is countable, as \( \mathcal{B}_x \) is countable. Let \( f(A) \in \mathcal{V}_y \), then

\[ [f(A)](y) = \bigvee_{z \in f^{-1}(y)} A(z) > \alpha, \text{ since } x \in f^{-1}(y) \text{ and } A(x) > \alpha \text{ for each } A \in \mathcal{B}_x. \]

Further, let \( G \in S \) with \( G(y) > \alpha \). Then \( f^{-1}(G) \in T \), as \( f \) is \( F \)-continuous and \( [f^{-1}(G)](x) = G(f(x)) = G(y) > \alpha \). Therefore \( f^{-1}(G) \in T \) and \( f^{-1}(G)(x) > \alpha \).

Since \( \mathcal{B}_x \) is \( \alpha \)-local base at \( x \), there exists \( A_0 \in \mathcal{B}_x \) such that \( A_0 \le f^{-1}(G) \) where \( A_0(x) > \alpha \). Therefore \( f(A_0) \le f[f^{-1}(G)] = G \) as \( f \) is onto. And \( f(A_0)(y) \rangle \alpha \) = \( \bigvee_{z \in f^{-1}(y)} A(z) > \alpha \). Therefore \( f(A_0) \rangle \alpha \). Thus for \( G \in S \) with \( G(y) > \alpha \), there exists \( f(A_0) \) in \( \mathcal{V}_y \) such that \( f(A_0) \le G \) and \( f(A_0)(y) \rangle \alpha \). Therefore \( \mathcal{V}_y \) is countable \( \alpha \)-local base at \( y \) in \( S \). Hence \((Y, S)\) is a \( \alpha - C_1 \) fts.

The following result gives an invariant property for \( \alpha \)-separability.
2.6 Theorem: Let \( f: (X, T) \rightarrow (Y, S) \) be an \( F \)-continuous surjection. If \((X, T)\) is \( \alpha \)-separable (resp. \( \alpha^* \)-separable) fts then \((Y, S)\) is also \( \alpha \)-separable (resp. \( \alpha^* \)-separable) fts.

Proof: Let \((X, T)\) be a \( \alpha \)-separable fts. Then there exists a sequence of points \( \{ x_i : i = 1, 2, 3, \ldots \} \) in \( X \) such that for every member \( A \) of \( T \) with \( A \neq \emptyset \) there exists an \( x_i \) such that \( A(x_i) > \alpha \). To prove \((Y, S)\) is also \( \alpha \)-separable. Consider \( \{ f(x_i) : i = 1, 2, 3, \ldots \} \) which is a sequence of points in \( Y \). Let \( B \in S \) and \( B \neq \emptyset \).

Then \( f^{-1}(B) \in T \) and \( f^{-1}(B) \neq \emptyset \). For \( B \neq \emptyset \) implies that there exists \( y \in Y \) such that \( B(y) > 0 \) and for \( x \in X \) such that \( f(x) = y \), \( f^{-1}(B)(x) = B(f(x)) = B(y) > 0 \). That is \( f^{-1}(B)(x) > 0 \). Therefore there exists \( x \) in \( X \) such that \( f^{-1}(B)(x) > 0 \).

Therefore \( f^{-1}(B) \neq \emptyset \). Since \((X, T)\) is \( \alpha \)-separable there exists \( x_{i_0} \in \{ x_i : i = 1, 2, 3, \ldots \} \) such that \( f^{-1}(B)(x_{i_0}) > \alpha \). Therefore \( B[f(x_{i_0})] > \alpha \). Thus for \( B \in S \) and \( B \neq \emptyset \), there exists a point \( f(x_{i_0}) \) in \( \{ f(x_i) : i = 1, 2, 3, \ldots \} \) such that \( B[f(x_{i_0})] > \alpha \). Hence \((Y, S)\) is \( \alpha \)-separable fts. The proof is similar for \( \alpha^* \)-case.

The concept of a base is introduced in the following.

2.7 Definition: Let \( \alpha \in [0, 1) \) (resp. \( \alpha \in (0, 1] \)). Let \((X, T)\) be a fts. A subfamily \( \mathcal{B} \) defined by \( \mathcal{B} = \{ B : B \in T, B \text{ is nonempty of order } \alpha \} \) (resp. \( \mathcal{B}^* = \{ B : B \in T, B \text{ is nonempty of order } \alpha^* \} \)) of \( T \) is said to be a
\( \alpha \) - base (resp. \( \alpha^* \) - base) if every member \( A \) of \( T \) with \( A \neq 0 \) is expressed as a union of members of \( \emptyset \) (resp. \( \emptyset^* \)).

The class of \( \alpha \) - C_{11} fts is introduced in the following.

**2.8 Definition:** A fts \( (X, T) \) is said to be \( \alpha \) - C_{11} (resp. \( \alpha^* \) - C_{11}) iff there exists a countable \( \alpha \) - base (resp. \( \alpha^* \) - base) for \( T \).

The following result gives an interrelation between \( \alpha \) - C_{11} and \( \alpha \) - C_{1} fts.

**2.9 Theorem:** Every \( \alpha \) - C_{11} (resp. \( \alpha^* \) - C_{11}) fts is \( \alpha \) - C_{1} (resp. \( \alpha^* \) - C_{1}).

**Proof:** Let \( (X, T) \) be a \( \alpha \) - C_{11} fts. Let \( x \in X \), to prove that there exists a countable \( \alpha \) - local base at \( x \). Since \( (X, T) \) is \( \alpha \) - C_{11}, it follows that \( T \) has a countable \( \alpha \) - base say \( \emptyset = \{ B : B \in T, B \text{ is nonempty of order } \alpha \} \). Let \( \emptyset_x \subseteq \emptyset \) be defined by \( \emptyset_x = \{ B : B \in \emptyset, B(x) > \alpha \} \) clearly \( \emptyset_x \) is countable. Let \( A \in T \) with \( A(x) > \alpha \).

Since \( A \in T \) and \( \emptyset \) is a \( \alpha \) - base for \( T \), \( A \) can be expressed as the union of some member of \( \emptyset \) say \( A = \bigvee_{B_i \in \emptyset} B_i \). But \( A(x) > \alpha \). Therefore \( ( \bigvee_{B_i \in \emptyset} B_i )(x) > \alpha \). That is \( B_i(x) > \alpha \) for some \( B_i \in \emptyset \). Therefore \( B_i \in \emptyset_x \) and \( B_i \leq \bigvee_{B_i \in \emptyset} B_i = A \). That is, \( B_i \leq A \). Therefore \( \emptyset_x \) is an \( \alpha \) - local base for \( T \) at \( x \). But \( x \in X \) is arbitrary. Therefore every \( x \in X \) has a countable \( \alpha \) - local base. Hence \( (X, T) \) is \( \alpha \) - C_{1} fts. The proof of \( \alpha^* \) - case is similar.

The following result gives an interrelation between \( \alpha \) - C_{11} and \( \alpha \) - separability.

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2.10 Theorem: If a fts \((X, T)\) is an \(\alpha\)-\(C_{11}\) (resp. \(\alpha^*\)-\(C_{11}\)) then it is \(\alpha\)-separable (resp. \(\alpha^*\)-separable).

Proof: Let \((X, T)\) be an \(\alpha\)-\(C_{11}\) fts. Therefore \(T\) has a countable \(\alpha\)-base say \(B = \{B_i: i \in \mathbb{N}, B_i \in T, B_i \text{ is nonempty of order } \alpha\}\). Since \(B_i\) is nonempty of order \(\alpha\), there is a point in \(X\) say \(x_i\) such that \(B_i(x_i) > \alpha\). Then \(\{x_i: B_i(x_i) > \alpha, B_i \in B\}\) is a sequence of points in \(X\). Let \(A \in T\) and \(A \neq 0\). Since \(B\) is a \(\alpha\)-base, \(A\) can be expressed as a union of members of \(B\) say, \(A = \bigvee_{B_i \in B} B_i\). Let \(x_{i_0} \in \{x_i: i \in \mathbb{N}\}\). Then \(A(x_{i_0}) = (\bigvee_{B_i \in B} B_i)(x_{i_0}) > \alpha\) as \(B_i(x_{i_0}) > \alpha\). Thus for every \(A \in T\) with \(A \neq 0\), there exists a point \(x_{i_0}\) such that \(A(x_{i_0}) > \alpha\). It follows that \((X, T)\) is \(\alpha\)-separable.

A hereditary property for \(\alpha\)-separability is contained in the following.

2.11 Theorem: Every open crisp subspace of a \(\alpha\)-separable fts is \(\alpha\)-separable (resp. \(\alpha^*\)-separable).

Proof: Let \((X, T)\) be a \(\alpha\)-separable space and \(Y\) be an open crisp subspace of \((X, T)\). Since \((X, T)\) is \(\alpha\)-separable, there exists a countable sequence of points say \(S = \{x_i: i \in \mathbb{N}\}\) such that for each \(A \in T\) with \(A \neq 0\) there exists \(x_i\) such that \(A(x_i) > \alpha\).

Now let \(S_1 = \{x_n \in S: n \in \mathbb{N}\}\), which is a countable sequence of points in \(Y\). Let \(U\) be any open fuzzy set in \(Y\), with \(U \neq 0\). Then there is an open
fuzzy set \( V \) in \( T \) such that \( U = Y \wedge V \). Since \( X \) is \( \alpha \)-separable fts and \( Y \) is also open fuzzy set in \( X \), there exists \( x_{i_0} \in S \) such that \( V(x_{i_0}) > \alpha \). Also \( Y(x_{i_0}) = 1 > \alpha \).

For if \( Y(x_{i_0}) = 0 \) then \( U(x_{i_0}) = Y(x_{i_0}) \wedge V(x_{i_0}) = 0 \wedge V(x_{i_0}) = 0 \) which contradicts \( U \neq 0 \). Therefore \( U(x_{i_0}) = Y(x_{i_0}) \wedge V(x_{i_0}) > \alpha \). Thus for each open fuzzy set \( U \) in \( Y \) with \( U \neq 0 \), there exists a point \( x_{i_0} \) in \( S \) which is countable sequence of points in \( Y \) such that \( U(x_{i_0}) > \alpha \). It follows that \( Y \) is \( \alpha \)-separable fts.

A hereditary property for \( \alpha \)-C\(_{11} \) is confined in the following.

**2.12 Theorem**: Every crisp subspace of a \( \alpha \)-C\(_{11} \) fts is \( \alpha \)-C\(_{11} \).

**Proof**: Let \( (X, T) \) be a \( \alpha \)-C\(_{11} \) fts and \( Y \) be a crisp subspace of \( (X, T) \). Since \( (X, T) \) is \( \alpha \)-C\(_{11} \) there exists a countable \( \alpha \)-base for \( T \), say \( \mathcal{B} = \{ B_i : i \in \mathbb{N} \text{ and } \text{B}_i \text{ is nonempty of order } \alpha \} \). Then \( \mathcal{B}_y = \{ B_i \wedge Y : i \in \mathbb{N} \} \) is a countable \( \alpha \)-base for crisp subspace \( Y \); For if \( U \) is an open fuzzy set in \( Y \) with \( U \neq 0 \) then \( U = Y \wedge G \), where \( G \) is open fuzzy set in \( X \). Now \( G \in T \) and \( \mathcal{B} \) is a \( \alpha \)-base for \( T \), it follows that \( G = \bigvee_{B_n \in \mathcal{B}} B_n \). Therefore \( U = \left( \bigvee_{B_n \in \mathcal{B}} B_n \wedge Y \right) = \bigvee_{B_n \in \mathcal{B}} (B_n \wedge Y) \), where \( B_n \wedge Y \in \mathcal{B}_y \) for each \( n \). Since \( B_i \) is nonempty of order \( \alpha \), there exists \( x \in X \) such that \( B_i(x) > \alpha \). Also \( Y(x) = 1 > \alpha \). Therefore \( Y \wedge B_i \) is nonempty of order \( \alpha \). Thus every open fuzzy set \( U \) with \( U \neq 0 \) in \( Y \), can be expressed as the union of members of \( \mathcal{B}_y \). Hence \( \mathcal{B}_y \) is a \( \alpha \)-base for \( Y \). Therefore \( Y \) is a \( \alpha \)-C\(_{11} \) fts.

Similar result can be proved for \( \alpha^* \)-case.
The next result shows that every \( \alpha - C_{11} \) fts is \( \alpha - \text{Lindelof} \).

2.13 Theorem: Let \( \alpha \in [0,1] \). If a fts \((X, T)\) is \( \alpha - C_{11} \) (resp. \( \alpha^* - C_{11} \)) then it is \( \alpha - \text{Lindelof} \) (resp. \( \alpha^* - \text{Lindelof} \)).

Proof: Let \((X, T)\) be a \( \alpha - C_{11} \) fts. Then \( T \) has a countable \( \alpha \)-base say \( \mathcal{B} = \{ B_i : B_i \in T, i \in \mathbb{N}, B_i \text{ is nonempty of order } \alpha \} \). To prove that \((X, T)\) is \( \alpha - \text{Lindelof} \) fts. Let \( \mathcal{U} = \{ U_\lambda : \lambda \in \Lambda \} \) be any open \( \alpha \)-shading of \( X \). Since \( \mathcal{B} \) is a \( \alpha \)-base for \( T \), each \( U_\lambda \) can be expressed as a union of member of \( \mathcal{B} \).

Let \( U_\lambda = \bigvee \{ B_{1, \lambda} \in \mathcal{B} : i = 1,2,3, \ldots, \lambda \} \). Let \( \mathcal{B}_n = \{ B_{i, \lambda} : \lambda \in \Lambda, \ i = 1,2,3, \ldots, \lambda_n \} \), clearly \( \mathcal{B}_n \) is a countable family of open fuzzy sets in \( X \). Also \( \mathcal{B}_n \) is an \( \alpha \)-shading of \( X \); Let \( x \in X \). Since \( \mathcal{U} \) is an \( \alpha \)-shading of \( X \), there exists some \( U_\mu \) in \( \mathcal{U} \) such that \( U_\mu(x) > \alpha \). Therefore \( \bigvee \{ B_{i, \lambda} : i = 1,2,3, \ldots, \lambda_n \}(x) > \alpha \).

Therefore \( B_{i', \lambda'}(x) > \alpha \) for some \( i', \lambda' \in \{ 1,2,3, \ldots, \lambda_n \} \) and \( B_{i', \lambda'} \in \mathcal{B}_n \). Therefore \( \mathcal{B}_n \) is an \( \alpha \)-shading of \( X \). Now each member of \( \mathcal{B}_n \) is less than or equal to some member of \( \mathcal{U} \); Let \( B_{1, \lambda} \in \mathcal{B}_n \) for some \( \lambda \in \Lambda \) and \( i \in \{ 1,2,3, \ldots, \lambda_n \} \). Then \( B_{1, \lambda} \leq \bigvee \{ B_{1, \lambda} : i = 1,2,3, \ldots, \lambda_n \} = U_\lambda \in \mathcal{U} \). Let \( \mathcal{J} = \{ U_\lambda : U_\lambda \geq B_{1, \lambda} \in \mathcal{B}_n \} \).

Since \( \mathcal{B}_n \) is countable, \( \mathcal{J} \) is a countable subfamily of \( \mathcal{U} \).

\( \mathcal{J} \) is an \( \alpha \)-shading of \( X \); Let \( x \in X \). Since \( \mathcal{B}_n \) is an \( \alpha \)-shading of \( X \), there exists a \( B_{1, \lambda} \) in \( \mathcal{B}_n \) such that \( B_{1, \lambda}(x) > \alpha \). But, for \( B_{1, \lambda} \in \mathcal{B}_n \) there exists \( U_\lambda \) in \( \mathcal{U} \) such that \( B_{1, \lambda} \leq U_\lambda \). Therefore \( U_\lambda(x) \geq B_{1, \lambda}(x) > \alpha \). That is, \( U_\lambda(x) > \alpha \). Also since
Therefore $\mathcal{J}$ is an $\alpha$-shading of $X$. Thus, every open $\alpha$-shading $\mathcal{U}$ of $X$ has a countable $\alpha$-subshading. Hence $X$ is $\alpha$-Lindelöf fts. The proof of $\alpha^*$-case is similar.

**2.14 Corollary:** If $(X, T)$ is a $\alpha$-$C_{11}$ (resp. $\alpha^*$-$C_{11}$) fts then it is nearly $\alpha$-Lindelöf (resp. nearly $\alpha^*$-Lindelöf) fts.

### 3. $\alpha$-PARALINDELÖF FTS

Paralindelöf spaces in general topology were studied by D. K. Burke [7&8], S. W. Davis et. al [11], Fleissner and Reed [15] and many others.

In the present section $\alpha$-paralindelöf fts have been introduced and studied. It is proved that every $\alpha$-Lindelöf fts is $\alpha$-paralindelöf, every $\alpha$-paracompact fts is $\alpha$-paralindelöf, every $\alpha$-$C_{11}$ fts is $\alpha$-paralindelöf, every closed crisp subspace of $\alpha$-paralindelöf fts is $\alpha$-paralindelöf and that $\alpha$-paralindelöf property is invariant under $F$-continuous, $F$-open surjective functions.

The concept of $\alpha$-locally countable collection of fuzzy sets is introduced in the following.

**3.1 Definition:** Let $\alpha \in [0, 1)$ (resp. $\alpha \in (0, 1]$). A family $(A_\lambda : \lambda \in \Lambda)$ of fuzzy sets in a fts $X$ is said to be $\alpha$-locally countable (resp. $\alpha^*$-locally countable) in $X$ if for each $x \in X$ there exists an open fuzzy set $U$ in $X$ such that $U(x) = 1$ and
$U \wedge A_{\lambda}$ is nonempty of order $\alpha$ (resp. $\alpha^*$) for atmost countably many $\lambda \in \Lambda$. That is $\{ \lambda \in \Lambda : U \wedge A_{\lambda}$ is nonempty of order $\alpha \}$ is atmost countable.

3.2 Remark: 1) Every $\alpha$ - locally finite family (resp. $\alpha^*$) of fuzzy sets is $\alpha$ - locally countable (resp. $\alpha^*$).

2) Every countable family of fuzzy sets in a fts is $\alpha$ - locally countable.

Paralindelof property in fuzzy topology is introduced in the following.

3.3 Definition: Let $0 \leq \alpha < 1$ (resp. $0 < \alpha \leq 1$). A fts $X$ is said to be $\alpha$ - paralindelof (resp. $\alpha^*$ - paralindelof) if each open $\alpha$ - shading (resp. $\alpha^*$ - shading) of $X$ has a $\alpha$ - locally countable (resp. $\alpha^*$ - locally countable) open refinement.

The next result gives an interrelation between $\alpha$ - Lindelof $\alpha$ - paracompact and $\alpha$ - paralindelof property.

3.4 Theorem: Every $\alpha$ - Lindelof fts is $\alpha$ - paralindelof fts.

Proof: Let $(X, T)$ be a $\alpha$ - Lindelof fts. To prove that $(X, T)$ is $\alpha$ - paralindelof fts. Let $\mathcal{U}$ be any open $\alpha$ - shading of $X$. Since $X$ is $\alpha$ - Lindelof, $\mathcal{U}$ has a countable $\alpha$ - subshading say $\mathcal{V}$. Then by Remark 3.2 (2), it follows that $\mathcal{V}$ is $\alpha$ - locally countable family. Also $\mathcal{V}$ is refinement of $\mathcal{U}$. Thus every open $\alpha$ - shading $\mathcal{U}$ of $X$ has a $\alpha$ - locally countable open refinement. Hence $X$ is $\alpha$ - paralindelof fts.

Similar result can be proved for $\alpha^*$ - case.
3.5 **Theorem**: Every $\alpha$-paracompact (resp. $\alpha^*$-paracompact) ft is $\alpha$-paralindelof (resp. $\alpha^*$-paralindelof) ft.

**Proof**: The proof follows immediately by Remark 3.2 (1).

The next result shows that every $\alpha$-$C_{11}$ ft is $\alpha$-paralindelof.

3.6 **Theorem**: Let $\alpha \in [0, 1)$ (resp. $\alpha \in (0, 1]$). If a ft $(X, T)$ is $\alpha$-$C_{11}$ (resp. $\alpha^*$-$C_{11}$) then it is $\alpha$-paralindelof (resp. $\alpha^*$-paralindelof) ft.

**Proof**: Let $(X, T)$ be a $\alpha$-$C_{11}$ ft. Then it has a countable $\alpha$-base say $\mathcal{B} = \{ B_i : B_i \in T, i \in \mathbb{N}, B_i \text{ is nonempty of order } \alpha \}$. To prove that $(X, T)$ is $\alpha$-paralindelof ft. Let $\mathcal{U} = \{ U_\lambda : \lambda \in \Lambda \}$ be any open $\alpha$-shading of $X$. Since $\mathcal{B}$ is a $\alpha$-base for $T$, each member of $\mathcal{U}$ can be expressed as union of member of $\mathcal{B}$.

Therefore $U_\lambda = \bigvee \{ B_i : i = 1, 2, 3, \ldots, \lambda_n \}$, where $\lambda_n$ may be infinite. Let

$\mathcal{B}_n = \{ B_{i, \lambda} : \lambda \in \Lambda, i = 1, 2, 3, \ldots, \lambda_n \}$. Clearly $\mathcal{B}_n$ is a countable family of open fuzzy sets in $X$. Also $\mathcal{B}_n$ is $\alpha$-shading of $X$. Let $x \in X$. Since $\mathcal{U}$ is an $\alpha$-shading of $X$, there exists $U_\lambda$ in $\mathcal{U}$ such that $U_\lambda(x) > \alpha$. Therefore $(\bigvee \{ B_{i, \lambda} : i = 1, 2, 3, \ldots, \lambda_n \})(x) > \alpha$. Therefore $B_{i', \lambda}(x) > \alpha$ for some $i' \in \{ 1, 2, 3, \ldots, \lambda_n \}$ and $B_{i', \lambda} \in \mathcal{B}_n$.

Now each member of $\mathcal{B}_n$ is less than or equal to some member of $\mathcal{U}$; For, let $B_{i, \lambda} \in \mathcal{B}_n$ for some $\lambda \in \Lambda$ and $i \in \{ 1, 2, 3, \ldots, \lambda_n \}$. Then $B_{i, \lambda} \leq \bigvee \{ B_{i, \lambda} : i = 1, 2, 3, \ldots, \lambda_n \} = U_\lambda \in \mathcal{U}$.
Let $\mathcal{I} = \{ U_\lambda : B_{i, \lambda} \subseteq U_\lambda, B_{i, \lambda} \in \mathcal{B}_n \}$. Since $\mathcal{B}_n$ is countable family, $\mathcal{I}$ is also countable and $\mathcal{I}$ is subfamily of $\mathcal{U}$. Also $\mathcal{I}$ is an $\alpha$-shading of $X$. Thus $\mathcal{I}$ is a $\alpha$-locally countable open refinement of $\mathcal{U}$. Hence $(X, T)$ is $\alpha$-paralindelof ft's.

The proof of $\alpha^*$-case is analogous.

A hereditary property of $\alpha$-paralindelof ft's is contained in the following.

**3.7 Theorem**: Every closed crisp subspace of a $\alpha$-paralindelof ft's is $\alpha$-paralindelof.

**Proof**: Let $(X, T)$ be a $\alpha$-paralindelof ft's and $Y$ be a closed crisp subspace of $(X, T)$. To prove that $Y$ is $\alpha$-paralindelof ft's. Let $\mathcal{V} = \{ V_\lambda : \lambda \in \Lambda \}$ be an open $\alpha$-shading of $Y$. For each $\lambda \in \Lambda$ there exists $U_\lambda \in T$ such that $V_\lambda = Y \setminus U_\lambda$. Then clearly $\mathcal{U} = \{ U_\lambda : \lambda \in \Lambda \} \cup \{ 1 - Y \}$ is an open $\alpha$-shading of $X$. Since $X$ is $\alpha$-paralindelof, $\mathcal{U}$ has a $\alpha$-locally countable open refinement say $\{ W_\gamma : \gamma \in \Gamma \}$.

Now the family $\{ Y \setminus W_\gamma : \gamma \in \Gamma \}$ is the required $\alpha$-locally countable open refinement of $\mathcal{V}$ for $Y$; For each $\gamma \in \Gamma$, $Y \setminus W_\gamma$ is an open fuzzy set in $Y$ and if $y \in Y$ then $y \in X$. Since $\{ W_\gamma : \gamma \in \Gamma \}$ is $\alpha$-locally countable, there exists an open fuzzy set $N$ with $N(y) = 1$ and $\{ \gamma \in \Gamma : N \setminus W_\gamma \text{ is nonempty of order } \alpha \}$ is atmost countable. That is $\{ \gamma \in \Gamma : Y \setminus N \setminus W_\gamma \text{ is nonempty of order } \alpha \}$ is atmost countable. That is $\{ \gamma \in \Gamma : (N \setminus Y) \setminus (Y \setminus W_\gamma) \text{ is nonempty of order } \alpha \}$ is atmost

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countable. Note that \( N(y) = 1 \) and \( Y(y) = 1 \). Therefore \( \langle N \land Y \rangle (y) = 1 \). Hence \( \{ Y \land W_y : y \in \Gamma \} \) is \( \alpha \) - locally countable family of fuzzy sets in \( Y \).

Finally, \( \{ Y \land W_y : y \in \Gamma \} \) is refinement of \( U \); Let \( Y \land W_y \in \{ Y \land W_y : y \in \Gamma \} \). Then for \( W_y \) there exists some \( U_y \) or \( 1 - Y \) such that \( W_y \leq U_y \) or \( W_y \leq 1 - Y \), as \( \{ W_y : y \in \Gamma \} \) is refinement of \( U \). That is \( Y \land W_y \leq Y \land U_y \) or \( Y \land W_y \leq Y \land (1 - Y) \). That is \( Y \land W_y \leq V_y \) or \( Y \land W_y \leq 0 \). Thus for \( Y \land W_y \in \{ Y \land W_y : y \in \Gamma \} \) there exists \( V_y \) in \( U \) such that \( Y \land W_y \leq V_y \). It follows that \( \{ Y \land W_y : y \in \Gamma \} \) is a refinement of \( U \). Hence \( Y \) is \( \alpha \) - paralindelof fts.

Similar result can be proved for \( \alpha^* \) - case.

The next result gives an invariance under maps, of \( \alpha \) - paralindelof property.

3.8 Theorem : Let \( f : X \to Y \) be an \( F \)- continuous, \( F \)- open surjection. If \( X \) is \( \alpha \) - paralindelof fts then \( Y \) is \( \alpha \) - paralindelof fts.

Proof : Let \( f : X \to Y \) be an \( F \)- continuous, \( F \)- open surjection and \( X \) be a \( \alpha \) - paralindelof fts. To prove that \( Y \) is \( \alpha \) - paralindelof fts. Let \( U \) be an open \( \alpha \) - shading of \( Y \). Then clearly \( f^{-1}(U) = \{ f^{-1}(U) : U \in U \} \) is an open \( \alpha \) - shading of \( X \). Since \( X \) is \( \alpha \) - paralindelof, it follows that \( f^{-1}(U) \) has a \( \alpha \) - locally countable open refinement say \( \{ V_y : y \in \Gamma \} \). Then \( \{ f(V_y) : y \in \Gamma \} \) is the required \( \alpha \) - locally countable open refinement of \( U \); Since \( V_y \) is open in \( X \) and \( f \)
is an $F$-open map it follows that $f(V_y)$ is an open fuzzy set in $Y$. \{ f(V_y) : y \in \Gamma \} is \alpha$-locally countable in $Y$; Let $y \in Y$. Then there exists $x \in X$ such that $f(x) = y$. Now $x \in X$ and \{ $V_y : y \in \Gamma \} is \alpha$-locally countable family in $X$. Therefore there exists an open fuzzy set $H$ in $X$ such that $H(x) = 1$ and \{ $\gamma \in \Gamma : H \wedge V_y is nonempty of order $\alpha$ \} is atmost countable. That is \{ $\gamma \in \Gamma : f( H \wedge V_y ) is nonempty of order $\alpha$ \} is atmost countable. Therefore \{ $\gamma \in \Gamma : f( H ) \wedge f( V_y ) is nonempty of order $\alpha$ \} is atmost countable. Thus each $y \in Y$, there exists an open fuzzy set $f(H)$ in $X$ such that $[f(H)](y) = \sup \{ H(z) : z \in f^{-1}(y) \} = 1$ as $H(x) = 1$ and $f(x) = y$. Also the set \{ $\gamma \in \Gamma : f( H ) \wedge f( V_y ) is nonempty of order $\alpha$ \} is atmost countable. Therefore the family \{ $f(V_y) : y \in \Gamma \} is \alpha$-locally countable in $Y$.

Finally \{ $f(V_y) : y \in \Gamma \} is refinement of $\mathcal{U}$; Let $\gamma \in \Gamma$. Since \{ $V_y : y \in \Gamma \} is refinement of $f(\mathcal{U})$, for $V_y, V' \in \{ V_y : y \in \Gamma \}$, there exists $f^{-1}(U_\delta)$ in $f^{-1}(\mathcal{U})$ such that $V_y \leq f^{-1}(U_\delta)$ which implies that $f(V_y) \leq f[f^{-1}(U_\delta)] = U_\delta$, as $f$ is onto. Thus each open $\alpha$-shading $\mathcal{U}$ of $Y$ has an $\alpha$-locally countable open refinement. Hence $Y$ is $\alpha$-paralindelof fts.
4. WEAKLY $\alpha$ - PARALINDELOF FTS

In this section, a weaker form of $\alpha$ - paralindelof property in fuzzy topology is introduced and studied, using the concept of $\alpha$ - point countable family of fuzzy sets. It is proved that a closed crisp subspace of weakly $\alpha$ - paralindelof fts is weakly $\alpha$ - paralindelof and that weakly $\alpha$ - paralindelof property is invariant under $F$ - continuous, $F$ - open surjections.

The concept of $\alpha$ - point countable collection of fuzzy sets is introduced in the following.

4.1 Definition : Let $\alpha \in [0,1)$ (resp. $\alpha \in (0,1]$). A family $\{ A_{\lambda} : \lambda \in \Lambda \}$ of fuzzy sets in a fuzzy topological space $(X, T)$ is said to be $\alpha$ - point countable (resp. $\alpha^*$ - point countable) if for each $x \in X$, $A_{\lambda}(x) > \alpha$ (resp. $A_{\lambda}(x) \geq \alpha$) for at most countably many $\lambda \in \Lambda$.

It is clear that every $\alpha$ - point finite family is $\alpha$ - point countable family and every $\alpha$ - locally finite family is $\alpha$ - point finite and hence $\alpha$ - point countable.

The class of weakly $\alpha$ - paralindelof fts is introduced in the following.

4.2 Definition : A fts $X$ is said to be a weakly $\alpha$ - paralindelof (resp. weakly $\alpha^*$ - paralindelof) if each open $\alpha$ - shading (resp. $\alpha^*$ - shading) of $X$ has a $\alpha$ - point countable (resp. $\alpha^*$ - point countable) open refinement.
One can easily verify that every $\alpha$-paralindelof fts is weakly $\alpha$-paralindelof and that every weakly $\alpha$-paracompact fts is weakly $\alpha$-paralindelof fts.

4.3 **Theorem**: Every $\alpha - C_{11}$ (resp. $\alpha^* - C_{11}$) fts is weakly $\alpha$-paralindelof (resp. $\alpha^*$-paralindelof) fts.

**Proof**: The proof is analogous to that of Theorem 3.6.

A hereditary property of weakly $\alpha$-paralindelof fts is contained in the following.

4.4 **Theorem**: Every closed crisp subspace of a weakly $\alpha$-paralindelof fts is weakly $\alpha$-paralindelof.

**Proof**: Let $(X, T)$ be a weakly $\alpha$-paralindelof fts and $Y$ be a closed crisp subspace of $X$. To prove that $Y$ is weakly $\alpha$-paralindelof fts. Let $\Psi = \{V_\lambda : \lambda \in \Lambda\}$ be any open $\alpha$-shading of $Y$. Then for each $\lambda \in \Lambda$ there is an open fuzzy set $U_\lambda$ in $X$ such that $V_\lambda = Y \cap U_\lambda$. Then it can be verified that $\mathcal{U} = \{U_\lambda : \lambda \in \Lambda\} \cup \{1 - Y\}$ is an open $\alpha$-shading of $X$. Since $X$ is weakly $\alpha$-paralindelof, it follows that $\mathcal{U}$ has a $\alpha$-point countable open refinement say $\{W_\gamma : \gamma \in \Gamma\}$. Then the family $\mathcal{V} = \{Y \cap W_\gamma : \gamma \in \Gamma\}$ is the required $\alpha$-point countable open refinement of $\Psi$ for $Y$; Consider for each $\gamma$, $Y \cap W_\gamma$ is an open fuzzy set in $Y$ and $\{Y \cap W_\gamma : \gamma \in \Gamma\}$ is refinement of $\Psi$. 

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Further, let \( y \in Y \), then \( y \in X \). Since \( \{ W_\gamma : \gamma \in \Gamma \} \) is \( \alpha \) - point countable in \( X \), \( W_\gamma(y) > \alpha \) for atmost countably many \( \gamma \in \Gamma \). That is \( \{ \gamma \in \Gamma : W_\gamma(y) > \alpha \} \) is atmost countable. That is \( \{ \gamma \in \Gamma : (Y \wedge W_\gamma)(y) > \alpha \} \) is atmost countable family for \( Y \). Thus every open \( \alpha \) - shading \( \mathcal{V} \) of \( Y \), has a \( \alpha \) - point countable open refinement \( \mathcal{V}^1 \). Hence \( Y \) is weakly \( \alpha \) - paralindelf tfs.

The next result shows that the class of weakly \( \alpha \) - paralindelf tfs is invariant under \( F \) - continuous, \( F \) - open surjections.

4.5 Theorem : Let \( f : X \to Y \) be a \( F \) - continuous, \( F \) - open surjection. If \( X \) is weakly \( \alpha \) - paralindelf tfs then \( Y \) is also weakly \( \alpha \) - paralindelf tfs.

Proof: Let \( f : X \to Y \) be a \( F \) - continuous, \( F \) - open surjection and \( X \) is weakly \( \alpha \) - paralindelf tfs. To prove that \( Y \) is weakly \( \alpha \) - paralindelf tfs. Let \( \mathcal{U} = \{ U_\lambda : \lambda \in \Lambda \} \) be any open \( \alpha \) - shading family of \( Y \). Since \( f \) is \( F \) - continuous, \( f^{-1}(\mathcal{U}) = \{ f^{-1}(U_\lambda) : \lambda \in \Lambda \} \) is an open \( \alpha \) - shading of \( X \). Since \( X \) is weakly \( \alpha \) - paralindelf, it follows that \( f^{-1}(\mathcal{U}) \) has a \( \alpha \) - point countable open refinement say \( \mathcal{V} = \{ V_\lambda : \lambda \in \Lambda \} \). Then \( f^{-1}(\mathcal{V}) = \{ f(V_\lambda) : \lambda \in \Lambda \} \) is the required \( \alpha \) - point countable open refinement of \( \mathcal{U} \) for \( Y \); For each \( \lambda \), \( f(V_\lambda) \) is open in \( Y \), as \( f \) is \( F \) - open and \( V_\lambda \) is open in \( X \). Clearly, \( \mathcal{V} \) is a refinement of \( \mathcal{U} \).

Finally, let \( y \in Y \) then \( y = f(x) \) for some \( x \in X \). Since \( \mathcal{V} \) is \( \alpha \) - point countable in \( X \), for \( x \in X \), \( V_\lambda(x) > \alpha \) for atmost countably many \( \lambda \in \Lambda \). That is
\{ \lambda \in \Lambda : V_\lambda(x) > \alpha \} \text{ is atmost countable, which implies} \\
\{ \lambda \in \Lambda : \sup_{z \in f^{-1}(y)} \{ V_\lambda(z) \} > \alpha \} \text{ is atmost countable. That is} \\
\{ \lambda \in \Lambda : f(V_\lambda)(y) > \alpha \} \text{ is atmost countable. It follows that} \{ f(V_\lambda) : \lambda \in \Lambda \} \text{ is} \\
\alpha \cdot \text{-point countable family in} Y. \text{ Hence} Y \text{ is weakly} \alpha \cdot \text{-paralindelof fts.} \\

Similar result can be proved for} \alpha^* \text{-case.} \\

5. STRONGLY \alpha \cdot \text{PARALINDELOF FTS} \\

In this section, a stronger form of \alpha \cdot \text{-paralindelof property in fuzzy topology} 

is introduced and studied using the concept of \alpha \cdot \text{-star countable collection of fuzzy} 

sets. Among other results, it is proved that a closed crisp subspace of a strongly \alpha \cdot \text{-paralindelof fts is strongly} \alpha \cdot \text{-paralindelof and that such class of fts is invariant} 

under F \cdot \text{-continuous, F \cdot open surjections.} 

The concept of \alpha \cdot \text{-star countable collection of fuzzy sets is introduced in the} 

following. 

5.1 Definition: Let \( 0 < \alpha < 1 \) (resp. \( 0 < \alpha \leq 1 \)). A family \( \{ A_\lambda : \lambda \in \Lambda \} \) of fuzzy 

sets in a fts \( X \) is said to be \( \alpha \cdot \text{-star countable (resp.} \alpha^* \cdot \text{-star countable) in} X \) if for each \( \lambda_0 \in \Lambda \) the set \( A_{\lambda_0} \land A_\lambda \) is nonempty of order \( \alpha \) (resp. \( \alpha^* \)) for atmost 
countably many \( \lambda \in \Lambda \). That is \( \{ \lambda \in \Lambda : A_{\lambda_0} \land A_\lambda \text{ is nonempty of order} \alpha \) (resp. \( \alpha^* \)) \} \text{ is atmost countable.} 

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5.2 Remark: The following results can easily be verified.

1) Every countable family of fuzzy sets is $\alpha$-star countable.

2) Every $\alpha$-star finite family of fuzzy sets is $\alpha$-star countable family.

3) Every $\alpha$-star countable family of fuzzy sets is $\alpha$-point countable.

The class of strongly $\alpha$-paralindelof fts is introduced in the following.

5.3 Definition: Let $0 < \alpha < 1$ (resp. $0 < \alpha \leq 1$). A fts $(X, \mathcal{T})$ is said to be strongly $\alpha$-paralindelof fts (resp. strongly $\alpha^*$-paralindelof) if each open $\alpha$-shading (resp. $\alpha^*$-shading) of $X$ has a $\alpha$-star countable (resp. $\alpha^*$-star countable) open refinement.

5.4 Theorem: Every strongly $\alpha$-paracompact fts is strongly $\alpha$-paralindelof.

Proof: Proof follows from Remark 5.2 (2).

5.5 Theorem: Every strongly $\alpha$-paralindelof fts is weakly $\alpha$-paralindelof.

Proof: Proof follows from Remark 5.2 (3).

The next result gives a hereditary property of strongly $\alpha$-paralindelof fts.

5.6 Theorem: Every closed crisp subspace of a strongly $\alpha$-paralindelof fts is strongly $\alpha$-paralindelof.

Proof: Let $Y$ be a closed crisp subspace of a strongly $\alpha$-paralindelof fts. Let $\mathcal{V} = \{ V_\lambda : \lambda \in \Lambda \}$ be any open $\alpha$-shading of $Y$. Then for each $\lambda \in \Lambda$ there is an open fuzzy set $U_\lambda$ in $X$ such that $V_\lambda = Y \cup U_\lambda$. Then the family $\mathcal{U} = \{ U_\lambda : \lambda \in \Lambda \} \cup \{ 1 - Y \}$ is an open $\alpha$-shading of $X$. Since $X$ is strongly...
\(\alpha\)-paralindelof fts, \(\mathcal{U}\) has a \(\alpha\)-star countable open refinement say \(\{ W_\gamma : \gamma \in \Gamma \}\).

Now the family \(\mathcal{V} = \{ Y \land W_\gamma : \gamma \in \Gamma \}\) is the required \(\alpha\)-star countable open refinement of \(\mathcal{V}\) for \(Y\); For each \(\gamma \in \Gamma\), \(Y \land W_\gamma\) is open fuzzy set in \(Y\), also \(\mathcal{V}\) is refinement of \(\mathcal{V}\).

Further \(\mathcal{V}\) is \(\alpha\)-star countable in \(Y\); Since \(\{ W_\gamma : \gamma \in \Gamma \}\) is \(\alpha\)-star countable in \(X\), for each \(\gamma_0 \in \Gamma\) the set \(W_{r_{\gamma_0}} \land W_\gamma\) is nonempty of order \(\alpha\) for atmost countably many \(\gamma \in \Gamma\). That is \(\{ \gamma \in \Gamma : W_{r_{\gamma_0}} \land W_\gamma\) is nonempty of order \(\alpha\}\) is atmost countable. That is \(\{ \gamma \in \Gamma : (Y \land W_{r_{\gamma_0}}) \land (Y \land W_\gamma)\) is nonempty of order \(\alpha\}\) is atmost countable. That is \(\{ \gamma \in \Gamma : (Y \land W_{r_{\gamma_0}}) \land (Y \land W_\gamma)\) is nonempty of order \(\alpha\}\) is atmost countable. It follows that \(\{ Y \land W_\gamma : \gamma \in \Gamma \}\) is \(\alpha\)-star countable family of fuzzy sets in \(Y\). Thus every open \(\alpha\)-shading \(\mathcal{V}\) of \(Y\) has a \(\alpha\)-star countable open refinement \(\mathcal{V}\). Hence \(Y\) is strongly \(\alpha\)-paralindelof fts.

Now the invariance under maps, of strongly \(\alpha\)-paralindelof fts is given in the following result.

**5.7 Theorem:** Let \(f : X \rightarrow Y\) be a \(F\)-continuous, \(F\)-open surjection. If \(X\) is strongly \(\alpha\)-paralindelof fts then \(Y\) is also strongly \(\alpha\)-paralindelof fts.

**proof:** Let \(f : X \rightarrow Y\) be a \(F\)-continuous, \(F\)-open surjection. Let \(X\) be a strongly \(\alpha\)-paralindelof fts. To prove that \(Y\) is strongly \(\alpha\)-paralindelof fts. Let
\[ U = \{ U_\lambda : \lambda \in \Lambda \} \] be any open \( \alpha \)-shading of \( Y \). Then clearly 
\[ f^{-1}(U) = \{ f^{-1}(U_\lambda) : \lambda \in \Lambda \} \] is an open \( \alpha \)-shading of \( X \). Since \( X \) is strongly 
\( \alpha \)-paralindelof, \( f^{-1}(U) \) has a \( \alpha \)-star countable open refinement say 
\[ V = \{ V_\lambda : \lambda \in \Lambda \}. \] Then \( V = \{ f(V_\lambda) : \lambda \in \Lambda \} \) is the required \( \alpha \)-star countable 
open refinement of \( U \); For each \( \lambda \in \Lambda \), \( f(V_\lambda) \) is open fuzzy set in \( Y \). It can be 
verified that \( \{ f(V_\lambda) : \lambda \in \Lambda \} \) is refinement of \( U \). Further, since \( V \) is \( \alpha \)-star countable it follows that for each \( \lambda_0 \in \Lambda \) the set 
\[ \{ \lambda \in \Lambda : V_{\lambda_0} \land V_\lambda \text{ is nonempty of order } \alpha \} \] is almost countable. This implies \( \{ \lambda \in \Lambda : f(V_{\lambda_0} \land V_\lambda) \text{ is nonempty of order } \alpha \} \) is almost countable. That is 
\[ \{ \lambda \in \Lambda : f(V_{\lambda_0}) \land f(V_\lambda) \text{ is nonempty of order } \alpha \} \] is almost countable. It follows that \( U = \{ f(V_\lambda) : \lambda \in \Lambda \} \) is \( \alpha \)-star countable. Hence \( Y \) is strongly \( \alpha \)-paralindelof fts.

6. NEARLY \( \alpha \)-PARALINDELOF FTS

The class of nearly \( \alpha \)-paralindelof fts is introduced and studied in this 
section. It is shown that the concept of nearly \( \alpha \)-paralindelof is a weaker form of 
\( \alpha \)-compactness, nearly \( \alpha \)-compactness, \( \alpha \)-paracompactness, nearly 
\( \alpha \)-paracompactness and \( \alpha \)-paralindelof property. Two characterizations and a 
invariance under maps, of such class of fts have also been obtained.
The concept of nearly \( \alpha \)-paralidelof property in fuzzy topology is introduced in the following.

**6.1 Definition:** Let \( 0 \leq \alpha < 1 \) (resp. \( 0 < \alpha \leq 1 \)). A fts \((X, T)\) is called nearly \( \alpha \)-paralindelof (resp. nearly \( \alpha^* \)-paralindelof) if every fuzzy regular open \( \alpha \)-shading (resp. \( \alpha^* \)-shading) of \( X \) has a \( \alpha \)-locally countable (resp. \( \alpha^* \)-locally countable) open \( \alpha \)-refinement.

The next five results give interrelationships.

**6.2 Theorem:** Every nearly \( \alpha \)-paracompact fts is nearly \( \alpha \)-paralindelof fts.

**Proof:** Let \((X, T)\) be a nearly \( \alpha \)-paracompact fts. To prove that \((X, T)\) is nearly \( \alpha \)-paralindelof fts. Let \( U \) be any fuzzy regular open \( \alpha \)-shading of \( X \). Since \( X \) is nearly \( \alpha \)-paracompact, \( U \) has a \( \alpha \)-locally finite open refinement say \( \mathcal{V} \). We know that every finite family is countable, therefore \( \mathcal{V} \) is the required \( \alpha \)-locally countable open refinement of \( U \). Hence \( X \) is nearly \( \alpha \)-paralindelof fts.

**6.3 Theorem:** Every \( \alpha \)-paracompact fts is nearly \( \alpha \)-paralindelof fts.

**Proof:** The proof follows from Theorems 5.3 of Ch.III and 6.2.

**6.4 Theorem:** Every \( \alpha \)-compact fts is nearly \( \alpha \)-paralindelof fts.

**Proof:** The proof follows from Theorems 5.4 of Ch.III and 6.2.

**6.5 Theorem:** Every nearly \( \alpha \)-compact fts is nearly \( \alpha \)-paralindelof fts.

**Proof:** The proof follows from Theorems 5.5 of Ch.III and 6.2.

**6.6 Theorem:** Every \( \alpha \)-paralindelof fts is nearly \( \alpha \)-paralindelof fts.
Proof: Let \((X, T)\) be a \(\alpha\)-paralindelof fts. To prove that \((X, T)\) is nearly \(\alpha\)-paralindelof fts. Let \(\mathcal{U} = \{ U_\lambda : \lambda \in \Lambda \}\) be any fuzzy regular open \(\alpha\)-shading of \(X\), then \(\mathcal{U}\) is also a fuzzy open \(\alpha\)-shading of \(X\). Since \(X\) is \(\alpha\)-paralindelof, it follows that \(\mathcal{U}\) has a \(\alpha\)-locally countable open refinement say \(\mathcal{V} = \{ V_\lambda : \lambda \in \Lambda \}\). Then \(\mathcal{V}^* = \{ \overline{V_\lambda} : \lambda \in \Lambda \}\) is the required \(\alpha\)-locally countable open refinement of \(\mathcal{U}\); Let \(x \in X\). Since \(\mathcal{V}\) is \(\alpha\)-locally countable, there exists an open fuzzy set \(N\) in \(X\) with \(N(x) = 1\) such that \(N \wedge V_\lambda\) is nonempty of order \(\alpha\) for atmost countably many \(\lambda \in \Lambda\). That is \(\{ \lambda \in \Lambda : N \wedge V_\lambda\) is nonempty of order \(\alpha\}\) is atmost countable. Now \(N \wedge V_\lambda\) is nonempty of order \(\alpha\) implies there exists some \(x_0 \in X\) such that \((N \wedge V_\lambda)(x_0) \geq \alpha\), and \(V_\lambda \supseteq N \wedge V_\lambda\) is always true. That is \(\overline{V_\lambda} \geq N \wedge V_\lambda\).

Also \(N \supseteq N \wedge V_\lambda\). Therefore it follows that \(N \wedge \overline{V_\lambda} \geq (N \wedge V_\lambda) \wedge (N \wedge V_\lambda) = N \wedge V_\lambda\) as \(N \wedge V_\lambda \leq N \wedge \overline{V_\lambda}\). But for some \(x_0 \in X\), \((N \wedge V_\lambda)(x_0) > \alpha\) for countably many \(\lambda \in \Lambda\). Therefore \((N \wedge \overline{V_\lambda})(x_0) > \alpha\) for atmost countably many \(\lambda \in \Lambda\). It follows that \(\mathcal{V}\) is \(\alpha\)-locally countable family. Also it can be show that \(\mathcal{V}\) is refinement of \(\mathcal{U}\). Hence \(X\) is nearly \(\alpha\)-paralindelof fts.

Similar result can be proved for \(\alpha^*\)-case.

6.7 Remark: The following diagram shows the interrelationships.
The following result contains a characterization of nearly $\alpha$-paralindelof fts.

6.8 Theorem: A fts $X$ is nearly $\alpha$-paralindelof if and only if every fuzzy regular open $\alpha$-shading of $X$ has an $\alpha$-locally countable, fuzzy regular open refinement.

Proof: Let $(X, T)$ be a nearly $\alpha$-paralindelof fts. Let $\mathcal{U} = \{ U_\lambda : \lambda \in \Lambda \}$ be any fuzzy regular open $\alpha$-shading of $X$. Since $X$ is nearly $\alpha$-paralindelof, it follows that $\mathcal{U}$ has an $\alpha$-locally countable open refinement say $\mathcal{V} = \{ V_\lambda : \lambda \in \Lambda \}$. Then $\mathcal{V}' = \{ V_\lambda^0 : \lambda \in \Lambda \}$ is the required fuzzy regular open, $\alpha$-locally countable open refinement of $\mathcal{U}$. Converse is obvious, because every fuzzy regular open set is fuzzy open set. Hence the theorem.

The next result gives an invariance under maps, of the class of nearly $\alpha$-paralindelof fts.

6.9 Theorem: Let $f : X \to Y$ be a fuzzy almost continuous, fuzzy almost open surjection. If $X$ is nearly $\alpha$-paralindelof fts then $Y$ is also nearly $\alpha$-paralindelof fts.

Proof: The proof of this theorem is similar to that of Theorem 5.8 of Ch.III.
The following result gives another characterization in terms of semiregularization fts.

6.10 Theorem: A fts is nearly $\alpha$-paralindelof iff its semiregular fts is $\alpha$-paralindelof fts.

Proof: The proof of this theorem is similar to that of Theorem 5.9 of Ch.III.

7. NEARLY WEAKLY $\alpha$-PARALINDELOF FTS

In this section, the class of nearly weakly $\alpha$-paralindelof fts is introduced and studied. It is shown that this class of fts generalizes the classes of $\alpha$-compactness, nearly $\alpha$-compactness, $\alpha$-paracompactness, nearly $\alpha$-paracompactness, weakly $\alpha$-paracompactness and nearly weakly $\alpha$-paracompactness. A characterization and an invariance under maps, of such a class of fts are also obtained.

The concept of nearly weakly $\alpha$-paralindelof property in fuzzy topology is introduced in the following.

7.1 Definition: Let $\alpha \in [0,1)$ (resp. $\alpha \in (0,1]$). A fts $(X,T)$ is said to be nearly weakly $\alpha$-paralindelof (resp. nearly weakly $\alpha^*$-paralindelof) fts if every fuzzy regular open $\alpha$-shading (resp. $\alpha^*$-shading) of $X$ has a $\alpha$-point countable (resp. $\alpha^*$-point countable) open refinement.
The following result shows that nearly weakly $\alpha$-paralindelof property generalizes nearly weakly $\alpha$-paracompactness.

**7.2 Theorem**: Every nearly weakly $\alpha$-paracompact fts is nearly weakly $\alpha$-paralindelof fts.

**Proof**: Let $(X, T)$ be a nearly weakly $\alpha$-paracompact fts. To prove that $(X, T)$ is nearly weakly $\alpha$-paralindelof fts. Let $U$ be any fuzzy regular open $\alpha$-shading of $X$. Since $X$ is nearly weakly $\alpha$-paracompact, $U$ has a $\alpha$-point finite open refinement say $V$. We Know that every finite family is countable so $V$ is the required $\alpha$-point countable open refinement of $U$. Hence $(X, T)$ is nearly weakly $\alpha$-paralindelof fts.

The next result says that nearly weakly $\alpha$-paralindelof property generalizes weakly $\alpha$-paracompactness.

**7.3 Theorem**: Every weakly $\alpha$-paracompact fts is nearly weakly $\alpha$-paralindelof fts.

**Proof**: The proof follows by Theorems 6.3 of Ch.III and 7.2.

The next four results also give the various interrelations.

**7.4 Theorem**: Every $\alpha$-paracompact fts is nearly weakly $\alpha$-paralindelof fts.

**Proof**: The proof follows by Theorems 6.2 of Ch.III and 7.2.

**7.5 Theorem**: Every nearly $\alpha$-paracompact fts is nearly weakly $\alpha$-paralindelof fts.

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**Proof**: The proof follows by Theorems 6.4 of Ch.III and 7.2.

7.6 **Theorem**: Every $\alpha$-compact fts is nearly weakly $\alpha$-paralindelof fts.

**Proof**: The proof follows by Theorems 6.6 of Ch.III and 7.2.

7.7 **Theorem**: Every nearly $\alpha$-compact fts is nearly weakly $\alpha$-paralindelof fts.

**Proof**: The proof follows by Theorems 6.5 of Ch.III and 7.2.

7.8 **Remark**: The following diagram shows the interrelationships.

```
    $\alpha$-COMPACT
        \downarrow
    NEARLY $\alpha$-COMPACT
        \downarrow
    $\alpha$-PARACOMPACT
        \downarrow
    NEARLY $\alpha$-PARACOMPACT
      \downarrow
    WEAKLY $\alpha$-PARACOMPACT
      \downarrow
    NEARLY WEAKLY $\alpha$ PARACOMPACT
```

The next result is a characterization of such fts.

7.9 **Theorem**: A fts $X$ is nearly weakly $\alpha$-paralindelof iff every fuzzy regular open $\alpha$-shading of $X$ has a $\alpha$-point countable, fuzzy regular open $\alpha$-refinement.

**Proof**: The proof is analogous to that of Theorem 6.8 of Ch.III.

The invariance under maps of nearly weakly $\alpha$-paralindelof property is contained in the following.
7.10 Theorem: Let $f: X \to Y$ be a fuzzy almost continuous, fuzzy almost open surjection mapping. If $X$ is nearly weakly $\alpha$-paralindelof fts, then $Y$ is also nearly weakly $\alpha$-paralindelof fts.

**Proof:** The Proof is analogous to that of Theorem 6.9 of Ch.III.

8. NEARLY STRONGLY $\alpha$-PARALINDELOF FTS

In this section a new class of fts called nearly strongly $\alpha$-paralindelof fts is introduced and studied. It is shown that such a class is a weaker form of strongly $\alpha$-paracompactness, nearly strongly $\alpha$-paracompactness and strongly $\alpha$-paralidelof property.

The new class is introduced in the following.

8.1 Definition: Let $0 < \alpha < 1$ (resp. $0 < \alpha \leq 1$). A fts $(X, T)$ is said to be nearly strongly $\alpha$-paralindelof (resp. nearly strongly $\alpha^*$-paralindelof) if every fuzzy regular open $\alpha$-shading (resp. $\alpha^*$-shading) of $X$ has a $\alpha$-star (resp. $\alpha^*$-star) countable open refinement.

The next result gives an interrelationship.

8.2 Theorem: Every nearly strongly $\alpha$-paracompact fts is nearly strongly $\alpha$-paralindelof fts.

**Proof:** Let $(X, T)$ be a nearly strongly $\alpha$-paracompact fts. To prove that $(X, T)$ is nearly strongly $\alpha$-paralindelof fts. Let $U$ be any fuzzy regular open $\alpha$ shading of
X. Since X is nearly strongly $\alpha$-paracompact, it follows that $\mathcal{U}$ has a $\alpha$-star finite open refinement say $\mathcal{V}$. But we know that every finite family is countable. Therefore $\mathcal{V}$ is the required $\alpha$-star countable open refinement of $\mathcal{U}$. Hence X is nearly strongly $\alpha$-paralindelof fts.

Another interrelationship is given in the following.

8.3 Theorem: Every strongly $\alpha$-paracompact fts is nearly strongly $\alpha$-paralindelof fts.

Proof: The proof follows by Theorems 6.11 of Ch. III and 8.2.

Yet another interrelationship is given in the following.

8.4 Theorem: Every strongly $\alpha$-paralindelof fts is nearly strongly $\alpha$-paralindelof fts.

Proof: Let $(X, T)$ be a strongly $\alpha$-Paralindelof fts. To prove that $(X, T)$ is nearly strongly $\alpha$-paralindelof fts. Let $\mathcal{U} = \{ U_\lambda : \lambda \in \Lambda \}$ be any fuzzy regular open $\alpha$-shading of X. Then $\mathcal{U}$ is fuzzy open $\alpha$-shading of X. Since X is strongly $\alpha$-paralindelof, $\mathcal{U}$ has a $\alpha$-star countable open refinement say $\mathcal{V} = \{ V_\lambda : \lambda \in \Lambda \}$.

Then $\mathcal{V}' = \{ V_\lambda^\alpha : \lambda \in \Lambda \}$ is the required $\alpha$-star countable open refinement of $\mathcal{U}$.

Hence $(X, T)$ is nearly strongly $\alpha$-paralindelof fts.

8.5 Remark: The following diagram shows the interrelationships.
STRONGLY $\alpha$ - PARACOMPACT

NEARLY STRONGLY $\alpha$-PARACOMPACT \rightarrow \{NEARLY STRONGLY $\alpha$ - PARALINDELOF

STRONGLY $\alpha$ - PARALINDELOF

8.6 Theorem: Every $\alpha$ - regular nearly strongly $\alpha$ - paralindelof fts is strongly $\alpha$ - paralindelof fts.

Proof: The proof is similar to that of Theorem 6.12 of Ch.III.

The following result gives an invariance under maps, of such a class of fts.

8.7 Theorem: Let $f: X \rightarrow Y$ be a fuzzy almost continuous, fuzzy almost open surjection mapping. If $X$ is nearly strongly $\alpha$ - paralindelof fts then $Y$ is so.

Proof: The proof is analogous to that of Theorem 6.13 of Ch.III.