CHAPTER 5

(τ_i, τ_j) -SEMI WEAKLY g*- CLOSED SETS IN BITOPOLOGICAL SPACES

5.1 INTRODUCTION

Khedr, El.Areefi and Noiri [50] defined pre-continuity and semi - pre-continuity in bitopological spaces. In 1986, Fukutake [35] introduced and studied generalized closed sets in bitopological spaces. In 1990, Arya and Nour [6] defined generalized semi closed sets in bitopological spaces. Khedr and Al.Shibani [49] introduced pairwise super continuous mappings. Khedr [48] introduced and investigated Cα-continuous functions in bitopological spaces. Pervin [87] investigated connectedness in bitopological space

In this chapter (τ_i, τ_j)-semi weakly g*- closed sets, (τ_i, τ_j)-semi weakly open sets, (τ_i, τ_j)-Quasi semi weakly g*-closed sets, (τ_i, τ_j)-Quasi semi weakly open sets, (τ_i, τ_j)-semi weakly g*- continuous functions, (τ_i, τ_j)-semi weakly g*- strongly continuous functions and (τ_i, τ_j)-semi weakly g*- irresolute functions are introduced and some of their properties are investigated.

5.2 (τ_i, τ_j) - SEMI WEAKLY g*-CLOSED SETS

In this section the concept of (τ_i, τ_j) semi weakly g*- closed sets in a bitopological space are defined and study some of their properties.

Definition 5.2.1: Let $(\tau_i, \tau_j) \in \{1,2\}$ be fixed integers. In a bitopological space (X, τ_i, τ_j), a subset $A \subseteq X$ is said to be (τ_i, τ_j) -semi weakly g*-closed (briefly(τ_i,τ_j)-swg* - closed), if τ_j-gcl (A) $\subseteq$ U whenever A $\subseteq$ U and U$\in \tau_i$-semi open.

Definition 5.2.2: The set of all τ_j-swg*-closed sets in X is denoted by τ_j-SWG*C (X, τ_i, τ_j) and the set of all τ_j-swg*-open set in X is denote by τ_j-SWG*O (X, τ_i,τ_j).

Theorem 5.2.3: Let A be (τ_i, τ_j)- swg*-closed set in a bitopological space(X, τ_i, τ_j). Then τ_j-gcl (A)-A contain no non-empty τ_i-semi closed set in (X, τ_i, τ_j).

Proof: Suppose that F is a τ_i-semi closed subset of τ_j - gcl(A) - A. This implies F$\subseteq \tau_j$-gcl(A) and F$\subseteq A^c$. Since F^c is τ_i-semi open set and A is (τ_i, τ_j)-swg*-closed set, τ_j-gcl(A) $\subseteq F^c$.Therefore F $\subseteq \tau_j$-gcl(A) $\cap$ $(\tau_j\text{-gcl}(A))^c = \phi$. Hence τ_j-gcl(A)-A contain no non-empty τ_i-semi closed set in (X, τ_i, τ_j).

Corollary 5..2.4: If subset A in bitopological space (X, τ_i, τ_j) is (τ_i, τ_j)-swg*-closed set then τ_j-gcl (A)-A = ϕ.

Proof: Assume that A is (τ_i, τ_j)- swg*-closed set. Since τ_j - gcl(A) = A therefore τ_j-gcl (A)-A = ϕ.

Theorem 5.2.5: Suppose B$\subseteq$ A$\subseteq$ X, B is (τ_i, τ_j)-swg*- closed set relative to A and that A is (τ_i, τ_j)-swg*-closed subset of (X, τ_i, τ_j). Then B is (τ_i, τ_j)- swg*-closed relative to X.

Proof: Let $B \subseteq U$ and U is τ_i-semi open in X. Then $B \subseteq A \cap U$ and hence τ_j-gcl $_A(B) \subseteq A \cap U$. It follows that $A \cap \tau_j$-gcl(B) $\subseteq A \cap U$ and $A \subseteq U \cup (\tau_j$-gcl $(B))^c$. Since A is (τ_i, τ_j)-swg*- closed in X, τ_j-gcl (A) $\subseteq U \cup (\tau_{j-}$ gcl(B)$)^c$. Therefore $(\tau_j$- gcl (B)) $\subseteq \tau_j$-gcl(A) $\subseteq U \cup (\tau_j$-gcl(B)$)^c$ and τ_j-gcl (B) $\subseteq U$. Then B is (τ_i, τ_j)-swg*closed set relative to X.

Theorem 5.2.6: Let $A \subseteq Y \subseteq X$ and suppose that A is (τ_i, τ_j)- swg*-closed in X. Then A is (τ_i, τ_j)-swg*-closed relative to Y.

Proof: Let $A \subseteq Y \cap U$ and U is τ_i-semi open in Y.Then $A \subseteq U$ and hence τ_j- gcl(A) $\subseteq$ U. It follows that $Y \cap \tau_j$-gcl(A) $\subseteq Y \cap U$. Then A is (τ_i, τ_j)- swg*- closed relative to Y.

Theorem 5.2.7: Every τ_j-closed set in bitopological space X is a (τ_i, τ_j)- swg*-closed in X.

Proof: Assume A is τ_j-closed in X. Let U be a τ_i- semi open set in X. Such that $A \subseteq U$, and $A \subseteq (\tau_j$-cl(τ_i-int (A)) $\subseteq$ U. Implies $A \subseteq (\tau_j$-cl(τ_i-int(A))) $\subseteq \tau_j$-cl(A) $\subseteq$ U and U is τ_i- semi open. Thus $A \subseteq \tau_j$-gcl(A) $\subseteq$ U. Therefore A is (τ_i, τ_j)-semi weakly g*-closed set.

Remarks 5.2.8: The converse of the above theorem need not be true as seen from the following example.

Example 5.2.9: Let X ={ a, b, c} with the bitopologies τ_i = { X, ϕ, {a, c}} and τ_j ={ X, ϕ, {c}, {b, c}}. In this bitopological space (X, τ_i, τ_j), the subset {b, c} is (τ_i, τ_j)-swg*-closed which is not τ_j-closed set.

Remarks 5.2.10: (τ_i, τ_j) - g - closed set and (τ_i, τ_j)-wg*-closed set are independent to each other as seen from the following examples .

Example 5.2.11: Let X ={a, b, c} with the bitopologies τ_i={X, ϕ, {a}} and τ_j= { X, ϕ, {a, b}}. In this bitopological space (X, τ_i, τ_j) the subset {b} is (τ_i, τ_j)-g - closed set which is not (τ_i, τ_j)-swg*-closed set.

Example 5.2.12: Let X ={a, b, c} with the bitopologies τ_i= {X, ϕ, {a}} and τ_j = { X, ϕ, {b}}. In this bitopological space (X, τ_i, τ_j) the subset {a} is (τ_i, τ_j) - swg*- closed set which is not (τ_i, τ_j)- g - closed set.

Remark 5.2.13: (τ_i, τ_j)- swg*- closed set and (τ_i, τ_j)- semi closed set are independent to each other as seen from the following examples.

Example 5.2.14: Let X ={ a, b, c} with the bitopologies τ_i= { X, ϕ, {a, b}} and τ_j = {X, ϕ, {a}, {a, b}}. In this bitopological space (X, τ_i, τ_j), the subset {b} is (τ_i, τ_j) - semi closed set which is not (τ_i, τ_j)- swg*-closed set.

Example 5.2.15: Let X= {a, b, c} with the bitopologies τ_i= { X, ϕ, {b, c}} and τ_j={X, ϕ, {b}, {b, c}}. In this bitopological space (X, τ_i, τ_j), the subset {a, b} is (τ_i, τ_j) -swg*-closed set which is not (τ_i, τ_j)-semi-closed set.

Remark 5.2.16: (τ_i, τ_j)- swg*-closed set and (τ_i, τ_j)- pre- closed set are independent to each other as seen from the following examples.

Example 5.2.17: Let X = {a, b, c} with the bitopologies τ_i = {X, ϕ, {a, b}}and τ_j = {X, ϕ, {a}, {a, b}, {a, c}}. In this bitopological space (X, τ_i, τ_j) the subset {a} is (τ_i, τ_j)- pre - closed set which is not (τ_i, τ_j) - swg*- closed set..

Example 5.2.18: Let X = {a, b, c} with the bitopologies τ_i = { X, ϕ, {a}} and τ_j = {X, ϕ, {a, b}}. In this bitopological space (X, τ_i, τ_j), the subset {a,c} is (τ_i, τ_j)- swg*- closed set which is not (τ_i, τ_j)- pre- closed set .

Remark 5.2.19 : (τ_i, τ_j) -swg*- closed set and (τ_i, τ_j)-α-closed set are independent to each other as seen from the following examples.

Example 5.2.20: Let X = {a, b, c} with the bitopologies τ_i= {X, ϕ, {a, b}} and τ_j = {X, ϕ, {a}, {a, b}}. In this bitopological space (X, τ_i, τ_j) the subset {b} is (τ_i, τ_j)-α -closed set which is not (τ_i, τ_j) swg*- closed set .

Example 5.2.21: Let X ={ a, b, c} with the bitopologies τ_i={X, ϕ, {a}, {a, b}} and τ_j = {X, ϕ, {c}, {b, c}}. In this bitopological space (X, τ_i, τ_j), the subset {b,c} is (τ_i, τ_j)-swg*-closed set which is not (τ_i, τ_j)-α-closed set .

Remark 5.2.22 : (τ_i, τ_j)-swg*- closed set and (τ_i, τ_j)- sg-closed set are independent to each other as seen from the following examples.

Example 5.2.23: Let X = {a, b, c} with the bitopologies τ_i= {X,ϕ, {a}} and τ_j = {X, ϕ, {b}}. In this bitopological space (X, τ_i, τ_j), the subset {b} is (τ_i,τ_j)-swg* - closed set which is not (τ_i, τ_j)-sg-closed set.

Example 5.2.24: Let $X = \{a, b, c\}$ with the bitopologies $\tau_i = \{X, \phi, \{a, b\}\}$ and $\tau_j = \{X, \phi, \{a\}, \{a, b\}\}$. In this bitopological space (X, τ_i, τ_j) , the subset $\{b\}$ is (τ_i, τ_j)- sg-closed set which is not (τ_i, τ_j)-swg*-closed set.

Remark 5.2.25 : (τ_i, τ_j)-swg*-closed set and (τ_i, τ_j)-gs -closed set are independent to each other as seen from the following examples.

Example 5.2.26: Let $X = \{a, b, c\}$ with the bitopologies $\tau_i = \{X, \phi, \{a\}\}$ and $\tau_j = \{X, \phi, \{a, b\}\}$. In this bitopological space (X, τ_i, τ_j), the subset $\{b\}$ is (τ_i, τ_j) - sg -closed set which is not (τ_i, τ_j)- swg*- closed set.

Example 5.2.27: Let $X = \{a, b, c\}$ with the bitopologies $\tau_i = \{X, \phi, \{b, c\}\}$ and $\tau_j = \{X, \phi, \{a, b\}\}$. In this bitopological space (X, τ_i, τ_j), the subset $\{b\}$ is (τ_i, τ_j)-swg*-closed set which is not (τ_i, τ_j)-gs-closed set.

Remark 5.2.28: (τ_i, τ_j)-swg*-closed set and (τ_i, τ_j)-αg-closed set are independent to each other as seen from the following examples.

Example 5.2.29: Let $X = \{a, b, c\}$ with the bitopologies $\tau_i = \{X, \phi, \{a\}\}$ and $\tau_j = \{X, \phi, \{a, b\}\}$. In this bitopological space (X, τ_i, τ_j), the subset $\{b\}$ is (τ_i, τ_j) αg - closed set which is not (τ_i, τ_j)-swg*-closed set.

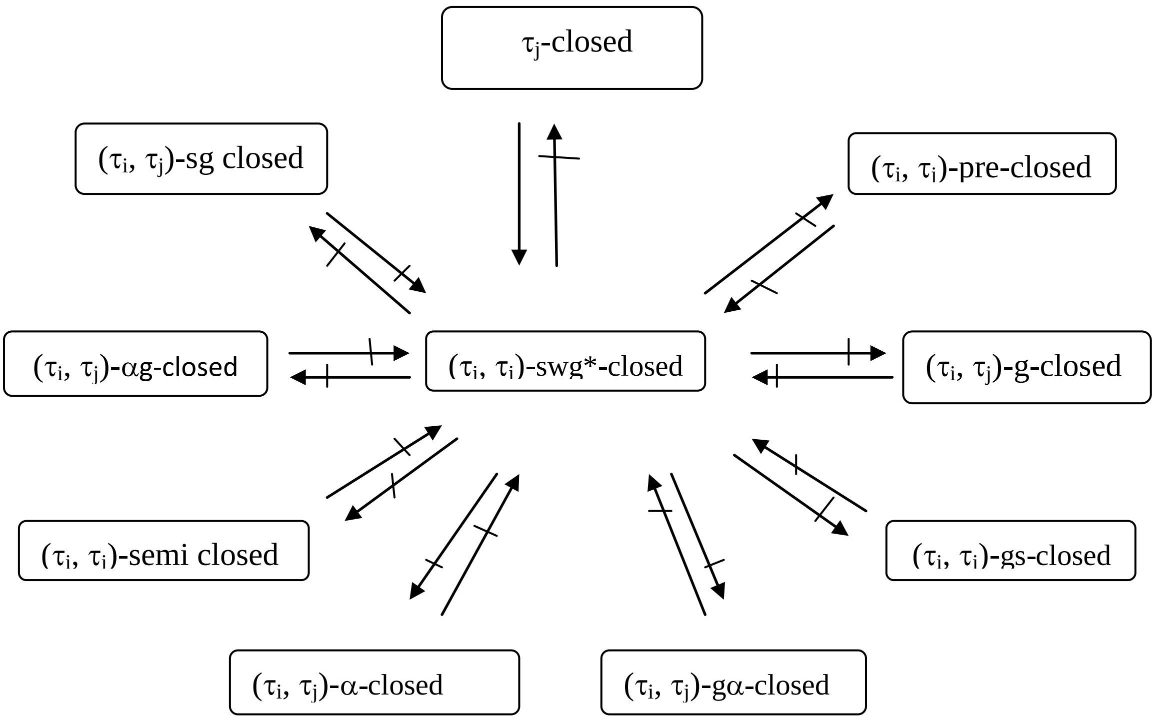
Example 5.2.30: Let $X = \{a, b, c\}$ with the bitopologies $\tau_i = \{X, \phi, \{a, b\}\}$ and $\tau_j = \{X, \phi, \{a, c\}\}$. In this bitopological space (X, τ_i, τ_j), the subset $\{a,b\}$ is (τ_i, τ_j)-swg*-closed set which is not (τ_i, τ_j) - αg-closed set.

Remark 5.2.31: (τ_i, τ_j)-swg*-closed set and (τ_i, τ_j) -gα-closed set are independent to each other as seen from the following examples.

Example 5.2.32: Let X = {a, b, c} with the bitopologies τ_i = {X, ϕ, {a, b}} and τ_j ={X, ϕ, {a}, {a, b}}. In this bitopological space (X, τ_i, τ_j), the subset {b} is (τ_i, τ_j)-gα-closed set which is not (τ_i, τ_j)-swg*-closed set.

Example 5.2.33: Let X = {a, b,c} with the bitopologies τ_i= {X, ϕ, {a},{a, b}} and τ_j= {X, ϕ, {c}, {b, c}}. In this bitopological space (X, τ_i, τ_j), the subset {c} is (τ_i, τ_j)-swg*-closed set which is not (τ_i, τ_j)- gα-closed set.

Remark 5.2.34: From the above results the following relation is obtained.



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5.3 (τ_i, τ_j)-SEMI WEAKLY g*- OPEN SETS

In this section the concept of (τ_i,τ_j)-swg*- open sets in bitopological spaces are introduced and study some of their properties.

Definition 5.3.1: A subset A of a bitopological space (X, τ_i, τ_j) is called (τ_i,τ_j) semi weakly g*-open (briefly (τ_i,τ_j) swg*-open) if and only if A^c is (τ_i,τ_j)-semi weakly g*-closed set.

Theorem 5.3.2: A subset A in a bitopological space (X, τ_i, τ_j) is (τ_i,τ_j)-swg*-open if and only if $F \subseteq \tau_j$-g int (A), whenever F is τ_i -semi-closed and $F \subseteq A$.

Proof: Assume that A is (τ_i,τ_j)-swg*-open in (X, τ_i, τ_j). Let F be τ_i semi-closed and $F \subseteq A$. This implies F^c is τ_i-semi open and $A^c \subseteq F^c$. Since A^c is (τ_i,τ_j)-swg*-closed, τ_j-gcl $(A^c) \subseteq F^c$. Since τ_j-gcl $(A^c) = (\tau_j\text{-(g int(A))}^c$, $(\tau_j\text{-g int(A)})^c \subseteq F^c$. Therefore $F \subseteq \tau_j$-gint(A). Conversely assume that $F \subseteq \tau_j$-gint (A). Whenever F is τ_i-semi closed, and $F \subseteq A$. Let U be a τ_i-semi open set in (X, τ_i, τ_j) containing A^c. Therefore U^c is τ_i-semi closed set contained in A by hypothesis $U^c \subseteq \tau_j$-gint (A) taking complements $U \supseteq \tau_j$-gcl (A^c). Therefore A^c is (τ_i,τ_j)-swg*-closed in (X, τ_i, τ_j). Hence A is (τ_i,τ_j)-swg*- open in (X, τ_i, τ_j).

Theorem 5.3.3: If $A \subseteq B \subseteq X$ where A is (τ_i,τ_j)-swg*-open relative to B and B is (τ_i,τ_j)-swg*-open relative to X then A is (τ_i,τ_j)-swg*-open relative to X.

Proof: Let F be a τ_i-semi closed set and suppose that F $\subseteq$ A. Then F is τ_i-semi closed relative to B and hence F $\subseteq$ τ_j-g int $_B$(A). Therefore there exists a τ_i-semi-open set U such that F $\subseteq$ U $\cap$ B $\subseteq$ A. But F $\subseteq$ U*$\subseteq$ B for τ_i-semi-open set U*. Since B is swg*-open in X. Thus F $\subseteq$ U*$\cap$ U $\subseteq$ B $\cap$ U $\subseteq$ A. It follows that F $\subseteq$ τ_j-g int (A), because set A is (τ_i, τ_j)-swg*-open set. This implies F $\subseteq$ τ_j-g int (A). Whenever F is τ_i -semi-closed set and F $\subseteq$ A. Therefore A is (τ_i, τ_j)-semi weakly g*-open in X.

Theorem 5.3.4: If τ_j-g-int(A) $\subseteq$ B $\subseteq$ A and if A is (τ_i, τ_j)-swg*- open then B is (τ_i, τ_j)- swg*-open.

Proof: $A^c \subseteq B^c \subseteq$ τ_j-gcl (A^c) and since A^c is (τ_i, τ_j)-swg*-closed set. It follows that B^c is (τ_i, τ_j)-swg*-closed set because A is (τ_i, τ_j)-swg*-closed and A $\subseteq$ B $\subseteq$ τ_j-gcl(A). Then B is (τ_i, τ_j)-swg*-open set.

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5.4 (τ_i, τ_j)-QUASI SEMI WEAKLY g*- OPEN FUNCTIONS AND (τ_i, τ_j)-QUASI SEMI WEAKLY g*-CLOSED FUNCTIONS

In this section (τ_i, τ_j)-Quasi semi weakly g*-open and (τ_i, τ_j)-Quasi semi weakly g*- closed functions in bitopological spaces are introduced and study some of their properties.

Definition 5.4.1: Let (X, τ_i, τ_j) and (Y, σ, σ_j) be any two bitopological spaces. A function $f : (X, \tau_i, \tau_j) \rightarrow (Y, \sigma_i, \sigma_j)$ is said to be (τ_i,τ_j)- quasi semi weakly g*-open if the image of every (τ_i, τ_j)-semi weakly g*-open set in X is σ_i-open in Y.

Theorem 5.4.2: Let $f : (X, \tau_i, \tau_j) \rightarrow (Y, \sigma_i, \sigma_j)$ be a function . Then the following are equivalent:

(i) f is (τ_i, τ_j)-quasi swg*-open;

(ii) For each subset U of X, $f((\tau_i,\tau_j)\text{-g int}(U)) \subset \sigma_i\text{- int}(f(U))$;

(iii) For each $x \in X$ and each (τ_i, τ_j)- swg*- neighbourhood U of x in X, there exists a σ_i -neighbourhood V of f(x) such that $V \subset f(U)$.

Proof:(i)⇒(ii) : Let f be an (τ_i, τ_j)-quasi swg*-open function. Since (τ_i, τ_j)-g int (U) is an (τ_i, τ_j)-swg*-open set contained in U, that implies $f((\tau_i, \tau_j)\text{-g int}(U)) \subset f(U)$. As $f((\tau_i, \tau_j)\text{-g int}(U))$ is σ_i-open, $f((\tau_i,\tau_j)\text{-g int}(U)) \subset \sigma_i\text{-int}(f(U))$.

(ii) ⇒ (iii): Let $x \in X$ and U be an (τ_i, τ_j)-swg*-neighbourhood of x in X. Then there exist an (τ_i, τ_j)-swg*-open set V in X such that $x \in V \subset U$. Thus by (ii), $f(V) = f((\tau_i, \tau_j)\text{-g int}(V)) \subset \sigma_i\text{-int}(f(V))$, and hence $f(V)= \sigma_i\text{-int}(f(V))$. Therefore it follows that f(V) is σ_i-open such that $f(x) \in f(V) \subset f(U)$.

(iii)⇒ (i) : Let U be an (τ_i, τ_j)-swg*-open set in X. Then by (iii), for each $y \in f(U)$, there exists a σ_i-neighbourhood V_y of y such that $V_y \subset f(U)$. As V_y is a σ_i-neighbourhood of y, there exists a σ_i-open set W_y such that $Y \in W_y \subset V_y$. Thus $f(U) = \cup\{W_y : Y \in f(U)\}$ is σ_i -open . Hence , f is (τ_i,τ_j)-quasi swg*-open.

Theorem 5.4.3: A function $f : (X, \tau_i, \tau_j) \to (Y, \sigma_i, \sigma_j)$ is (τ_i,τ_j)-quasi swg*-open set, if and only if for any subset B of Y and for any (τ_i, τ_j)-swg*-closed set F in X such that $f^{-1}(B) \subset F$, there exists a σ_i-closed set G containing B such that $f^{-1}(G) \subset F$.

Proof: Suppose that f is (τ_i, τ_j)-quasi swg*- open set. Let $B \subset Y$ and F be an (τ_i,τ_j)-swg*- closed set in X such that $f^{-1}(B) \subset F$. Now, put G= Y- f (X-F). It is clear that $B \subset G$ as $f^{-1}(B) \subset F$, and that $f^{-1}(G) \subset F$. Also G is σ_i-closed, since f is (τ_i, τ_j)-quasi- swg*-open. Conversely, let U be an $((\tau_i,\tau_j)$-swg*-open set in X, and put B=Y-f(U). Then X-U is an (τ_i,τ_j)-swg*-closed set in X such that $f^{-1}(B) \subset X-U$. By hypothesis, there exists a σ_i-closed set G such that $B\subset G$ and $f^{-1}(G) \subset X-U$. Hence, $f(U) \subset Y-G$. On the other hand $B \subset G$, $Y-G \subset Y-B = f(U)$. Thus f(U) = Y- G is σ_i-open and hence f is a (τ_i,τ_j)-quasi swg*-open.

Theorem 5.4.4: Let $f : (X, \tau_i, \tau_j) \to (Y, \sigma_i, \sigma_j)$ be a function . Then the following are equivalent:

(i) f is (τ_i, τ_j)-quasi swg*-open;

(ii)$f^{-1}(\sigma_i\text{-cl}(B)) \subset (\tau_i, \tau_j)\text{-gcl}(f^{-1}(B))$ for every subset B of Y ;

(iii)$(\tau_i, \tau_j)\text{-g int}(f^{-1}(B)) \subset f^{-1}(\sigma_i\text{-int}(B))$ for every subset B of Y.

Proof:

(i) ⇒(ii) : Suppose that f is (τ_i, τ_j)-quasi swg*-open set. Now for any subset B of Y, $f^{-1}(B) \subset (\tau_i, \tau_j)\text{-gcl}(f^{-1}(B))$.Therefore by theorem 5.4.3, there exists σ_i-closed set G such that $B \subset G$ and $f^{-1}(G) \subset (\tau_i, \tau_j)$- gcl$(f^{-1}(B))$. Hence, $f^{-1}(\sigma_i\text{-cl}(B)) \subset f^{-1}(G) \subset (\tau_i,\tau_j)$-gcl$(f^{-1}(B))$.

(ii) ⇒ (i) : Let $B \subset Y$ and F be an (τ_i, τ_j)- swg*- closed set in X such that $f^{-1}(B) \subset F$. Put $G = \sigma_i$-cl(B), then $B \subset G$, G is σ_i-closed, and $f^{-1}(G) \subset (\tau_i, \tau_j)$ gcl $(f^{-1}(B)) \subset F$. Thus by theorem 5.4.3, f is (τ_i,τ_j)-quasi swg*-open set .

(ii)⇔(iii): It is Clear, because $f^{-1}(\sigma_i\text{-cl }(B)) \subset (\tau_i,\tau_j)$ -gcl$(f^{-1}(B))$ for every subset B of Y is equal to (τ_i, τ_j)-g int$(f^{-1}(B)) \subset f^{-1}(\sigma_i\text{-int}(B))$ for every subset B of Y.

Theorem 5.4.5: Let $f : (X, \tau_i ,\tau_j) \rightarrow (Y, \sigma_i, \sigma_j)$ and $g : (Y, \sigma_i, \sigma_j) \rightarrow (Z, \eta_i, \eta_j)$ be two functions such that $g \circ f : X \rightarrow Z$ is (τ_i, τ_j)-quasi swg*-open. If g is a pairwise continuous injection then f is (τ_i, τ_j)-quasi swg*-open set .

Proof: Let U be an (τ_i, τ_j)-swg*-open set in X. Then $g \circ f$ (U) is η_i-open as $g \circ f$ is (τ_i,τ_j)-quasi swg*-open. Since g is a pairwise continuous injection, $f(U) = g^{-1}(g \circ f (U))$ is σ_i-open. Hence, f is (τ_i, τ_j)-quasi swg*- open set.

Definition 5.4.6:A function $f : (X, \tau_i, \tau_j) \rightarrow (Y, \sigma_i, \sigma_j)$ is said to be (τ_i, τ_j)-quasi swg* closed if the image of each (τ_i, τ_j)-swg*- closed set in X is σ_i-closed in Y.

Theorem 5.4.7: A function $f : (X, \tau_i, \tau_j) \rightarrow (Y, \sigma_i, \sigma_j)$ is (τ_i, τ_j)- quasi swg*-closed set if and only if σ_i-cl$(f(A)) \subset f((\tau_i, \tau_j)$-gcl(A)) for every subset A of X.

Proof: Let f be (τ_i, τ_j)- quasi swg*- closed set, there exist σ_i-cl$(f(A)) \subset f((\tau_i,\tau_j)$-gcl(A)) for every subset A of X. Conversely, every σ_i-cl$(f(A)) \subset f((\tau_i,\tau_j)$ - gcl (A)) is (τ_i, τ_j)- quasi swg*- closed.

Theorem 5.4.8: Let f: (X, τ_i, τ_j) $\rightarrow$(Y, σ_i, σ_j) be a function.Then the following are equivalent:

(i) f is (τ_i, τ_j)-quasi swg*- closed ;

(ii) For any subset B of Y and for any (τ_i, τ_j)- swg*-open set G in X such that $f^{-1}(B) \subset G$, there exists a σ_i-open set U containing B such that $f^{-1}(U) \subset G$;

(iii) For each $y \in Y$ and for any (τ_i, τ_j)-swg*-open set G in X such that $f^{-1}(\{y\}) \subset G$, there exists a σ_i-open set U containing {y} such that $f^{-1}(U) \subset G$.

Proof:

(i)⇒ (ii): Suppose f is (τ_i, τ_j)-quasi swg* closed set. Now there exist for any subset B of Y and for (τ_i, τ_j)-swg*-open set G in X such that $f^{-1}(B) \subset G$, there exist a σ_i-open set U containing B such that $f^{-1}(U) \subset G$.

(ii)⇒ (iii) : For any subset B of Y and for any (τ_i, τ_j)- swg*-open set G in X such that $f^{-1}(B) \subset G$, there exists a σ_i-open set U containing B such that $f^{-1}(V) \subset G$. Also there exist for each $y \in Y$ and for any (τ_i, τ_j)-swg*-open set G in X such that $f^{-1}(\{y\}) \subset G$, there exists a σ_i-open set containing {y} such that $f^{-1}(U) \subset G$.

(iii)⇒(i): For each $y \in Y$ and for any (τ_i, τ_j)-swg*-open set G in X such that $f^{-1}((Y)) \subset G$, there exists a σ_i-open set U containing B such that $f^{-1}(U) \subset G$. Then f is (τ_i, τ_j)-quasi swg*-closed set .

Definition 5.4.9: A function f: (X, τ_i, τ_j) $\rightarrow$(Y, σ_i, σ_j) is called pairwise swg**-closed if the image of every (τ_i, τ_j)- swg*-closed set in X is (τ_i, τ_j)- swg*-closed set in Y.

Theorem 5.4.10: Let f : (X, τ_i, τ_j) $\rightarrow$(Y, σ_i, σ_j) be a function. Then the following as equivalent:

(i) f is pairwise swg**- closed;

(ii) For any subset B of Y and for any (τ_i, τ_j)- swg*-open set G in X such that $f^{-1}(B) \subset G$, there exists an (τ_i,τ_j)- swg*-open set U in Y such that $B \subset U$ and $f^{-1}(U) \subset G$;

(iii) For each $y \in Y$ and for any (τ_i, τ_j)- swg*-open set G in X such that $f^{-1}(\{Y\}) \subset G$, there exists an (τ_i, τ_j)- swg*-open set U in Y such that $y \in U$ and $f^{-1}(U) \subset G$;

(iv) (τ_i, τ_j)-gcl(f(A)) $\subset$ f ((τ_i, τ_j)-gcl(A)) for every subset A of X.

Proof:

(i) $\Rightarrow$ (ii): Let f be an pairwise swg**- closed. By definition 5.4.9, f : (X, τ_i, τ_j) $\rightarrow$ (Y, σ_i, σ_j) is called pairwise swg**-closed if the image of every (τ_i, τ_j)-swg*-closed set in X is (τ_i, τ_j)- swg*- closed set in Y. There exists for any subset B of Y and for any (τ_i, τ_j)-swg*-open set G in X such that $f^{-1}(B) \subset G$. Also there exists an (τ_i, τ_j)- swg*-open set U in Y, such that $B \subset U$ and $f^{-1}(U) \subset G$.

(ii)$\Rightarrow$(iii): For any subset B of Y and for any (τ_i, τ_j)- swg*-open set G in X such that $f^{-1}(B) \subset G$, there exists an (τ_i, τ_j)- swg*open set U in Y such that $B \subset U$ and $f^{-1}(U) \subset G$.There exist for $y \in Y$ and for any (τ_i, τ_j)- swg* -open set G in X, such that $f^{-1}(\{y\}) \subset G$. Also there exists an (τ_i, τ_j)- wg*-open set U in Y such that $y \in U$ and $f^{-1}(U) \subset G$.

(iii)⇒(iv) : Let each $y \in Y$ and for any (τ_i, τ_j)-swg*-open set G in X such that $f^{-1}(\{y\}) \subset G$, there exists an (τ_i, τ_j)- swg*-open set U in Y such that $y \in U$ and $f^{-1}(U) \subset G$. This implies (τ_i, τ_j)-gcl (f (A)) ⊂ f(τ_i, τ_j)-gcl(A)) for every subset A of X.

(iv)⇒(i) : Let (τ_i, τ_j)-gcl(f(A)) ⊂ f ((τ_i, τ_j)-gcl(A)) for every subset A of X. There exist f is pairwise swg**-closed.

Theorem 5.4.11: Let f : (X, τ_i, τ_j) → (Y, σ_i, σ_j) and g : (Y, σ_i, σ_j) → (Z, η_i, η_j) are two (τ_i, τ_j)-quasi swg*-closed functions, then g∘f : (X, τ_i, τ_j) → (Z, η_i, η_j) is (τ_i, τ_j)-quasi swg*- closed set.

Proof: If f : (X, τ_i, τ_j) → (Y, σ_i, σ_j) and g : (Y, σ_i, σ_j) → (Z, η_i, η_j) are two (τ_i,τ_j)- quasi swg*-closed set. Let U be an (τ_i, τ_j)-swg*-closed set in X. Then g∘f (U) is σ_i -closed as g∘f is (τ_i, τ_j)- quasi swg*-closed set.

Theorem 5.4.12: Let f : (X, τ_i ,τ_j) → (Y, σ_i, σ_j) and g : (Y, σ_i, σ_j) → (Z, η_i, η_j) be any two functions. Then if f is pairwise swg*-closed and g is (τ_i,τ_j)-quasi swg*-closed set the g∘f is pairwise closed.

Proof: If f is pairwise swg* -closed and g is (τ_i, τ_j)-quasi swg*-closed set then g∘f is pairwise closed.

Theorem 5.4.13 : Let f : (X, τ_i, τ_j) → (Y, σ_i, σ_j) and g : (Y, σ_i, σ_j) → (Z, η_i, η_j) be any two functions. Then if f is pairwise swg**-closed and g is (τ_i, τ_j)-quasi swg*-closed then g∘f is (τ_i, τ_j)- quasi swg*-closed.

Proof: If f is pairwise swg**-closed and g is (τ_i, τ_j)-quasi swg*-closed then $g \circ f$ is (τ_i,τ_j)-quasi swg*-closed set.

Definition 5.4.14: A function $f : (X, \tau_i, \tau_j) \to (Y, \sigma_i, \sigma_j)$ is called pairwise swg*-irresolute, if $f^{-1}(V)$ is (τ_i, τ_j)- swg*- open in (X, τ_i, τ_j) for every (τ_i, τ_j)-swg*- open set V in Y.

Definition 5.4.15: A function $f : (X, \tau_i, \tau_j) \to (Y, \sigma_i, \sigma_j)$ is called pairwise swg*-continuous, if $f^{-1}(V)$ is (τ_i, τ_j)-swg*- open in (X, τ_i, τ_j) for every σ_i- open set V in Y.

Theorem 5.4.16:

Let $f : (X, \tau_i, \tau_j) \to (Y, \sigma_i, \sigma_j)$ and $g : (Y, \sigma_i, \sigma_j) \to (Z, \eta_i ,\eta_j)$ be two functions such that $g \circ f : X \to Z$ is (τ_i, τ_j)- quasi swg*-closed . Then

(i) If f is a pairwise swg*-irresolute surjection, then g is (τ_i, τ_j)- quasi swg*-closed.

(ii) If g is a pairwise swg*-continuous injection, then f is pairwise swg**-closed.

Proof:

(i) Suppose that F is (τ_i, τ_j)-swg*-closed set in Y. Then $f^{-1}(F)$ is (τ_i, τ_j)- swg*-closed in X as f is pairwise swg*- irresolute. Since $g \circ f$ is (τ_i, τ_j)- quasi swg*-

closed and f is surjective $(g\circ f\,(f^{-1}(F))) = g(F)$ is η_i-closed . Hence g is (τ_i, τ_j)-quasi-swg*-closed.

(ii) Suppose that F is an (τ_i, τ_j)-swg*-closed set in X. Since $g\circ f$ is (τ_i, τ_j)- quasi swg*-closed set, $(g\circ f(F)$ is η_i-closed, but g is a pairwise swg*-continuous injection, so $g^{-1}(g\circ f(F)) = f(F)$ is (τ_i, τ_j) - swg*- closed set in Y. Hence f is pairwise swg**- closed.

Theorem 5.4.17: Let $g : (Y, \sigma_i, \sigma_j) \rightarrow (Z, \sigma_i, \sigma_j)$ be a function . Then g is (τ_i, τ_j)-quasi swg*-closed if and only if g(X) is σ_i-closed , and g(V) - g(X-V) is σ_i-open in g(X) whenever V is (τ_i, τ_j)- swg*-open in X.

Proof: Necessity: Let g is (τ_i, τ_j)- quasi swg*-closed. Then g(X) is σ_i- closed as X is (τ_i, τ_j)-swg*-closed and g(V) –g (X-V) = g(X) - g(X-V) is σ_i-open in g(X) when V is (τ_i, τ_j)-swg*-open in X.

Sufficiency: Suppose that g(X) is σ_i-closed and g(V) - g (X-V) is σ_i-open is g (X) when V is (τ_i, τ_j)- swg*-open set in X, and let C be (τ_i, τ_j)- swg*-closed set in X. Then g (C)=g(X) - (g(X-C)-g(C)) is σ_i-closed in g(X) and therefore g(C) is σ_i-closed. Hence, g is (τ_i, τ_j)-quasi swg*-closed set.

Corollary 5.4.18: Let $g : (X, \tau_i, \tau_j) \rightarrow (Y, \sigma_i, \sigma_j)$ be a surjection. Then g is (τ_i, τ_j)-quasi swg*-closed if and only if g(V) - g(X-V) is σ_i-open whenever V is (τ_i, τ_j)-swg*-open in X.

Definition 5.4.19: A Space (X, τ_i, τ_j) is said to be pairwise swg*- normal if for any disjoint subset $F_1 \in (\tau_i, \tau_j)$-SWG*C (X) and $F_2 \in (\tau_j, \tau_i)$- SWG*C (X), there exist disjoint subsets $U \in \tau_i$ and $V \in \tau_j$ such that $F_1 \subset U$ and $F_2 \subset V$.

Theorem 5.4.20: Let $(X,\tau_i ,\tau_j) \to (Y,\sigma_i,\sigma_j)$ be two spaces , where X is pairwise swg*-normal . Let $g : (X, \tau_i, \tau_j) \to (Y, \sigma_i, \sigma_j)$ be a pairwise swg*-continuous, (τ_i, τ_j)-quasi swg*-closed surjection . Then Y is pairwise normal.

Proof: Let X is pairwise swg*- normal. Let K be σ_i -closed and M be σ_j -closed disjoint subsets of Y. Then $g^{-1}(K) \in (\tau_i,\tau_j)$-SWG*C(X), $g^{-1}(M) \in (\tau_j,\tau_i)$ - SWG*C(X) and $g^{-1}(K) \cap g^{-1}(M) = \phi$. Since X is pairwise swg*- normal, there exist disjoint sets $V \in \tau_i$ and $W \in \tau_j$ such that $g^{-1}(K) \subset V$ and $g^{-1}(M) \subset W$. Thus $K \subset g(V) - g(X-V)$ and $M \subset g(W)-g(X-W)$. It follows also from corollary 5.4.18 that $g(V)-g(X-V) \in \sigma_i$ and $g(W) - g(X-W) \in \sigma_j$, and clearly $(g(V) - g(X-V)) \cap (g(w)-g(X-W)) = \phi$ because $V \cap W = \phi$. Hence Y is pairwise normal .

Theorem 5.4.21: Let $(X, \tau_i, \tau_j) \to (Y, \sigma_i, \sigma_j)$ be two spaces, where X is pairwise swg*-normal. Let $g : (X, \tau_i, \tau_j) \to (Y, \sigma_i, \sigma_j)$ be a pairwise swg*-irresolute, (τ_i, τ_j)-quasi swg*-closed surjection.Then Y is pairwise swg*-normal.

Proof: Let X is pairwise swg*- normal. Let K be σ_i-swg*- closed and M be σ_j-swg*- closed disjoint subsets of Y. Then $g^{-1}(K) \in (\tau_i, \tau_j)$- SWG*C(X), $g^{-1}(M) \in (\tau_j, \tau_i)$-SWG*C (X) and $g^{-1}(K) \cap g^{-1}(M) = \phi$. Since X is pairwise swg*- normal , there exists disjoint sets $V \in \tau_i$ and $W \in \tau_j$ such that $g^{-1}(K) \subset V$ and $g^{-1}(M) \subset W$.

Thus $K \subset g(V) - g(X-V)$ and $M \subset g(W) - g(X-W)$. It follows from corollary 5.2.18, that $g(V)-g(X-V) \in \sigma_i$ and $g(W) - g(X-W) \in \sigma_j$. Clearly $g(V) - g(X-V) \cap (g(W)- g(X-W) = \phi$ because $V \cap W = \phi$. Hence Y is pairwise swg*-normal.

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5.5 (τ_i,τ_j)-SEMI WEAKLY g*- CONTINUOUS FUNCTIONS IN BITOPOLOGICAL SPACES

In this section (τ_i ,τ_j)-swg*-continuous functions in bitopological spaces are introduced and study some of their properties.

Definition 5.5.1: A function $f : (X, \tau_i ,\tau_j) \to (Y, \sigma_i, \sigma_j)$ is said to be (τ_i, τ_j)- σ_i- semi weakly g*-continuous function, if the inverse image of every σ_i-closed set in Y is (τ_i, τ_j)-semi weakly g*-closed set in X.

Definition 5.5.2: A function $f : (X, \tau_i ,\tau_j) \to (Y, \sigma_i,\sigma_j)$ is called (τ_i, τ_j)- semi weakly g* - irresolute (briefly (τ_i, τ_j) -swg*- irresolute) function, if $f^{-1}(V)$ is (τ_i, τ_j)-swg*-closed set in X for every (τ_i, τ_j)- swg*- closed set V of Y.

Theorem 5.5.3: The following are equivalent for a function $f : (X, \tau_i ,\tau_j) \to (Y, \sigma_i, \sigma_j)$.

(i) f is pairwise swg*-continuous function

(ii) $f^{-1}(U)$ is (τ_i, τ_j)-swg*-closed for each σ_i-closed set U in Y, $i\neq j$ and $(\tau_i, \tau_j)= 1,2$.

Proof :

(i)⇒ (ii) : Suppose that f is pairwise swg*- continuous function. Let A be σ_j- closed in Y. Then A^c is σ_j- open in Y. Since f is pairwise swg*- continuous, $f^{-1}(A^c)$ is (τ_i, τ_j)-swg*-open in X, $i \neq j$ and (τ_i, τ_j)= 1,2.Consequently, $f^{-1}(A)$ is (τ_i, τ_j)- swg* - closed set in X.

(ii)⇒(i): Suppose that $f^{-1}(U)$ is (τ_i, τ_j)- swg*-closed for each σ_i-closed set U in Y, $i\neq j$ and (τ_i, τ_j)= 1,2. Let V be σ_j-open in Y. Then V^c is σ_j-closed in Y. Therefore by our assumption, $f^{-1}(V^c)$ is (τ_i, τ_j)- swg*-closed in X, $i\neq j$ and (τ_i, τ_j) = 1,2. Hence $f^{-1}(V)$ is (τ_i, τ_j)- swg*-open in X. This completes the proof.

Lemma 5.5.4: A function $f: (X, \tau_i, \tau_j) \rightarrow (Y, \sigma_i, \sigma_j)$ is (τ_j, τ_i)-swg*-irresolute, then for every subset B of Y, (τ_j, τ_i)-swg*cl$(f^{-1}(B)) \subset f^{-1}((\tau_j, \tau_i)$-swg*cl $(B))$.

Proof: Let $x \in (\tau_j, \tau_i)$- swg*cl $(f^{-1}(B))$. Suppose that V is (τ_j, τ_i)-swg* -open set of Y, containing f (x) , i.e. $f(x) \in V$, then $x \in f^{-1}(V)$. Since $f^{-1}(V)$ is (τ_j,τ_i)-swg*-open of X, then $f^{-1}(V) \cap f^{-1}(B) \neq \phi$ implies that $f^{-1}(V\cap B) \neq \phi$ and $V\cap B \neq \phi$. Thus $f(x) \in (\tau_j,\tau_i)$-swg*cl(B) and $x\in f^{-1}(f(x)) \in f^{-1}((\tau_j,\tau_i)$-swg*cl (B)), this means $x \in f^{-1}((\tau_j,\tau_i)$-swg*cl(B)). Hence $((\tau_j, \tau_i)$-swg*-cl$(f^{-1}(B)) \subset f^{-1}((\tau_j, \tau_i)$-swg*cl (B)).

Lemma 5.5.5: If a function $f : (X, \tau_i, \tau_j) \rightarrow (Y, \sigma_i, \sigma_j)$ is τ_i-closed, then for each subset $S \subset Y$ and each τ_i-open set U containing $f^{-1}(S)$, there is a σ_i-open set V containing S such that $f^{-1}(V) \subset U$.

Proof: Let $S \subset Y$ and U is τ_i-open containing $f^{-1}(S)$, Put V=Y-f (X-U). Then U is σ_i-open set in Y containing S. It follows that $f^{-1}(V) \subset U$.

Theorem 5.5.6: Let $f : (X, \tau_i, \tau_j) \to (Y, \sigma_i, \sigma_j)$ be a function and f is (τ_i, τ_j)- swg*-continuous, then for each $x \in X$ and for each σ_j-open set V containing f(x), there is an (τ_i, τ_j) -swg*-open set U containing x such that $f(U) \subset V$.

Proof: Let $x \in X$ and V be σ_j-open set containing f(x).Then f is (τ_i, τ_j)-swg*-continuous, so $f^{-1}(V)$ is (τ_i, τ_j)swg*-open set of X which containing X. If $U = f^{-1}(V)$, then $f(U) \subset V$.

Theorem 5.5.7: Let $f : (X, \tau_i , \tau_j) \to (Y, \sigma_i, \sigma_j)$ be a function and each $x \in X$ and for each σ_j- open set V containing f(x) , there is an (τ_i,τ_j)-swg* -open set U containing x such that $f(U) \subset V$. Then $f((\tau_i, \tau_j)\text{- swg * cl }(A)) \subset \tau_j\text{-cl }(f(A))$ for each subset A of X.

Proof: Let A be a subset of a space X and $f(x) \notin \tau_j\text{-cl }(f(A))$. Then there exists σ_j-open set V of Y containing f(x) such that $V \cap f(A) = \phi$.Then for each $x \in X$ and for each σ_j- open set V containing f(x), there is an (τ_i, τ_j)- swg*-open set U containing x such that $f(U) \subset V$, so there is an (τ_i, τ_j)- swg*open set U such that $f(x) \in f(U) \subset V$. Hence $f(U) \cap f(A) = \phi$ implies $U \cap A = \phi$. Consequently, $x \notin (\tau_i, \tau_j)$- swg*cl(A) and $f(x) \notin f((\tau_i, \tau_j)\text{- swg*cl}(A))$.

Theorem 5.5.8: Let f: $(X, \tau_i, \tau_j) \to (Y, \sigma_i, \sigma_j)$ be a function and f $((\tau_i, \tau_j)$- swg *-cl (A)) $\subset \tau_j$-cl (f(A)) for each subset A of X then (τ_i, τ_j)- swg* cl(f^{-1}(B))$\subset f^{-1}(\tau_j$-cl (B)) for each subset B of Y.

Proof: Let B be a subset of Y and A = f^{-1} (B).Then f $((\tau_i,\tau_j)$ swg *-cl(A)) $\subset \tau_j$-cl f(A)) for each subset A of X, so $((\tau_i, \tau_j)$-swg*-cl(f^{-1}(B)) $\subset \tau_j$-cl(B)) $\subset \tau_j$-cl (f(f^{-1}(B))) $\subset$ τ_j-cl (B). Thus (τ_i, τ_j)- swg*-cl(f^{-1}(B)) $\subset f^{-1}$ $(\tau_j$-cl(B)).

Theorem 5.5.9: If a map f: $(X, \tau_i, \tau_j) \to (Y, \sigma_i, \sigma_j)$ is τ_i-closed set and (τ_j, τ_i)-swg*-irresolute, then for each (τ_i, τ_j)-swg*-closed set B of Y, f^{-1}(B) is (τ_i,τ_j)-swg*-closed in X.

Proof: Let B be an (τ_i, τ_j)- swg*- closed subset of Y and f^{-1} (B) $\subset$ U , where U is a τ_i-open set of X. Since f is τ_i-closed set, by lemma 5.5.5 there is a σ_i-open set V such that B$\subset$ V and f^{-1} (V) $\subset$ U. Since B is (τ_i,τ_j)-swg*-closed set and B $\subset$ V, then (τ_j, τ_i)-gcl(B)$\subset$ V. Hence $f^{-1}((\tau_j,\tau_i)$-gcl (B)) $\subset f^{-1}$ (V) $\subset$ U. By lemma 5.5.4 (τ_j,τ_i)-gcl(f^{-1} (B)) $\subset$ U and hence f^{-1} (B) is (τ_i, τ_j)- swg* - closed set in X.

Theorem 5.5.10 : Let f: $(X, \tau_i, \tau_j) \to (Y, \sigma_i, \sigma_j)$ and g : $(Y, \sigma_i, \sigma_j) \to (Z, \upsilon_i, \upsilon_j)$ be two functions. Then ;

(i) If f is (τ_i, τ_j)- swg*-continuous and g is τ_j- continuous, then g∘f is (τ_i, τ_j)-swg*- continuous.

(ii) If f is τj-swg*-irresolute, τ_i- closed and g is (τ_i, τ_j)- swg*-continuous, then g∘f is (τ_i, τ_j)- swg*-continuous.

Proof:

(i) Let W be a υ_j- closed set of Z. Since g is σ_j - continuous , then g^{-1} (W) is σ_j - closed set of Y. Since f is (τ_i, τ_j)- swg*- continuous , then $(g\circ f)^{-1}$ (W) $=f^{-1}(g^{-1}(W))$ is (τ_i, τ_j)-swg*-closed set of X. Hence $g\circ f$ is (τ_i, τ_j)-swg*-continuous.

(ii) Let W be a υ_j - closed set of Z. Since g is an (τ_i, τ_j)- swg* - continuous , then g^{-1} (W) is (τ_i, τ_j)- swg* closed set of Y . By theorem 5.5.6 $(g\circ f)^{-1}(W) = f^{-1}(g^{-1}(W))$ is an (τ_i, τ_j)- swg*-closed of X. Hence $g\circ f$ is (τ_i, τ_j) swg*continuous

Remark 5.5.11: (τ_i, τ_j)- σ_i- semi continuous function and (τ_i,τ_j)-σ_i-swg*-continuous function are independent.

Example 5.5.12: Let (X, τ_i, τ_j) and (Y, σ_i, σ_j) be two bitopological spaces. Let X = Y = {a, b, c} with the bitopologies τ_i = {X, ϕ, {a}}, τ_j={X, ϕ, {a, b}} and σ_i={Y, ϕ, {b}} , σ_j = {Y, ϕ, {c}}. Define $f : (X, \tau_i, \tau_j) \to (Y, \sigma_i, \sigma_j)$ by f(a) = c, f(b) = b, f(c)=a. A function f is (τ_i, τ_j)- σ_i- swg*-continuous which is not (τ_i, τ_j)-σ_i-semi continuous function, since the inverse image of σ_i-closed set {a, c} in Y is not (τ_i, τ_j)- semi closed in X.

Example 5.5.13: Let (X, τ_i, τ_j) and (Y, σ_i, σ_j) be two bitopological spaces. Let X = Y= {a, b, c} with the bitopologies τ_i= {X, ϕ, {b, c}}, τ_j = {X, ϕ, {b}, {b, c}} and σ_i={Y, ϕ, {a, b}}, σ_j = {Y ,ϕ, {b}}. Let $f : (X, \tau_i, \tau_j) \to (Y, \sigma_i, \sigma_j)$ be defined by identity map. A function f is (τ_i, τ_j)- σ_i- semi continuous which is not (τ_i, τ_j)- σ_i-

swg*-continuous function, since the inverse image of σ_i-closed set{c} in Y is not (τ_i, τ_j)-swg*-closed in X.

Remark 5.5.14:(τ_i, τ_j)- -σ_i- pre- continuous function and (τ_i, τ_j)-σ_i-swg*-continuous function are independent.

Example 5.5.15: Let (X, τ_i, τ_j) and (Y, σ_i, σ_j) be two bitopological spaces**.** Let X = Y = {a, b, c} with the bitopologies τ_i = {X, ϕ, {b}}, τ_j = {X, ϕ, {c}} and σ_i={Y, ϕ, {a, c}}, σ_j = {Y, ϕ, {b}, {b, c}}. Define f : $(X, \tau_i, \tau_j) \rightarrow (Y, \sigma_i, \sigma_j)$ by f(a) = c, f(b) = b, f(c) = a. A function f is (τ_i, τ_j)- σ_i- swg*-continuous which is not (τ_i, τ_j)-σ_i-pre-continuous function, since the inverse image of σ_i-closed set{b} in Y is not (τ_i, τ_j)-pre-closed in X.

Example 5.5.16: Let (X, τ_i, τ_j) and (Y, σ_i, σ_j) be two bitopological spaces**.** Let X = Y={a, b, c} with the bitopologies τ_i={X, ϕ, {b, c}}, τ_j={X , ϕ, {b}, {b, c}} and σ_i={Y, ϕ, {a, b}}, σ_j= {Y, ϕ, {a}}. Let f : $(X, \tau_i, \tau_j) \rightarrow (Y, \sigma_i, \sigma_j)$ be defined by identity map. A function f is (τ_i, τ_j)- σ_i- pre- continuous which is not (τ_i,τ_j)-σ_i-swg*-continuous function, since the inverse image of σ_i-closed {c} in Y is not (τ_i, τ_j) swg*-closed in X.

Remark 5.5.17: (τ_i, τ_j)- σ_i-α- continuous function and (τ_i, τ_j)- σ_i-swg*-continuous function are independent.

Example 5.5.18: Let (X, τ_i, τ_j) and (Y, σ_i, σ_j) be two bitopological spaces. Let $X = Y= \{a, b, c\}$ with the bitopologies $\tau_i= \{X, \phi, \{a\}\}$, $\tau_j=\{X, \phi, \{b\}\}$ and $\sigma_i= \{Y, \phi, \{c\}\}$, $\sigma_j = \{Y, \phi, \{a, b\}\}$. Define $f : (X, \tau_i, \tau_j) \rightarrow (Y, \sigma_i, \sigma_j)$ by f(a) = b, f(b) = a, f(c) = c. A function f is (τ_i, τ_j)- σ_i- swg*- continuous which is not (τ_i, τ_j)-σ_i-α-continuous function, since the inverse image of σ_i-closed set {a, b} in Y is not (τ_i, τ_j)-α-closed in X.

Example 5.5.19: Let (X, τ_i, τ_j) and (Y, σ_i, σ_j) be two bitopological spaces. Let $X=Y= \{a, b, c\}$ with the bitopological $\tau_i = \{X, \phi, \{b, c\}\}$, $\tau_j = \{X, \phi, \{b\}, \{b, c\}\}$and $\sigma_i=\{Y, \phi, \{a, b\}\}$, $\sigma_j= \{Y, \phi, \{a\}, \{a, b\}\}$. Define $f : (X, \tau_i, \tau_j) \rightarrow (Y, \sigma_i, \sigma_j)$ by f(a) = b, f(b) = a, f(c) = c. A function f is (τ_i, τ_j)- σ_i-α-continuous which is not (τ_i, τ_j)- σ_i- swg*-continuous function, since the inverse image of σ_i-closed set {c} in Y is not (τ_i, τ_j)-swg*-closed in X.

Remark 5.5.20: (τ_i, τ_j)- σ_i- semi generalized continuous function and (τ_i, τ_j)- σ_i-swg*-continuous function are independent.

Example 5.5.21: Let (X, τ_i, τ_j) and (Y, σ_i, σ_j) be two bitopological spaces. Let $X=Y=\{a, b, c\}$ with the bitopologies $\tau_i= \{X, \phi, \{a\}\}$, $\tau_j = \{X, \phi, \{b\}\}$ and $\sigma_i= \{Y, \phi, \{c\}\}$, $\sigma_j= \{Y, \phi, \{a, b\}\}$. Define $f : (X, \tau_i, \tau_j) \rightarrow (Y, \sigma_i, \sigma_j)$ by f(a) = b, f(b) = a, f(c) = c. A function f is (τ_i, τ_j)- σ_i- swg*-continuous which is not (τ_i, τ_j)- σ_i- semi generalized continuous function, since the inverse image of σ_i-closed set {a, b} in Y is not (τ_i, τ_j)-semi generalized closed in X.

Example 5.5.22: Let (X, τ_i, τ_j) and (Y, σ_i, σ_j) be two bitopological spaces . Let $X = Y = \{a, b, c\}$ with the bitopologies $\tau_i = \{X, \phi, \{b, c\}\}$, $\tau_j = \{X, \phi, \{b\}, \{b, c\}\}$ and $\sigma_i = \{Y, \phi, \{a, b\}\}$, $\sigma_j = \{Y, \phi, \{a\}, \{a, b\}\}$. Define $f : (X, \tau_i, \tau_j) \rightarrow (Y, \sigma_i, \sigma_j)$ by $f(a) = b$, $f(b) = a$, $f(c) = c$. A function f is (τ_i, τ_j)- σ_i-semi generalized continuous which is not (τ_i, τ_j)-σ_i-swg*-continuous function, since the inverse image of σ_i-closed set $\{c\}$ in Y is not (τ_i, τ_j)-swg*-closed in X.

Remark 5.5.23: (τ_i, τ_j)- σ_i- generalized semi continuous function and (τ_i, τ_j)- σ_i-swg*-continuous function are independent.

Example 5.5.24: Let (X, τ_i, τ_j) and (Y, σ_i, σ_j) be two bitopological spaces. Let $X=Y=\{a, b, c\}$ with the bitopologies $\tau_i = \{X, \phi, \{b, c\}\}$, $\tau_j = \{X, \phi, \{b\}, \{b, c\}\}$ and $\sigma_i = \{Y, \phi, \{a, b\}\}$, $\sigma_j = \{Y, \phi, \{a\}, \{a, b\}\}$. Define $f : (X, \tau_i, \tau_j) \rightarrow (Y, \sigma_i, \sigma_j)$ by $f(a) = b$, $f(b) = a$, $f(c) = c$. A function f is (τ_i, τ_j)- σ_i- generalized semi continuous which is not (τ_i, τ_j)- σ_i- swg*-continuous function, since the inverse image of σ_i-closed set $\{c\}$ in Y is not (τ_i, τ_j)-swg*-closed in X.

Example 5.5.25: Let (X, τ_i, τ_j) and (Y, σ_i, σ_j) be two bitopological spaces . Let $X=Y= \{a, b, c\}$with the bitopologies $\tau_i = \{X, \phi, \{a, b\}\}$, $\tau_j = \{X, \phi, \{b, c\}\}$ and $\sigma_i = \{Y, \phi, \{c\}\}$, $\sigma_j = \{Y, \phi, \{b\}, \{a, c\}\}$. Define : $(X, \tau_i, \tau_j) \rightarrow (Y, \sigma_i, \sigma_j)$ by $f(a)=b$, $f(b)=a$, $f(c)=c$. A function f is (τ_i, τ_j)- σ_i- swg*- continuous which is not (τ_i, τ_j)- σ_i- generalized semi continuous function,since the inverse image of σ_i-closed set $\{a, b\}$ in Y is not (τ_i, τ_j)-generalized semi closed in X.

Remark 5.5.26:(τ_i, τ_j)- σ_i-α- generalized continuous function and (τ_i, τ_j)- σ_i-swg*-continuous function are independent.

Example 5.5.27: Let (X, τ_i, τ_j) and (Y, σ_i, σ_j) be two bitopological spaces. Let X = Y= {a, b, c} with the bitopologies τ_i = {X, ϕ, {a, b}}, τ_j = {X, ϕ, {a, c}} and σ_i= {Y, ϕ, {c}}, σ_j= {Y, ϕ, {b}}. Define f : $(X, \tau_i, \tau_j) \rightarrow (Y, \sigma_i, \sigma_j)$ by f(a) = b, f(b) = a,f(c) = c. A function f is (τ_i, τ_j)- σ_i- swg* continuous which is not (τ_i, τ_j)- σ_i- α- generalized continuous function, since the inverse image of σ_i-closed set {a, b} in Y is not (τ_i, τ_j)-α-generalized closed in X.

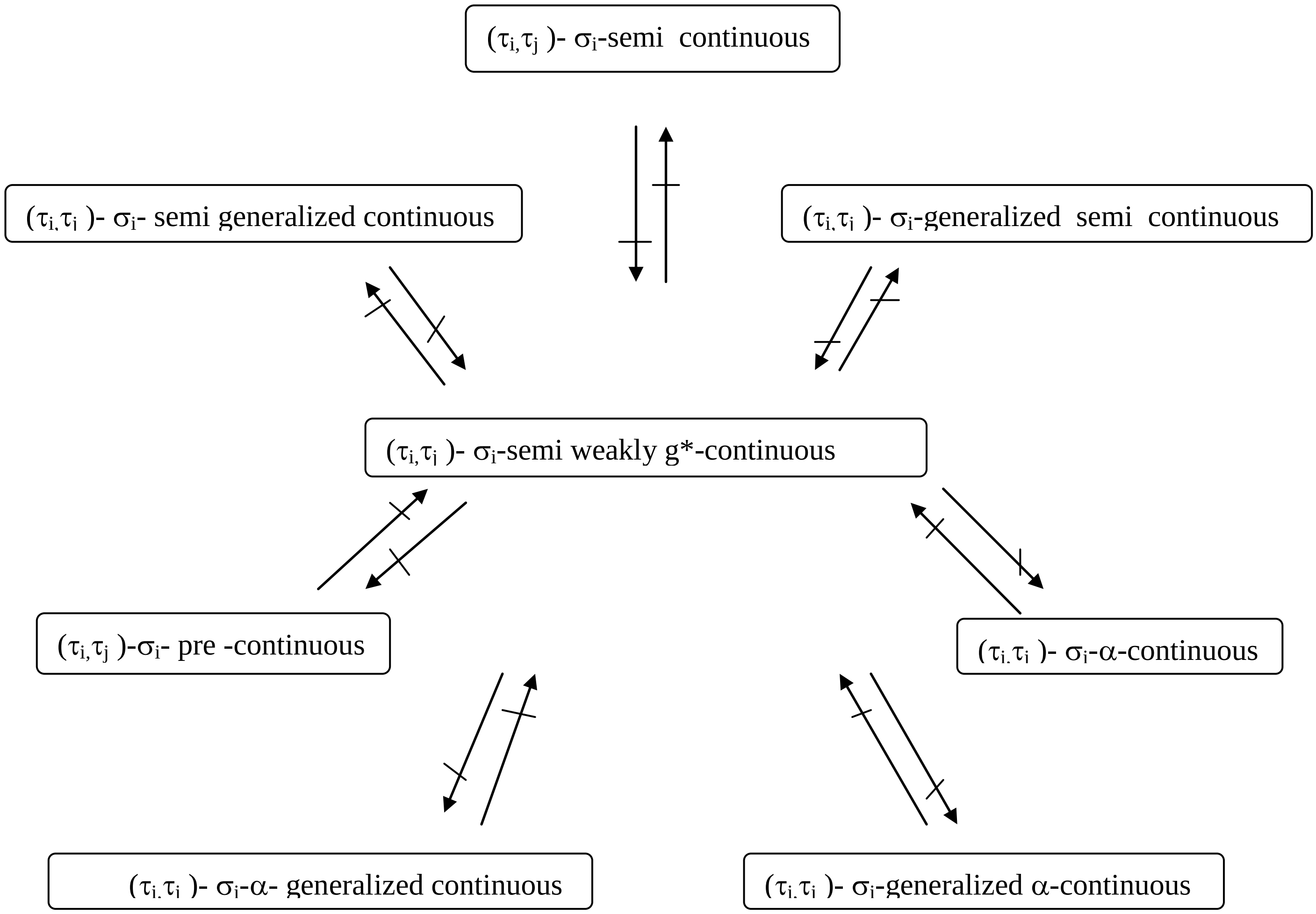
Example 5.5.28: Let (X, τ_i, τ_j) and (Y, σ_i, σ_j) be two bitopological spaces. Let X=Y={a, b, c}with the bitopologies τ_i= {X, ϕ, {a}}, τ_j = {X, ϕ, {a, b}} and σ_i= {Y, ϕ, {a, c}}, σ_j= {Y, ϕ, {c}, {a, c}}. Define f : $(X, \tau_i, \tau_j) \rightarrow (Y, \sigma_i, \sigma_j)$ by f(a) = c, f(b) = b, f (c) = a. A function f is (τ_i, τ_j)- σ_i- α - generalized - continuous which is not (τ_i, τ_j)- σ_i- swg*-continuous function, since the inverse image of σ_i-closed set {b} in Y is not (τ_i, τ_j)-swg*-closed in X.

Remark 5.5.29:(τ_i, τ_j)- σ_i- generalized α- continuous function and (τ_i, τ_j)- σ_i-swg*-continuous function are independent.

Example 5.5.30: Let (X, τ_i, τ_j) and (Y, σ_i, σ_j) be two bitopological spaces . Let X=Y= {a, b, c} with the bitopologies τ_i= {X, ϕ, {a}}, τ_j={X, ϕ, {c}} and σ_i= {Y, ϕ, {b}}, σ_j= {Y, ϕ, {a}, {b, c}}. Define f : $(X, \tau_i, \tau_j) \rightarrow (Y, \sigma_i, \sigma_j)$ by f(a) = c ,f(b) = b, f(c) = a. A function f is (τ_i, τ_j)- σ_i- swg* -continuous which is not (τ_i, τ_j)- σ_i- generalized α-continuous function, since the inverse image of σ_i- closed set {a, c} in Y is not (τ_i, τ_j)-generalized α- closed in X.

Example 5.5.31: Let (X, τ_i, τ_j) and (Y, σ_i, σ_j) be two bitopological spaces**.** Let X =Y= {a, b, c} with the bitopologies τ_i= {X, ϕ, {b, c}}, τ_j={X, ϕ, {b}, {b, c}} and σ_i= {Y, ϕ, {a, b}}, σ_j= {Y, ϕ, {b, c},{c}}. Let f : $(X, \tau_i, \tau_j) \rightarrow (Y, \sigma_i, \sigma_j)$ be defined by identity map. A function f is (τ_i, τ_j)- σ_i-generalized α- continuous which is not (τ_i, τ_j)- σ_i- swg*-continuous function, since the inverse image of σ_i-closed set {c} in Y is not (τ_i, τ_j)-swg*- closed in X.

Remark 5.5.32 : From the above results the following relation is obtained.



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5.6 (τ_i,τ_j)-SEMI WEAKLY g*-STRONGLY CONTINUOUS FUNCTIONS AND (τ_i,τ_j)-SEMI WEAKLY g*-IRRESOLUTE FUNCTIONS

In this section (τ_i, τ_j)-semi weakly g*-strongly continuous functions and (τ_i, τ_j)-semi weakly g*- irresolute functions are introduced and some of their properties are investigated.

Definition 5.6.1 : A function $f : (X, \tau_i, \tau_j) \rightarrow (Y, \sigma_i, \sigma_j)$ is called (τ_i, τ_j)- semi weakly g*- strongly -continuous (briefly (τ_i, τ_j)- swg*- s-continuous) if $f^{-1}(V)$ is (τ_i, τ_j)-semi weakly g* -strongly- closed in X for every σ_j-closed set V of Y.

Definition 5.6.2: A functionf: $(X, \tau_i, \tau_j) \rightarrow (Y, \sigma_i, \sigma_j)$ is called (τ_i, τ_j)- semi weakly g*- strongly- irresolute (briefly (τ_i, τ_j)- swg*- (τ_i, τ_j)- s-irresolute) if $f^{-1}(V)$ is (τ_i, τ_j)-semi weakly g*-strongly- closed in X for every semi weakly g*-strongly-closed set V of Y.

Definition 5.6.3: A subset A of a bitopological space X is called (τ_i, τ_j)- semi weakly g*-closed set (briefly (τ_i, τ_j)- swg*-closed) if τ_j-gcl $(A) \subset U$ whenever $A \subset U$ and U is τ_i-semi open in X . If $A \subset X$ is (τ_i, τ_j)- swg* closed and (τ_j, τ_i)-swg*-closed, then it is said to be (τ_i, τ_j)- semi weakly g*- strongly closed (briefly (τ_i, τ_j)- swg* -s-closed) set.

Lemma 5.6.4: If a function f: $(X, \tau_i, \tau_j) \rightarrow (Y, \sigma_i, \sigma_j)$ is an τ_i- closed, then for each subset $S \subset Y$ and each τ_i- open set U containing $f^{-1}(S)$, there is a σ_i- open set V containing S such that $f^{-1}(V) \subset U$.

Proof: Let $S \subset Y$ and U is τ_i- open containing $f^{-1}(S)$, Put V=Y- f (X-U) .Then U is σ_i-open set in Y containing S. It follows that $f^{-1}(V) \subset U$.

Lemma 5.6.5: If a function f: $(X, \tau_{i,} \tau_j) \rightarrow (Y, \sigma_{i,} \sigma_j)$ is surjective τ_j- continuous, then for every subset B of Y, τ_j-cl $(f^{-1}(B)) \subset f^{-1}(\tau_j\text{-cl}(B))$.

Proof: Let $x \in \tau_j\text{-cl}(f^{-1}(B))$. Suppose that V be τ_j-open set of Y containing f(x), i.e. $f(x) \in V$,then $x \in f^{-1}(V)$. Since $f^{-1}(V)$ is τ_j-open of X, then $f^{-1}(V) \cap f^{-1}(B) \neq \phi$. This implies that $f^{-1}(V \cap B) \neq \phi$ and $V \cap B \neq \phi$. Thus $f(x) \in \tau_j\text{-cl}(B)$ and $x \in f^{-1}(f(x)) \in f^{-1}(\tau_j\text{-cl}(B))$. This means $x \in f^{-1}(\tau_j\text{-cl}(B))$. Hence $\tau_j\text{-cl}(f^{-1}(B)) \subset f^{-1}(\tau_j\text{-cl}(B))$.

Theorem 5.6.6: Let f : $(X, \tau_{i,} \tau_j) \rightarrow (Y, \sigma_{i,} \sigma_j)$ be a function and f is (τ_i, τ_j)- semi weakly g*- strongly-continuous then for each $x \in X$. Also σ_j-open set V containing f(x) , there is an (τ_i, τ_j)- swg* -strongly open set U containing x such that $f(U) \subset V$.

Proof: Let $x \in X$ and V be σ_j-open set containing f(x). Then f is (τ_i, τ_j)- semi weakly g*- strongly-continuous, so $f^{-1}(V)$ is (τ_i, τ_j)- semi weakly g*- strongly -open set of X which containing X. If $U = f^{-1}(V)$ then $f(U) \subset V$.

Theorem 5.6.7: Let f : $(X, \tau_{i,} \tau_j) \rightarrow (Y, \sigma_{i,} \sigma_j)$ be a function and for each $x \in X$. Also σ_i-open set V containing f(x), there is an (τ_i, τ_j)-swg*-strongly open set U containing x such that $f(U) \subset V$, then f $((\tau_i, \tau_j)\text{- swg*-s-cl}(A)) \subset \tau_j\text{-cl}(f(A))$ for each subset A of X.

Proof: Let A be a subset of a bitopological space X and f(x) $\notin$, τ_j-cl(f(A)), then there exists σ_j-open set V of Y containing f(x) such that V $\cap$ f(A) = ϕ. Then each x$\in$X and for each σ_j- open set V containing f(x). There is an (τ_i, τ_j)-swg*-strongly open set U containing x such that f(U) $\subset$ V. Also f(x) $\in$f(U) $\subset$ V. Hence f(U) $\cap$ f(A)= ϕ. Implies U $\cap$ A = ϕ. Consequently, x $\notin$(τ_i, τ_j)-semi weakly g* -strongly cl(A) and f(x) $\notin$ f((τ_i, τ_j)-semi weakly g*-strongly -cl(A)). Implies f ((τ_i, τ_j)-swg*-s cl(A)) $\subset$ τ_j -cl(f(A)) for each subset A of X.

Theorem 5.6.8: Let f : (X, τ_i, τ_j) $\to$ (Y, σ_i, σ_j) be a function and f ((τ_i, τ_j)- swg*-s-cl(A)) $\subset$ τ_j-cl(f(A)) for each subset A of X, then ((τ_i, τ_j)-swg*-s- cl(f^{-1}(B)) $\subset$ f^{-1} (τ_j -cl(B)) for each subset B of Y.

Proof : Let B be a subset of Y and A=f^{-1}(B).Then f(τ_i, τ_j) swg*-s cl(A))$\subset$$\tau_j$ -cl(f(A)) for each subset A of X. Therefore ((τ_i, τ_j)-semi weakly g*-strongly-cl (f^{-1}(B))) $\subset$ τ_j-cl (f (f^{-1}(B)). Thus (τ_i, τ_j)- strongly semi weakly g*-cl(f^{-1}(B)) $\subset$ f^{-1}(τ_j-cl(B)).

Theorem 5.6.9 : If f : (X, τ_i, τ_j) $\to$ (Y, σ_i, σ_j) is bijectives (τ_i, τ_j)-semi weakly g* -open and (τ_i, τ_j)-semi weakly g*- strongly -continuous, then f is (τ_i, τ_j)-semi weakly g*- strongly -irresolute.

Proof: Let V be (τ_i, τ_j)-semi weakly g*- strongly -closed set of Y and let $f^{-1}(V) \subset U$, where U be (τ_i, τ_j)-semi weakly g*-open set. Clearly $V \subset f(U)$. Since f(U) is (τ_i, τ_j)-semi weakly g*- open and V is (τ_i, τ_j)-semi weakly g* - strongly -closed set in Y. Then τ_j-cl(V) $\subset$ f(U) and $f^{-1}(\tau_j\text{-cl}(V)) \subset U$. Since f is (τ_i, τ_j)-semi weakly g*-strongly - continuous and τ_j-cl(V) is σ_j-closed in Y, then $\tau_j\text{-cl}(f^{-1}(\tau_j\text{-cl}(V))) \subset U$. Hence $\tau_j\text{-cl}(f^{-1}(V)) \subset U$. Therefore $f^{-1}(V)$ is (τ_i, τ_j)-semi weakly g* - strongly -closed of X. Hence f is (τ_i, τ_j)-semi weakly g*- strongly- irresolute.

Theorem 5.6.10 : If $f: (X, \tau_i, \tau_j) \to (Y, \sigma_i, \sigma_j)$ is (τ_i, τ_j)-semi weakly g*- irresolute and τ_j-closed, then every (τ_i, τ_j)-semi weakly g*- strongly - closed set A of X, f(A) is (τ_i, τ_j)-semi weakly g*-strongly-closed set of Y.

Proof: Let A be an (τ_i, τ_j)-semi weakly g*- strongly- closed set. Suppose that $f(A) \subset U$, where U is an (τ_i, τ_j)-semi weakly g*- open in Y. Then $A \subset f^{-1}(U)$ and $f^{-1}(U)$ is (τ_i, τ_j)-semi weakly g*-open and f is (τ_i, τ_j)-semi weakly g* - irresolute function. Since A is semi weakly g*-strongly closed, $\tau_j\text{-cl}(A) \subset f^{-1}(U)$ and hence $f(\tau_j\text{-cl}(A)) \subset U$. Therefore $\tau_j\text{-cl}(f(A) \subset \tau_j\text{-cl}(f(\tau_j\text{-cl}(A))) = f(\tau_j\text{-cl}(A)) \subset U$ as f is τ_j-closed . Hence f(A) is (τ_i, τ_j)-semi weakly g*-strongly-closed set in Y.

Theorem 5.6.11: If $f : (X, \tau_i, \tau_j) \to (Y, \sigma_i, \sigma_j)$ is surjective, τ_j- closed and τ_j- continuous, then for every (τ_i, τ_j)-semi weakly g*- strongly- closed set B of Y, $f^{-1}(B)$ is (τ_i, τ_j)-semi weakly g*- strongly- closed set in X.

Proof: Let B be an (τ_i, τ_j)-semi weakly g*- strongly -closed subset of Y and $f^{-1}(B) \subset U$, where U is a τ_j-open set of X. As f is τ_j-closed and by lemma 5.6.4, there is a σ_j-open set V such that $B \subset V$ and $f^{-1}(V) \subset U$. Since B is (τ_i, τ_j)-semi weakly g*-strongly-closed set and $B \subset V$. Then τ_j-cl(B) $\subset$ V. Hence $f^{-1}(\tau_j\text{-cl}(B)) \subset f^{-1}(V) \subset U$. By Lemma 5.6.5, $\tau_j\text{-cl}(f^{-1}(B)) \subset U$ and hence $f^{-1}(B)$ is (τ_i, τ_j)-semi weakly g* - strongly -closed set in X, since every τ_j-open set is (τ_i, τ_j)-semi weakly g*- open set.

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