CHAPTER 5

(\tau_i, \tau_j) -SEMI WEAKLY g*- CLOSED SETS

IN BITOPOLITICAL SPACES

5.1 INTRODUCTION


In this chapter \((\tau_i, \tau_j)\)-semi weakly \(g^*\)- closed sets, \((\tau_i, \tau_j)\)-semi weakly open sets, \((\tau_i, \tau_j)\)-Quasi semi weakly \(g^*\)-closed sets, \((\tau_i, \tau_j)\)-Quasi semi weakly open sets, \((\tau_i, \tau_j)\)-semi weakly \(g^*\)- continuous functions, \((\tau_i, \tau_j)\)-semi weakly \(g^*\)- strongly continuous functions and \((\tau_i, \tau_j)\)-semi weakly \(g^*\)- irresolute functions are introduced and some of their properties are investigated.

5.2 (\tau_i, \tau_j) - SEMI WEAKLY g*-CLOSED SETS

In this section the concept of \((\tau_i, \tau_j)\) semi weakly \(g^*\)- closed sets in a bitopological space are defined and study some of their properties.
Definition 5.2.1: Let \((\tau_i, \tau_j) \in \{1,2\}\) be fixed integers. In a bitopological space \((X, \tau_i, \tau_j)\), a subset \(A \subseteq X\) is said to be \((\tau_i, \tau_j)\)-semi weakly g*-closed (briefly \((\tau_i, \tau_j)\)-swg*-closed), if \(\tau_j\)-gcl \((A) \subseteq U\) whenever \(A \subseteq U\) and \(U \in \tau_i\)-semi open.

Definition 5.2.2: The set of all \(\tau_j\)-swg*-closed sets in \(X\) is denoted by \(\tau_j\)-SWG*C \((X, \tau_i, \tau_j)\) and the set of all \(\tau_j\)-swg*-open set in \(X\) is denote by \(\tau_j\)-SWG*O \((X, \tau_i, \tau_j)\).

Theorem 5.2.3: Let \(A\) be \((\tau_i, \tau_j)\)-swg*-closed set in a bitopological space \((X, \tau_i, \tau_j)\). Then \(\tau_j\)-gcl \((A) \subseteq A\) contain no non-empty \(\tau_i\)-semi closed set in \((X, \tau_i, \tau_j)\).

Proof: Suppose that \(F\) is a \(\tau_i\)-semi closed subset of \(\tau_j\)-gcl \((A) \subseteq A\). This implies \(F \subseteq \tau_j\)-gcl \((A)\) and \(F \subseteq A^c\). Since \(F^c\) is \(\tau_i\)-semi open set and \(A\) is \((\tau_i, \tau_j)\)-swg*-closed set, \(\tau_j\)-gcl \((A) \subseteq F^c\). Therefore \(F \subseteq \tau_j\)-gcl \((A) \cap (\tau_j\)-gcl \((A)\))^c = \phi\). Hence \(\tau_j\)-gcl \((A) \subseteq A\) contain no non-empty \(\tau_i\)-semi closed set in \((X, \tau_i, \tau_j)\).

Corollary 5.2.4: If subset \(A\) in bitopological space \((X, \tau_i, \tau_j)\) is \((\tau_i, \tau_j)\)-swg*-closed set then \(\tau_j\)-gcl \((A) \subseteq A = \phi\).

Proof: Assume that \(A\) is \((\tau_i, \tau_j)\)-swg*-closed set. Since \(\tau_j\)-gcl \((A) = A\) therefore \(\tau_j\)-gcl \((A) \subseteq A = \phi\).

Theorem 5.2.5: Suppose \(B \subseteq A \subseteq X\), \(B\) is \((\tau_i, \tau_j)\)-swg*-closed set relative to \(A\) and that \(A\) is \((\tau_i, \tau_j)\)-swg*-closed subset of \((X, \tau_i, \tau_j)\). Then \(B\) is \((\tau_i, \tau_j)\)-swg*-closed relative to \(X\).
Proof: Let $B \subseteq U$ and $U$ is $\tau_i$-semi open in $X$. Then $B \subseteq A \cap U$ and hence $\tau_j\text{-gcl}_A(B) \subseteq A \cap U$. It follows that $A \cap \tau_j\text{-}\text{gcl}(B) \subseteq A \cap U$ and $A \subseteq U \cup (\tau_j\text{-}\text{gcl}(B))^c$. Since $A$ is $(\tau_i, \tau_j)$-swg*-closed in $X$, $\tau_j\text{-}\text{gcl}(A) \subseteq U \cup (\tau_j\text{-}\text{gcl}(B))^c$. Therefore $(\tau_j\text{-}\text{gcl}(B)) \subseteq \tau_j\text{-}\text{gcl}(A) \subseteq U \cup (\tau_j\text{-}\text{gcl}(B))^c$ and $\tau_j\text{-}\text{gcl}(B) \subseteq U$. Then $B$ is $(\tau_i, \tau_j)$-swg*-closed set relative to $X$.

**Theorem 5.2.6:** Let $A \subseteq Y \subseteq X$ and suppose that $A$ is $(\tau_i, \tau_j)$-swg*-closed in $X$. Then $A$ is $(\tau_i, \tau_j)$-swg*-closed relative to $Y$.

**Proof:** Let $A \subseteq Y \cap U$ and $U$ is $\tau_i$-semi open in $Y$. Then $A \subseteq U$ and hence $\tau_j\text{-}\text{gcl}(A) \subseteq U$. It follows that $Y \cap \tau_j\text{-}\text{gcl}(A) \subseteq Y \cap U$. Then $A$ is $(\tau_i, \tau_j)$-swg*-closed relative to $Y$.

**Theorem 5.2.7:** Every $\tau_j$-closed set in bitopological space $X$ is a $(\tau_i, \tau_j)$-swg*-closed in $X$.

**Proof:** Assume $A$ is $\tau_j$-closed in $X$. Let $U$ be a $\tau_i$-semi open set in $X$. Such that $A \subseteq U$, and $A \subseteq (\tau_j\text{-}\text{cl}(\tau_i\text{-}\text{int}(A))) \subseteq U$. Implies $A \subseteq (\tau_j\text{-}\text{cl}(\tau_i\text{-}\text{int}(A))) \subseteq \tau_j\text{-}\text{cl}(A) \subseteq U$ and $U$ is $\tau_i$-semi open. Thus $A \subseteq \tau_j\text{-}\text{gcl}(A) \subseteq U$. Therefore $A$ is $(\tau_i, \tau_j)$-semi weakly g*-closed set.

**Remarks 5.2.8:** The converse of the above theorem need not be true as seen from the following example.

**Example 5.2.9:** Let $X = \{a, b, c\}$ with the bitopologies $\tau_i = \{X, \phi, \{a, c\}\}$ and $\tau_j = \{X, \phi, \{c\}, \{b, c\}\}$. In this bitopological space $(X, \tau_i, \tau_j)$, the subset $\{b, c\}$ is $(\tau_i, \tau_j)$-swg*-closed which is not $\tau_j$-closed set.
Remarks 5.2.10: \((\tau_i, \tau_j)\) - g - closed set and \((\tau_i, \tau_j)\)-wg*-closed set are independent to each other as seen from the following examples.

Example 5.2.11: Let \(X = \{a, b, c\}\) with the bitopologies \(\tau_i = \{X, \phi, \{a\}\}\) and \(\tau_j = \{X, \phi, \{a, b\}\}\). In this bitopological space \((X, \tau_i, \tau_j)\) the subset \(\{b\}\) is \((\tau_i, \tau_j)\)-g - closed set which is not \((\tau_i, \tau_j)\)-swg*-closed set.

Example 5.2.12: Let \(X = \{a, b, c\}\) with the bitopologies \(\tau_i = \{X, \phi, \{a\}\}\) and \(\tau_j = \{X, \phi, \{b\}\}\). In this bitopological space \((X, \tau_i, \tau_j)\) the subset \(\{a\}\) is \((\tau_i, \tau_j)\) - swg*-closed set which is not \((\tau_i, \tau_j)\)-g - closed set.

Remark 5.2.13: \((\tau_i, \tau_j)\)- swg*-closed set and \((\tau_i, \tau_j)\)- semi closed set are independent to each other as seen from the following examples.

Example 5.2.14: Let \(X = \{a, b, c\}\) with the bitopologies \(\tau_i = \{X, \phi, \{a, b\}\}\) and \(\tau_j = \{X, \phi, \{a\}, \{a, b\}\}\). In this bitopological space \((X, \tau_i, \tau_j)\), the subset \(\{b\}\) is \((\tau_i, \tau_j)\) - semi closed set which is not \((\tau_i, \tau_j)\)-swg*-closed set.

Example 5.2.15: Let \(X = \{a, b, c\}\) with the bitopologies \(\tau_i = \{X, \phi, \{b, c\}\}\) and \(\tau_j = \{X, \phi, \{b\}, \{b, c\}\}\). In this bitopological space \((X, \tau_i, \tau_j)\), the subset \(\{a, b\}\) is \((\tau_i, \tau_j)\) - swg*-closed set which is not \((\tau_i, \tau_j)\)-semi-closed set.

Remark 5.2.16: \((\tau_i, \tau_j)\)- swg*-closed set and \((\tau_i, \tau_j)\)- pre- closed set are independent to each other as seen from the following examples.
Example 5.2.17: Let $X = \{a, b, c\}$ with the bitopologies $\tau_i = \{X, \phi, \{a, b\}\}$ and $\tau_j = \{X, \phi, \{a\}, \{a, b\}, \{a, c\}\}$. In this bitopological space $(X, \tau_i, \tau_j)$ the subset $\{a\}$ is $(\tau_i, \tau_j)$-pre-closed set which is not $(\tau_i, \tau_j)$-swg*-closed set.

Example 5.2.18: Let $X = \{a, b, c\}$ with the bitopologies $\tau_i = \{X, \phi, \{a\}\}$ and $\tau_j = \{X, \phi, \{a, b\}\}$. In this bitopological space $(X, \tau_i, \tau_j)$ the subset $\{a, c\}$ is $(\tau_i, \tau_j)$-swg*-closed set which is not $(\tau_i, \tau_j)$-pre-closed set.

Remark 5.2.19: $(\tau_i, \tau_j)$-swg*-closed set and $(\tau_i, \tau_j)$-$\alpha$-closed set are independent to each other as seen from the following examples.

Example 5.2.20: Let $X = \{a, b, c\}$ with the bitopologies $\tau_i = \{X, \phi, \{a, b\}\}$ and $\tau_j = \{X, \phi, \{a\}, \{a, b\}\}$. In this bitopological space $(X, \tau_i, \tau_j)$ the subset $\{b\}$ is $(\tau_i, \tau_j)$-$\alpha$-closed set which is not $(\tau_i, \tau_j)$-swg*-closed set.

Example 5.2.21: Let $X = \{a, b, c\}$ with the bitopologies $\tau_i = \{X, \phi, \{a\}\}$ and $\tau_j = \{X, \phi, \{c\}, \{b, c\}\}$. In this bitopological space $(X, \tau_i, \tau_j)$ the subset $\{b, c\}$ is $(\tau_i, \tau_j)$-swg*-closed set which is not $(\tau_i, \tau_j)$-$\alpha$-closed set.

Remark 5.2.22: $(\tau_i, \tau_j)$-swg*-closed set and $(\tau_i, \tau_j)$-sg-closed set are independent to each other as seen from the following examples.

Example 5.2.23: Let $X = \{a, b, c\}$ with the bitopologies $\tau_i = \{X, \phi, \{a\}\}$ and $\tau_j = \{X, \phi, \{b\}\}$. In this bitopological space $(X, \tau_i, \tau_j)$ the subset $\{b\}$ is $(\tau_i, \tau_j)$-swg*-closed set which is not $(\tau_i, \tau_j)$-sg-closed set.
Example 5.2.24: Let $X = \{a, b, c\}$ with the bitopologies $\tau_i = \{X, \phi, \{a, b\}\}$ and $\tau_j = \{X, \phi, \{a\}, \{a, b\}\}$. In this bitopological space $(X, \tau_i, \tau_j)$, the subset $\{b\}$ is $(\tau_i, \tau_j)$-sg-closed set which is not $(\tau_i, \tau_j)$-swg*-closed set.

Remark 5.2.25: $(\tau_i, \tau_j)$-swg*-closed set and $(\tau_i, \tau_j)$-gs-closed set are independent to each other as seen from the following examples.

Example 5.2.26: Let $X = \{a, b, c\}$ with the bitopologies $\tau_i = \{X, \phi, \{a\}\}$ and $\tau_j = \{X, \phi, \{a, b\}\}$. In this bitopological space $(X, \tau_i, \tau_j)$, the subset $\{b\}$ is $(\tau_i, \tau_j)$-sg-closed set which is not $(\tau_i, \tau_j)$-swg*-closed set.

Example 5.2.27: Let $X = \{a, b, c\}$ with the bitopologies $\tau_i = \{X, \phi, \{b, c\}\}$ and $\tau_j = \{X, \phi, \{a, b\}\}$. In this bitopological space $(X, \tau_i, \tau_j)$, the subset $\{b\}$ is $(\tau_i, \tau_j)$-swg*-closed set which is not $(\tau_i, \tau_j)$-gs-closed set.

Remark 5.2.28: $(\tau_i, \tau_j)$-swg*-closed set and $(\tau_i, \tau_j)$-ag-closed set are independent to each other as seen from the following examples.

Example 5.2.29: Let $X = \{a, b, c\}$ with the bitopologies $\tau_i = \{X, \phi, \{a\}\}$ and $\tau_j = \{X, \phi, \{a, b\}\}$. In this bitopological space $(X, \tau_i, \tau_j)$, the subset $\{b\}$ is $(\tau_i, \tau_j)$-ag-closed set which is not $(\tau_i, \tau_j)$-swg*-closed set.

Example 5.2.30: Let $X = \{a, b, c\}$ with the bitopologies $\tau_i = \{X, \phi, \{a, b\}\}$ and $\tau_j = \{X, \phi, \{a, c\}\}$. In this bitopological space $(X, \tau_i, \tau_j)$, the subset $\{a,b\}$ is $(\tau_i, \tau_j)$-swg*-closed set which is not $(\tau_i, \tau_j)$-ag-closed set.
Remark 5.2.31: \((\tau_i, \tau_j)\)-swg*-closed set and \((\tau_i, \tau_j)\)-g\(\alpha\)-closed set are independent to each other as seen from the following examples.

Example 5.2.32: Let \(X = \{a, b, c\}\) with the bitopologies \(\tau_i = \{X, \phi, \{a, b\}\}\) and \(\tau_j = \{X, \phi, \{a\}, \{a, b\}\}\). In this bitopological space \((X, \tau_i, \tau_j)\), the subset \(\{b\}\) is \((\tau_i, \tau_j)\)-g\(\alpha\)-closed set which is not \((\tau_i, \tau_j)\)-swg*-closed set.

Example 5.2.33: Let \(X = \{a, b, c\}\) with the bitopologies \(\tau_i = \{X, \phi, \{a\}, \{a, b\}\}\) and \(\tau_j = \{X, \phi, \{c\}, \{b, c\}\}\). In this bitopological space \((X, \tau_i, \tau_j)\), the subset \(\{c\}\) is \((\tau_i, \tau_j)\)-swg*-closed set which is not \((\tau_i, \tau_j)\)-g\(\alpha\)-closed set.

Remark 5.2.34: From the above results the following relation is obtained.

\[
\begin{align*}
\text{\((\tau_i, \tau_j)\)-sg closed} & \quad \Downarrow \quad \text{\((\tau_i, \tau_j)\)-pre-closed} \\
\text{\((\tau_i, \tau_j)\)-\(\alpha\)-closed} & \quad \Downarrow \quad \text{\((\tau_i, \tau_j)\)-swg*-closed} \\
\text{\((\tau_i, \tau_j)\)-semi closed} & \quad \Downarrow \quad \text{\((\tau_i, \tau_j)\)-\(\alpha\)g-closed} \\
\text{\((\tau_i, \tau_j)\)-\(\alpha\)g-closed} & \quad \Downarrow \quad \text{\((\tau_i, \tau_j)\)-swg*-closed} \\
\text{\((\tau_i, \tau_j)\)-semi closed} & \quad \Downarrow \quad \text{\((\tau_i, \tau_j)\)-\(\alpha\)g-closed} \\
\text{\((\tau_i, \tau_j)\)-semi closed} & \quad \Downarrow \quad \text{\((\tau_i, \tau_j)\)-\(\alpha\)g-closed} \\
\end{align*}
\]
5.3 \((\tau_i, \tau_j)\)-SEMI WEAKLY \(g^*-\) OPEN SETS

In this section the concept of \((\tau_i, \tau_j)\)-swg*- open sets in bitopological spaces are introduced and study some of their properties.

**Definition 5.3.1:** A subset \(A\) of a bitopological space \((X, \tau_i, \tau_j)\) is called \((\tau_i, \tau_j)\) semi weakly \(g^*\)-open (briefly \((\tau_i, \tau_j)\) swg*-open) if and only if \(A^c\) is \((\tau_i, \tau_j)\)-semi weakly \(g^*\)-closed set.

**Theorem 5.3.2:** A subset \(A\) in a bitopological space \((X, \tau_i, \tau_j)\) is \((\tau_i, \tau_j)\)-swg*-open if and only if \(F \subseteq \tau_j\)-g int \((A)\), whenever \(F\) is \(\tau_i\)-semi-closed and \(F \subseteq A\).

**Proof:** Assume that \(A\) is \((\tau_i, \tau_j)\)-swg*-open in \((X, \tau_i, \tau_j)\). Let \(F\) be \(\tau_i\)-semi-closed and \(F \subseteq A\). This implies \(F^c\) is \(\tau_i\)-semi open and \(A^c \subseteq F^c\). Since \(A^c\) is \((\tau_i, \tau_j)\)-swg*-closed, \(\tau_j\)-gcl \((A^c)\) \(\subseteq F^c\). Since \(\tau_j\)-gcl \((A^c)\) = \((\tau_j\)-g int\((A))^c, \(\tau_j\)-g int\((A))^c \subseteq F^c\) Therefore \(F \subseteq \tau_j\)-gint\((A)\). Conversely assume that \(F \subseteq \tau_j\)-gint \((A)\). Whenever \(F\) is \(\tau_j\)-semi closed, and \(F \subseteq A\). Let \(U\) be a \(\tau_j\)-semi open set in \((X, \tau_i, \tau_j)\) containing \(A^c\). Therefore \(U^c\) is \(\tau_j\)-semi closed set contained in \(A\) by hypothesis \(U^c \subseteq \tau_j\)-gint \((A)\) taking complements \(U \supseteq \tau_j\)-gcl \((A^c)\). Therefore \(A^c\) is \((\tau_i, \tau_j)\)-swg*-closed in \((X, \tau_i, \tau_j)\). Hence \(A\) is \((\tau_i, \tau_j)\)-swg*- open in \((X, \tau_i, \tau_j)\).

**Theorem 5.3.3:** If \(A \subseteq B \subseteq X\) where \(A\) is \((\tau_i, \tau_j)\)-swg*-open relative to \(B\) and \(B\) is \((\tau_i, \tau_j)\)-swg*-open relative to \(X\) then \(A\) is \((\tau_i, \tau_j)\)-swg*-open relative to \(X\).
**Proof:** Let $F$ be a $\tau_i$-semi closed set and suppose that $F \subseteq A$. Then $F$ is $\tau_i$-semi closed relative to $B$ and hence $F \subseteq \tau_j g\text{-}\text{int}_B(A)$. Therefore there exists a $\tau_i$-semi-open set $U$ such that $F \subseteq U \cap B \subseteq A$. But $F \subseteq U^* \subseteq B$ for $\tau_i$-semi-open set $U^*$.

Since $B$ is $\text{swg}^*$-open in $X$. Thus $F \subseteq U^* \cap U \subseteq B \cap U \subseteq A$. It follows that $F \subseteq \tau_j g\text{-}\text{int}(A)$, because set $A$ is $(\tau_i, \tau_j)$-$\text{swg}^*$-open set. This implies $F \subseteq \tau_j g\text{-}\text{int}(A)$. Whenever $F$ is $\tau_i$-semi-closed set and $F \subseteq A$. Therefore $A$ is $(\tau_i, \tau_j)$-semi weakly $g^*$-open in $X$.

**Theorem 5.3.4:** If $\tau_j g\text{-}\text{int}(A) \subseteq B \subseteq A$ and if $A$ is $(\tau_i, \tau_j)$-$\text{swg}^*$-open then $B$ is $(\tau_i, \tau_j)$-$\text{swg}^*$-open.

**Proof:** $A^c \subseteq B^c \subseteq \tau_j g\text{-}\text{cl}(A^c)$ and since $A^c$ is $(\tau_i, \tau_j)$-$\text{swg}^*$-closed set. It follows that $B^c$ is $(\tau_i, \tau_j)$-$\text{swg}^*$-closed set because $A$ is $(\tau_i, \tau_j)$-$\text{swg}^*$-closed and $A \subseteq B \subseteq \tau_j \text{gcl}(A)$. Then $B$ is $(\tau_i, \tau_j)$-$\text{swg}^*$-open set.

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5.4 $(\tau_i, \tau_j)$-QUASI SEMI WEAKLY $g^*$-OPEN FUNCTIONS AND $(\tau_i, \tau_j)$-QUASI SEMI WEAKLY $g^*$-CLOSED FUNCTIONS

In this section $(\tau_i, \tau_j)$-Quasi semi weakly $g^*$-open and $(\tau_i, \tau_j)$-Quasi semi weakly $g^*$-closed functions in bitopological spaces are introduced and study some of their properties.
**Definition 5.4.1:** Let \((X, \tau_i, \tau_j)\) and \((Y, \sigma_i, \sigma_j)\) be any two bitopological spaces. A function \(f : (X, \tau_i, \tau_j) \to (Y, \sigma_i, \sigma_j)\) is said to be \((\tau_i, \tau_j)\)-quasi semi weakly \(g^*\)-open if the image of every \((\tau_i, \tau_j)\)-semi weakly \(g^*\)-open set in \(X\) is \(\sigma_i\)-open in \(Y\).

**Theorem 5.4.2:** Let \(f : (X, \tau_i, \tau_j) \to (Y, \sigma_i, \sigma_j)\) be a function. Then the following are equivalent:

(i) \(f\) is \((\tau_i, \tau_j)\)-quasi \(swg^*\)-open;

(ii) For each subset \(U\) of \(X\), \(f((\tau_i, \tau_j)\text{-}g\text{ int}(U)) \subset \sigma_i\text{- int}(f(U))\);

(iii) For each \(x \in X\) and each \((\tau_i, \tau_j)\)-\(swg^*\)-neighbourhood \(U\) of \(x\) in \(X\), there exists a \(\sigma_i\)-neighbourhood \(V\) of \(f(x)\) such that \(V \subset f(U)\).

**Proof:** (i) \(\Rightarrow\) (ii): Let \(f\) be an \((\tau_i, \tau_j)\)-quasi \(swg^*\)-open function. Since \((\tau_i, \tau_j)\)-\(g\text{ int}(U)\) is an \((\tau_i, \tau_j)\)-\(swg^*\)-open set contained in \(U\), that implies \(f((\tau_i, \tau_j)\text{-}g\text{ int}(U)) \subset f(U)\). As \(f((\tau_i, \tau_j)\text{-}g\text{ int}(U))\) is \(\sigma_i\)-open, \(f((\tau_i, \tau_j)\text{-}g\text{ int}(U)) \subset \sigma_i\text{- int}(f(U))\).

(ii) \(\Rightarrow\) (iii): Let \(x \in X\) and \(U\) be an \((\tau_i, \tau_j)\)-\(swg^*\)-neighbourhood of \(x\) in \(X\). Then there exist an \((\tau_i, \tau_j)\)-\(swg^*\)-open set \(V\) in \(X\) such that \(x \in V \subset U\). Thus by (ii), \(f(V) = f((\tau_i, \tau_j)\text{-}g\text{ int}(V)) \subset \sigma_i\text{- int}(f(V)),\) and hence \(f(V)\) = \(\sigma_i\text{- int}(f(V))\). Therefore it follows that \(f(V)\) is \(\sigma_i\)-open such that \(f(x) \in f(V) \subset f(U)\).

(iii) \(\Rightarrow\) (i): Let \(U\) be an \((\tau_i, \tau_j)\)-\(swg^*\)-open set in \(X\). Then by (iii), for each \(y \in f(U)\), there exists a \(\sigma_i\)-neighbourhood \(V_y\) of \(y\) such that \(V_y \subset f(U)\). As \(V_y\) is a \(\sigma_i\)-neighbourhood of \(y\), there exists a \(\sigma_i\)-open set \(W_y\) such that \(Y \in W_y \subset V_y\). Thus \(f(U) = \cup\{W_y : Y \in f(U)\}\) is \(\sigma_i\)-open. Hence, \(f\) is \((\tau_i, \tau_j)\)-quasi \(swg^*\)-open.
Theorem 5.4.3: A function \( f : (X, \tau_i, \tau_j) \to (Y, \sigma_i, \sigma_j) \) is \((\tau_i, \tau_j)\)-quasi swg* - open, if and only if for any subset \( B \) of \( Y \) and for any \((\tau_i, \tau_j)\)-swg*-closed set \( F \) in \( X \) such that \( f^{-1}(B) \subset F \), there exists a \( \sigma_i \)-closed set \( G \) containing \( B \) such that \( f^{-1}(G) \subset F \).

Proof: Suppose that \( f \) is \((\tau_i, \tau_j)\)-quasi swg* - open. Let \( B \subset Y \) and \( F \) be an \((\tau_i, \tau_j)\)-swg*-closed set in \( X \) such that \( f^{-1}(B) \subset F \). Now, put \( G = Y - f(X - F) \). It is clear that \( B \subset G \) as \( f^{-1}(B) \subset F \), and that \( f^{-1}(G) \subset F \). Also \( G \) is \( \sigma_i \)-closed, since \( f \) is \((\tau_i, \tau_j)\)-quasi- swg*-open. Conversely, let \( U \) be an \((\tau_i, \tau_j)\)-swg*-open set in \( X \), and put \( B = Y - f(U) \). Then \( X - U \) is an \((\tau_i, \tau_j)\)-swg*-closed set in \( X \) such that \( f^{-1}(B) \subset X - U \). By hypothesis, there exists a \( \sigma_i \)-closed set \( G \) such that \( B \subset G \) and \( f^{-1}(G) \subset X - U \). Hence, \( f(U) \subset Y - G \). On the other hand \( B \subset G \), \( Y - G \subset Y - B = f(U) \). Thus \( f(U) = Y - G \) is \( \sigma_i \)-open and hence \( f \) is a \((\tau_i, \tau_j)\)-quasi swg* - open.

Theorem 5.4.4: Let \( f : (X, \tau_i, \tau_j) \to (Y, \sigma_i, \sigma_j) \) be a function. Then the following are equivalent:

(i) \( f \) is \((\tau_i, \tau_j)\)-quasi swg*-open;

(ii) \( f^{-1}(\sigma_i\text{-cl}(B)) \subset (\tau_i, \tau_j)\)-gcl\( (f^{-1}(B)) \) for every subset \( B \) of \( Y \);

(iii) \((\tau_i, \tau_j)\)-g int\( (f^{-1}(B)) \subset f^{-1}(\sigma_i\text{-int}(B)) \) for every subset \( B \) of \( Y \).

Proof:

(i) \( \Rightarrow \) (ii): Suppose that \( f \) is \((\tau_i, \tau_j)\)-quasi swg* - open set. Now, for any subset \( B \) of \( Y \), \( f^{-1}(B) \subset (\tau_i, \tau_j)\)-gcl\( (f^{-1}(B)) \). Therefore by theorem 5.4.3, there exists \( \sigma_i \)-closed set \( G \)
such that $B \subseteq G$ and $f^{-1}(G) \subseteq (\tau_i, \tau_j)$-gcl$(f^{-1}(B))$. Hence, $f^{-1}(\sigma_i\text{-cl}(B)) \subseteq f^{-1}(G) \subseteq (\tau_i, \tau_j)$-gcl$(f^{-1}(B))$.

(ii) $\Rightarrow$ (i): Let $B \subseteq Y$ and $F$ be an $(\tau_i, \tau_j)$-swg*-closed set in $X$ such that $f^{-1}(B) \subseteq F$. Put $G = \sigma_i\text{-cl}(B)$, then $B \subseteq G$, $G$ is $\sigma_i$-closed, and $f^{-1}(G) \subseteq (\tau_i, \tau_j)$-gcl$(f^{-1}(B)) \subseteq F$. Thus by theorem 5.4.3, $f$ is $(\tau_i, \tau_j)$-quasi swg*-open set.

(ii)$\Leftrightarrow$(iii): It is clear, because $f^{-1}(\sigma_i\text{-cl}(B)) \subseteq (\tau_i, \tau_j)$-gcl$(f^{-1}(B))$ for every subset $B$ of $Y$ is equal to $(\tau_i, \tau_j)$-g int$(f^{-1}(B)) \subseteq f^{-1}(\sigma_i\text{-int}(B))$ for every subset $B$ of $Y$.

**Theorem 5.4.5:** Let $f : (X, \tau_i, \tau_j) \rightarrow (Y, \sigma_i, \sigma_j)$ and $g : (Y, \sigma_i, \sigma_j) \rightarrow (Z, \eta_i, \eta_j)$ be two functions such that $g \circ f : X \rightarrow Z$ is $(\tau_i, \tau_j)$-quasi swg*-open. If $g$ is a pairwise continuous injection then $f$ is $(\tau_i, \tau_j)$-quasi swg*-open set.

**Proof:** Let $U$ be an $(\tau_i, \tau_j)$-swg*-open set in $X$. Then $g \circ f(U)$ is $\eta_i$-open as $g \circ f$ is $(\tau_i, \tau_j)$-quasi swg*-open. Since $g$ is a pairwise continuous injection, $f(U) = g^{-1}(g \circ f(U))$ is $\sigma_i$-open. Hence, $f$ is $(\tau_i, \tau_j)$-quasi swg*-open set.

**Definition 5.4.6:** A function $f : (X, \tau_i, \tau_j) \rightarrow (Y, \sigma_i, \sigma_j)$ is said to be $(\tau_i, \tau_j)$-quasi swg* closed if the image of each $(\tau_i, \tau_j)$-swg*-closed set in $X$ is $\sigma_i$-closed in $Y$.

**Theorem 5.4.7:** A function $f : (X, \tau_i, \tau_j) \rightarrow (Y, \sigma_i, \sigma_j)$ is $(\tau_i, \tau_j)$-quasi swg*-closed set if and only if $\sigma_i\text{-cl}(f(A)) \subseteq f((\tau_i, \tau_j)\text{-gcl}(A))$ for every subset $A$ of $X$.

**Proof:** Let $f$ be $(\tau_i, \tau_j)$-quasi swg*-closed set, there exist $\sigma_i\text{-cl}(f(A)) \subseteq f((\tau_i, \tau_j)\text{-gcl}(A))$ for every subset $A$ of $X$. Conversely, every $\sigma_i\text{-cl}(f(A)) \subseteq f((\tau_i, \tau_j)\text{-gcl}(A))$ is $(\tau_i, \tau_j)$-quasi swg*-closed.

[79]
**Theorem 5.4.8:** Let $f: (X, \tau_i, \tau_j) \rightarrow (Y, \sigma_i, \sigma_j)$ be a function. Then the following are equivalent:

(i) $f$ is $(\tau_i, \tau_j)$-quasi swg*-closed;

(ii) For any subset $B$ of $Y$ and for any $(\tau_i, \tau_j)$-swg*-open set $G$ in $X$ such that $f^{-1}(B) \subseteq G$, there exists a $\sigma_i$-open set $U$ containing $B$ such that $f^{-1}(U) \subseteq G$;

(iii) For each $y \in Y$ and for any $(\tau_i, \tau_j)$-swg*-open set $G$ in $X$ such that $f^{-1}(\{y\}) \subseteq G$, there exists a $\sigma_i$-open set $U$ containing $\{y\}$ such that $f^{-1}(U) \subseteq G$.

**Proof:**

(i)$\Rightarrow$(ii): Suppose $f$ is $(\tau_i, \tau_j)$-quasi swg* closed set. Now there exist for any subset $B$ of $Y$ and for $(\tau_i, \tau_j)$-swg*-open set $G$ in $X$ such that $f^{-1}(B) \subseteq G$, there exist a $\sigma_i$-open set $U$ containing $B$ such that $f^{-1}(U) \subseteq G$.

(ii)$\Rightarrow$(iii) : For any subset $B$ of $Y$ and for any $(\tau_i, \tau_j)$-swg*-open set $G$ in $X$ such that $f^{-1}(B) \subseteq G$, there exists a $\sigma_i$-open set $U$ containing $B$ such that $f^{-1}(V) \subseteq G$. Also there exist for each $y \in Y$ and for any $(\tau_i, \tau_j)$-swg*-open set $G$ in $X$ such that $f^{-1}(\{y\}) \subseteq G$, there exists a $\sigma_i$-open set containing $\{y\}$ such that $f^{-1}(U) \subseteq G$.

(iii)$\Rightarrow$(i): For each $y \in Y$ and for any $(\tau_i, \tau_j)$-swg*-open set $G$ in $X$ such that $f^{-1}(\{y\}) \subseteq G$, there exists a $\sigma_i$-open set $U$ containing $\{y\}$ such that $f^{-1}(U) \subseteq G$. Then $f$ is $(\tau_i, \tau_j)$-quasi swg*-closed set.

**Definition 5.4.9:** A function $f: (X, \tau_i, \tau_j) \rightarrow (Y, \sigma_i, \sigma_j)$ is called pairwise swg***-closed if the image of every $(\tau_i, \tau_j)$-swg*-closed set in $X$ is $(\tau_i, \tau_j)$-swg*-closed set in $Y$. 
Theorem 5.4.10: Let \( f : (X, \tau_i, \tau_j) \rightarrow (Y, \sigma_i, \sigma_j) \) be a function. Then the following as equivalent:

(i) \( f \) is pairwise swg\(^{**}\)-closed;

(ii) For any subset \( B \) of \( Y \) and for any \((\tau_i, \tau_j)\)-swg*-open set \( G \) in \( X \) such that \( f^{-1}(B) \subset G \), there exists an \((\tau_i, \tau_j)\)-swg*-open set \( U \) in \( Y \) such that \( B \subset U \) and \( f^{-1}(U) \subset G \);

(iii) For each \( y \in Y \) and for any \((\tau_i, \tau_j)\)-swg*-open set \( G \) in \( X \) such that \( f^{-1}(\{y\}) \subset G \), there exists an \((\tau_i, \tau_j)\)-swg*-open set \( U \) in \( Y \) such that \( y \in U \) and \( f^{-1}(U) \subset G \);

(iv) \((\tau_i, \tau_j)\)-gcl\((f(A)) \subset f((\tau_i, \tau_j)\)-gcl\((A)) \) for every subset \( A \) of \( X \).

Proof:

(i) \( \Rightarrow \) (ii): Let \( f \) be an pairwise swg\(^{**}\)-closed. By definition 5.4.9, \( f : (X, \tau_i, \tau_j) \rightarrow (Y, \sigma_i, \sigma_j) \) is called pairwise swg\(^{**}\)-closed if the image of every \((\tau_i, \tau_j)\)-swg*-closed set in \( X \) is \((\tau_i, \tau_j)\)-swg*-closed set in \( Y \). There exists for any subset \( B \) of \( Y \) and for any \((\tau_i, \tau_j)\)-swg*-open set \( G \) in \( X \) such that \( f^{-1}(B) \subset G \). Also there exists an \((\tau_i, \tau_j)\)-swg*-open set \( U \) in \( Y \), such that \( B \subset U \) and \( f^{-1}(U) \subset G \).

(ii) \( \Rightarrow \) (iii): For any subset \( B \) of \( Y \) and for any \((\tau_i, \tau_j)\)-swg*-open set \( G \) in \( X \) such that \( f^{-1}(B) \subset G \), there exists an \((\tau_i, \tau_j)\)-swg*-open set \( U \) in \( Y \) such that \( B \subset U \) and \( f^{-1}(U) \subset G \). There exist for \( y \in Y \) and for any \((\tau_i, \tau_j)\)-swg*-open set \( G \) in \( X \), such that \( f^{-1}(\{y\}) \subset G \). Also there exists an \((\tau_i, \tau_j)\)-swg*-open set \( U \) in \( Y \) such that \( y \in U \) and \( f^{-1}(U) \subset G \).
(iii)⇒(iv) : Let each \( y \in Y \) and for any \( (\tau_i, \tau_j) \)-swg*-open set \( G \) in \( X \) such that 
\[ f^{-1}(\{y\}) \subset G, \]
there exists an \( (\tau_i, \tau_j) \)-swg*-open set \( U \) in \( Y \) such that \( y \in U \) and 
\[ f^{-1}(U) \subset G. \]
This implies \( (\tau_i, \tau_j)\)-gcl \((f(A)) \subset f(\tau_i, \tau_j)\)-gcl\((A)) \) for every subset \( A \) of \( X \).

(iv)⇒(i) : Let \( (\tau_i, \tau_j)\)-gcl\((f(A)) \subset f((\tau_i, \tau_j)\)-gcl\((A)) \) for every subset \( A \) of \( X \). There exist \( f \) is pairwise swg**-closed.

**Theorem 5.4.11:** Let \( f : (X, \tau_i, \tau_j) \to (Y, \sigma_i, \sigma_j) \) and \( g : (Y, \sigma_i, \sigma_j) \to (Z, \eta_i, \eta_j) \) are two \( (\tau_i, \tau_j)\)-quasi swg*-closed functions, then \( g\circ f : (X, \tau_i, \tau_j) \to (Z, \eta_i, \eta_j) \) is \( (\tau_i, \tau_j)\)-quasi swg*-closed.

**Proof:** If \( f : (X, \tau_i, \tau_j) \to (Y, \sigma_i, \sigma_j) \) and \( g : (Y, \sigma_i, \sigma_j) \to (Z, \eta_i, \eta_j) \) are two \( (\tau_i, \tau_j)\)-quasi swg*-closed set. Let \( U \) be an \( (\tau_i, \tau_j)\)-swg*-closed set in \( X \). Then \( g\circ f \) is \( (\tau_i, \tau_j)\)-quasi swg*-closed set.

**Theorem 5.4.12:** Let \( f : (X, \tau_i, \tau_j) \to (Y, \sigma_i, \sigma_j) \) and \( g : (Y, \sigma_i, \sigma_j) \to (Z, \eta_i, \eta_j) \) be any two functions. Then if \( f \) is pairwise swg*-closed and \( g \) is \( (\tau_i, \tau_j)\)-quasi swg*-closed set the \( g\circ f \) is pairwise closed.

**Proof:** If \( f \) is pairwise swg*-closed and \( g \) is \( (\tau_i, \tau_j)\)-quasi swg*-closed set then \( g\circ f \) is pairwise closed.

**Theorem 5.4.13:** Let \( f : (X, \tau_i, \tau_j) \to (Y, \sigma_i, \sigma_j) \) and \( g : (Y, \sigma_i, \sigma_j) \to (Z, \eta_i, \eta_j) \) be any two functions. Then if \( f \) is pairwise swg***-closed and \( g \) is \( (\tau_i, \tau_j)\)-quasi swg*-closed then \( g\circ f \) is \( (\tau_i, \tau_j)\)-quasi swg*-closed.
Proof: If \( f \) is pairwise \( \text{swg}^{**} \)-closed and \( g \) is \((\tau_i, \tau_j)\)-quasi \( \text{swg}^* \)-closed then \( g \circ f \) is \((\tau_i, \tau_j)\)-quasi \( \text{swg}^* \)-closed set.

Definition 5.4.14: A function \( f : (X, \tau_i, \tau_j) \to (Y, \sigma_i, \sigma_j) \) is called pairwise \( \text{swg}^* \)-irresolute, if \( f^{-1}(V) \) is \((\tau_i, \tau_j)\)-\( \text{swg}^* \)-open in \((X, \tau_i, \tau_j)\) for every \((\tau_i, \tau_j)\)-\( \text{swg}^* \)-open set \( V \) in \( Y \).

Definition 5.4.15: A function \( f : (X, \tau_i, \tau_j) \to (Y, \sigma_i, \sigma_j) \) is called pairwise \( \text{swg}^* \)-continuous, if \( f^{-1}(V) \) is \((\tau_i, \tau_j)\)-\( \text{swg}^* \)-open in \((X, \tau_i, \tau_j)\) for every \( \sigma_i \)-open set \( V \) in \( Y \).

Theorem 5.4.16:

Let \( f : (X, \tau_i, \tau_j) \to (Y, \sigma_i, \sigma_j) \) and \( g : (Y, \sigma_i, \sigma_j) \to (Z, \eta_i, \eta_j) \) be two functions such that \( g \circ f : X \to Z \) is \((\tau_i, \tau_j)\)-quasi \( \text{swg}^* \)-closed. Then

(i) If \( f \) is a pairwise \( \text{swg}^* \)-irresolute surjection, then \( g \) is \((\tau_i, \tau_j)\)-quasi \( \text{swg}^* \)-closed.

(ii) If \( g \) is a pairwise \( \text{swg}^* \)-continuous injection, then \( f \) is pairwise \( \text{swg}^{**} \)-closed.

Proof:

(i) Suppose that \( F \) is \((\tau_i, \tau_j)\)-\( \text{swg}^* \)-closed set in \( Y \). Then \( f^{-1}(F) \) is \((\tau_i, \tau_j)\)-\( \text{swg}^* \)-closed in \( X \) as \( f \) is pairwise \( \text{swg}^* \)-irresolute. Since \( g \circ f \) is \((\tau_i, \tau_j)\)-quasi \( \text{swg}^* \)-closed.
closed and f is surjective \((g \circ f)(f^{-1}(F)) = g(F)\) is \(\eta_i\)-closed. Hence g is \((\tau_i, \tau_j)\)-quasi-sw\(g^\ast\)-closed.

(ii) Suppose that F is an \((\tau_i, \tau_j)\)-sw\(g^\ast\)-closed set in X. Since \(g \circ f\) is \((\tau_i, \tau_j)\)-quasi-sw\(g^\ast\)-closed set, \((g \circ f)(F)\) is \(\eta_i\)-closed, but g is a pairwise sw\(g^\ast\)-continuous injection, so \(g^{-1}(g \circ f(F)) = f(F)\) is \((\tau_i, \tau_j)\)-sw\(g^\ast\)-closed set in Y. Hence f is pairwise sw\(g^\ast\)-closed.

**Theorem 5.4.17:** Let \(g : (Y, \sigma_i, \sigma_j) \to (Z, \sigma_i, \sigma_j)\) be a function. Then g is \((\tau_i, \tau_j)\)-quasi-sw\(g^\ast\)-closed if and only if g(X) is \(\sigma_i\)-closed, and \(g(V) - g(X-V)\) is \(\sigma_i\)-open in g(X) whenever V is \((\tau_i, \tau_j)\)-sw\(g^\ast\)-open in X.

**Proof: Necessity:** Let g is \((\tau_i, \tau_j)\)- quasi-sw\(g^\ast\)-closed. Then g(X) is \(\sigma_i\)-closed as X is \((\tau_i, \tau_j)\)-sw\(g^\ast\)-closed and \(g(V) - g(X-V) = g(X) - g(X-V)\) is \(\sigma_i\)-open in g(X) when V is \((\tau_i, \tau_j)\)-sw\(g^\ast\)-open in X.

**Sufficiency:** Suppose that g(X) is \(\sigma_i\)-closed and \(g(V) - g(X-V)\) is \(\sigma_i\)-open is g(X) when V is \((\tau_i, \tau_j)\)-sw\(g^\ast\)-open set in X, and let C be \((\tau_i, \tau_j)\)-sw\(g^\ast\)-closed set in X. Then g(C) = g(X) - (g(X-C) - g(C)) is \(\sigma_i\)-closed in g(X) and therefore g(C) is \(\sigma_i\)-closed. Hence, g is \((\tau_i, \tau_j)\)-quasi-sw\(g^\ast\)-closed set.

**Corollary 5.4.18:** Let g : \((X, \tau_i, \tau_j) \to (Y, \sigma_i, \sigma_j)\) be a surjection. Then g is \((\tau_i, \tau_j)\)-quasi-sw\(g^\ast\)-closed if and only if \(g(V) - g(X-V)\) is \(\sigma_i\)-open whenever V is \((\tau_i, \tau_j)\)-sw\(g^\ast\)-open in X.
Definition 5.4.19: A Space \((X, \tau_i, \tau_j)\) is said to be pairwise swg*-normal if for any disjoint subset \(F_1 \in (\tau_i, \tau_j)\)-SWG*C \((X)\) and \(F_2 \in (\tau_j, \tau_i)\)-SWG*C \((X)\), there exist disjoint subsets \(U \in \tau_i\) and \(V \in \tau_j\) such that \(F_1 \subset U\) and \(F_2 \subset V\).

Theorem 5.4.20: Let \((X, \tau_i, \tau_j) \rightarrow (Y, \sigma_i, \sigma_j)\) be two spaces, where \(X\) is pairwise swg*-normal. Let \(g : (X, \tau_i, \tau_j) \rightarrow (Y, \sigma_i, \sigma_j)\) be a pairwise swg*-continuous, \((\tau_i, \tau_j)\)-quasi swg*-closed surjection. Then \(Y\) is pairwise normal.

Proof: Let \(X\) be pairwise swg*-normal. Let \(K\) be \(\sigma_i\)-closed and \(M\) be \(\sigma_j\)-closed disjoint subsets of \(Y\). Then \(g^{-1}(K) \in (\tau_i, \tau_j)\)-SWG*C \((X), g^{-1}(M) \in (\tau_j, \tau_i)\)-SWG*C \((X)\) and \(g^{-1}(K) \cap g^{-1}(M) = \phi\). Since \(X\) is pairwise swg*-normal, there exist disjoint sets \(V \in \tau_i\) and \(W \in \tau_j\) such that \(g^{-1}(K) \subset V\) and \(g^{-1}(M) \subset W\). Thus \(K \subset g(V) - g(X - V)\) and \(M \subset g(W) - g(X - W)\). It follows also from corollary 5.4.18 that \(g(V) - g(X - V) \in \sigma_i\) and \(g(W) - g(X - W) \in \sigma_j\), and clearly \((g(V) - g(X - V)) \cap (g(W) - g(X - W)) = \phi\) because \(V \cap W = \phi\). Hence \(Y\) is pairwise normal.

Theorem 5.4.21: Let \((X, \tau_i, \tau_j) \rightarrow (Y, \sigma_i, \sigma_j)\) be two spaces, where \(X\) is pairwise swg*-normal. Let \(g : (X, \tau_i, \tau_j) \rightarrow (Y, \sigma_i, \sigma_j)\) be a pairwise swg*-irresolute, \((\tau_i, \tau_j)\)-quasi swg*-closed surjection. Then \(Y\) is pairwise swg*-normal.

Proof: Let \(X\) be pairwise swg*-normal. Let \(K\) be \(\sigma_i\)-swg*-closed and \(M\) be \(\sigma_j\)-swg*-closed disjoint subsets of \(Y\). Then \(g^{-1}(K) \in (\tau_i, \tau_j)\)-SWG*C \((X), g^{-1}(M) \in (\tau_j, \tau_i)\)-SWG*C \((X)\) and \(g^{-1}(K) \cap g^{-1}(M) = \phi\). Since \(X\) is pairwise swg*-normal, there exists disjoint sets \(V \in \tau_i\) and \(W \in \tau_j\) such that \(g^{-1}(K) \subset V\) and \(g^{-1}(M) \subset W\).
Thus $K \subset g(V) - g(X-V)$ and $M \subset g(W) - g(X-W)$. It follows from corollary 5.2.18, that $g(V) - g(X-V) \in \sigma_i$ and $g(W) - g(X-W) \in \sigma_j$. Clearly $g(V) - g(X-V) \cap (g(W) - g(X-W)) = \emptyset$ because $V \cap W = \emptyset$. Hence $Y$ is pairwise $\text{swg}^*$-normal.

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5.5 $(\tau_i,\tau_j)$-SEMI WEAKLY $g^*$-CONTINUOUS FUNCTIONS IN BITOPOLOGICAL SPACES

In this section $(\tau_i,\tau_j)$-$\text{swg}^*$-continuous functions in bitopological spaces are introduced and study some of their properties.

**Definition 5.5.1:** A function $f : (X, \tau_i, \tau_j) \rightarrow (Y, \sigma_i, \sigma_j)$ is said to be $(\tau_i, \tau_j)$-$\sigma_i$-semi weakly $g^*$-continuous function, if the inverse image of every $\sigma_i$-closed set in $Y$ is $(\tau_i, \tau_j)$-semi weakly $g^*$-closed set in $X$.

**Definition 5.5.2:** A function $f : (X, \tau_i, \tau_j) \rightarrow (Y, \sigma_i, \sigma_j)$ is called $(\tau_i, \tau_j)$-semi weakly $g^*$-irresolute (briefly $(\tau_i, \tau_j)$-$\text{swg}^*$-irresolute) function, if $f^{-1}(V)$ is $(\tau_i, \tau_j)$-$\text{swg}^*$-closed set in $X$ for every $(\tau_i, \tau_j)$-$\text{swg}^*$-closed set $V$ of $Y$.

**Theorem 5.5.3:** The following are equivalent for a function $f : (X, \tau_i, \tau_j) \rightarrow (Y, \sigma_i, \sigma_j)$.

(i) $f$ is pairwise $\text{swg}^*$-continuous function

(ii) $f^{-1}(U)$ is $(\tau_i, \tau_j)$-$\text{swg}^*$-closed for each $\sigma_i$-closed set $U$ in $Y$, $i \neq j$ and $(\tau_i, \tau_j)= 1,2$. 

86
Proof:

(i)⇒(ii): Suppose that $f$ is pairwise $\text{swg}^*$-continuous. Let $A$ be $\sigma_j$-closed in $Y$. Then $A^c$ is $\sigma_j$-open in $Y$. Since $f$ is pairwise $\text{swg}^*$-continuous, $f^{-1}(A^c)$ is $(\tau_i, \tau_j)$-$\text{swg}^*$-open in $X$, $i \neq j$ and $(\tau_i, \tau_j)=1,2$. Consequently, $f^{-1}(A)$ is $(\tau_i, \tau_j)$-$\text{swg}^*$-closed set in $X$.

(ii)⇒(i): Suppose that $f^{-1}(U)$ is $(\tau_i, \tau_j)$-$\text{swg}^*$-closed for each $\sigma_i$-closed set $U$ in $Y$, $i \neq j$ and $(\tau_i, \tau_j)=1,2$. Let $V$ be $\sigma_j$-open in $Y$. Then $V^c$ is $\sigma_j$-closed in $Y$. Therefore by our assumption, $f^{-1}(V^c)$ is $(\tau_i, \tau_j)$-$\text{swg}^*$-closed in $X$, $i \neq j$ and $(\tau_i, \tau_j)=1,2$. Hence $f^{-1}(V)$ is $(\tau_i, \tau_j)$-$\text{swg}^*$-open in $X$. This completes the proof.

**Lemma 5.5.4:** A function $f: (X, \tau_i, \tau_j) \rightarrow (Y, \sigma_i, \sigma_j)$ is $(\tau_j, \tau_i)$-$\text{swg}^*$-irresolute, then for every subset $B$ of $Y$, $(\tau_j, \tau_i)$-$\text{swg}^*$-$\text{cl}(f^{-1}(B)) \subseteq f^{-1}((\tau_j, \tau_i)$-$\text{swg}^*$-$\text{cl}(B))$.

**Proof:** Let $x \in (\tau_j, \tau_i)$-$\text{swg}^*$-$\text{cl}(f^{-1}(B))$. Suppose that $V$ is $(\tau_j, \tau_i)$-$\text{swg}^*$-open set of $Y$, containing $f$ $(x)$, i.e. $f$ $(x) \in V$, then $x \in f^{-1}(V)$. Since $f^{-1}(V)$ is $(\tau_j, \tau_i)$-$\text{swg}^*$-open of $X$, then $f^{-1}(V) \cap f^{-1}(B) \neq \emptyset$ implies that $f^{-1}(V \cap B) \neq \emptyset$ and $V \cap B \neq \emptyset$. Thus $f(x) \in (\tau_j, \tau_i)$-$\text{swg}^*$-$\text{cl}(B)$ and $x \in f^{-1}(f(x)) \in f^{-1}((\tau_j, \tau_i)$-$\text{swg}^*$-$\text{cl}(B))$, this means $x \in f^{-1}((\tau_j, \tau_i)$-$\text{swg}^*$-$\text{cl}(B))$. Hence $((\tau_j, \tau_i)$-$\text{swg}^*$-$\text{cl}(f^{-1}(B)) \subseteq f^{-1}((\tau_j, \tau_i)$-$\text{swg}^*$-$\text{cl}(B))$.

**Lemma 5.5.5:** If a function $f: (X, \tau_i, \tau_j) \rightarrow (Y, \sigma_i, \sigma_j)$ is $\tau_i$-closed, then for each subset $S \subseteq Y$ and each $\tau_i$-open set $U$ containing $f^{-1}(S)$, there is a $\sigma_i$-open set $V$ containing $S$ such that $f^{-1}(V) \subseteq U$. 

87
Proof: Let $S \subset Y$ and $U$ is $\tau_i$-open containing $f^{-1}(S)$, Put $V=Y-f(X-U)$. Then $U$ is $\sigma_i$-open set in $Y$ containing $S$. It follows that $f^{-1}(V) \subset U$.

Theorem 5.5.6: Let $f: (X, \tau_1, \tau_j) \to (Y, \sigma_1, \sigma_j)$ be a function and $f$ is $(\tau_i, \tau_j)$-swg*-continuous, then for each $x \in X$ and for each $\sigma_j$-open set $V$ containing $f(x)$, there is an $(\tau_i, \tau_j)$-swg*-open set $U$ containing $x$ such that $f(U) \subset V$.

Proof: Let $x \in X$ and $V$ be $\sigma_i$-open set containing $f(x)$. Then $f$ is $(\tau_i, \tau_j)$-swg*-continuous, so $f^{-1}(V)$ is $(\tau_i, \tau_j)$swg*-open set of $X$ containing $X$. If $U=f^{-1}(V)$, then $f(U) \subset V$.

Theorem 5.5.7: Let $f: (X, \tau_i, \tau_j) \to (Y, \sigma_i, \sigma_j)$ be a function and each $x \in X$ and for each $\sigma_j$-open set $V$ containing $f(x)$, there is an $(\tau_i, \tau_j)$-swg*-open set $U$ containing $x$ such that $f(U) \subset V$. Then $f((\tau_i, \tau_j)$-swg* cl $(A)) \subset \tau_j$-cl $(f(A))$ for each subset $A$ of $X$.

Proof: Let $A$ be a subset of a space $X$ and $f(x) \not\in \tau_j$-cl $(f(A))$. Then there exists $\sigma_i$-open set $V$ of $Y$ containing $f(x)$ such that $V \cap f(A) = \emptyset$. Then for each $x \in X$ and for each $\sigma_j$-open set $V$ containing $f(x)$, there is an $(\tau_i, \tau_j)$-swg*-open set $U$ containing $x$ such that $f(U) \subset V$, so there is an $(\tau_i, \tau_j)$-swg*-open set $U$ such that $f(x) \in f(U) \subset V$. Hence $f(U) \cap f(A) = \emptyset$ implies $U \cap A = \emptyset$. Consequently, $x \not\in (\tau_i, \tau_j)$-swg*cl$(A)$ and $f(x) \not\in f((\tau_i, \tau_j)$-swg*cl$(A))$. 

88
Theorem 5.5.8: Let \( f : (X, \tau_i, \tau_j) \to (Y, \sigma_i, \sigma_j) \) be a function and \( f((\tau_i, \tau_j)\text{-swg } \ast \text{-cl} \,(A)) \subset \tau_j\text{-cl} \,(f(A)) \) for each subset \( A \) of \( X \) then \( (\tau_i, \tau_j)\text{-swg } \ast \text{-cl}(f^{-1}(B)) \subset f^{-1}(\tau_j\text{-cl} \,(B)) \) for each subset \( B \) of \( Y \).

Proof: Let \( B \) be a subset of \( Y \) and \( A = f^{-1}(B) \). Then \( f((\tau_i, \tau_j)\text{-swg } \ast \text{-cl}(A)) \subset \tau_j\text{-cl} \,(f(A)) \) for each subset \( A \) of \( X \), so \( ((\tau_i, \tau_j)\text{-swg } \ast \text{-cl}(f^{-1}(B)) \subset \tau_j\text{-cl} \,(f(f^{-1}(B))) \subset \tau_j\text{-cl} \,(B) \). Thus \( (\tau_i, \tau_j)\text{-swg } \ast \text{-cl}(f^{-1}(B)) \subset f^{-1}(\tau_j\text{-cl} \,(B)) \).

Theorem 5.5.9: If a map \( f : (X, \tau_i, \tau_j) \to (Y, \sigma_i, \sigma_j) \) is \( \tau_i\text{-closed} \) set and \( (\tau_j, \tau_i)\text{-swg} \ast \text{-irresolute} \), then for each \( (\tau_i, \tau_j)\text{-swg} \ast \text{-closed} \) set \( B \) of \( Y \), \( f^{-1}(B) \) is \( (\tau_i, \tau_j)\text{-swg} \ast \text{-closed} \) set in \( X \).

Proof: Let \( B \) be an \( (\tau_i, \tau_j)\text{-swg} \ast \text{-closed} \) subset of \( Y \) and \( f^{-1}(B) \subset U \), where \( U \) is a \( \tau_i\text{-open} \) set of \( X \). Since \( f \) is \( \tau_i\text{-closed} \) set, by lemma 5.5.5 there is a \( \sigma_i\text{-open} \) set \( V \) such that \( B \subset V \) and \( f^{-1}(V) \subset U \). Since \( B \) is \( (\tau_i, \tau_j)\text{-swg} \ast \text{-closed} \) set and \( B \subset V \), then \( (\tau_j, \tau_i)\text{-gcl}(B) \subset V \). Hence \( f^{-1}((\tau_j, \tau_i)\text{-gcl} \,(B)) \subset f^{-1}(V) \subset U \). By lemma 5.5.4 \( (\tau_j, \tau_i)\text{-gcl}(f^{-1}(B)) \subset U \) and hence \( f^{-1}(B) \) is \( (\tau_i, \tau_j)\text{-swg} \ast \text{-closed} \) set in \( X \).

Theorem 5.5.10: Let \( f : (X, \tau_i, \tau_j) \to (Y, \sigma_i, \sigma_j) \) and \( g : (Y, \sigma_i, \sigma_j) \to (Z, \upsilon_i, \upsilon_j) \) be two functions. Then:

(i) If \( f \) is \( (\tau_i, \tau_j)\text{-swg} \ast \text{-continuous} \) and \( g \) is \( \tau_j\text{-continuous} \), then \( g \circ f \) is \( (\tau_i, \tau_j)\text{-swg} \ast \text{-continuous} \).

(ii) If \( f \) is \( \tau_j\text{-swg} \ast \text{-irresolute} \), \( \tau_i\text{-closed} \) and \( g \) is \( (\tau_i, \tau_j)\text{-swg} \ast \text{-continuous} \), then \( g \circ f \) is \( (\tau_i, \tau_j)\text{-swg} \ast \text{-continuous} \).
Proof:

(i) Let \( W \) be a \( \nu_j \)-closed set of \( Z \). Since \( g \) is \( \sigma_j \)-continuous, then \( g^{-1}(W) \) is \( \sigma_j \)-closed set of \( Y \). Since \( f \) is \((\tau_i, \tau_j)\)-swg* continuous, then \( (g \circ f)^{-1}(W) = f^{-1}(g^{-1}(W)) \) is \((\tau_i, \tau_j)\)-swg*-closed set of \( X \). Hence \( g \circ f \) is \((\tau_i, \tau_j)\)-swg*-continuous.

(ii) Let \( W \) be a \( \nu_j \)-closed set of \( Z \). Since \( g \) is an \((\tau_i, \tau_j)\)-swg* continuous, then \( g^{-1}(W) \) is \((\tau_i, \tau_j)\)-swg* closed set of \( Y \). By theorem 5.5.6 \((g \circ f)^{-1}(W) = f^{-1}(g^{-1}(W)) \) is an \((\tau_i, \tau_j)\)-swg*-closed of \( X \). Hence \( g \circ f \) is \((\tau_i, \tau_j)\)swg* continuous.

Remark 5.5.11: \((\tau_i, \tau_j)\)-\( \sigma_i \)-semi continuous function and \((\tau_i, \tau_j)\)-\( \sigma_i \)-swg*-continuous function are independent.

Example 5.5.12: Let \((X, \tau_i, \tau_j)\) and \((Y, \sigma_i, \sigma_j)\) be two bitopological spaces. Let \( X = Y = \{a, b, c\} \) with the bitopologies \( \tau_i = \{X, \phi, \{a\}\}, \tau_j = \{X, \phi, \{a, b\}\} \) and \( \sigma_i = \{Y, \phi, \{b\}\}, \sigma_j = \{Y, \phi, \{c\}\} \). Define \( f : (X, \tau_i, \tau_j) \to (Y, \sigma_i, \sigma_j) \) by \( f(a) = c, f(b) = b, f(c) = a \). A function \( f \) is \((\tau_i, \tau_j)\)-\( \sigma_i \)-swg*-continuous which is not \((\tau_i, \tau_j)\)-\( \sigma_i \)-semi continuous function, since the inverse image of \( \sigma_i \)-closed set \( \{a, c\} \) in \( Y \) is not \((\tau_i, \tau_j)\)-semi closed in \( X \).

Example 5.5.13: Let \((X, \tau_i, \tau_j)\) and \((Y, \sigma_i, \sigma_j)\) be two bitopological spaces. Let \( X = Y = \{a, b, c\} \) with the bitopologies \( \tau_i = \{X, \phi, \{b, c\}\}, \tau_j = \{X, \phi, \{b\}, \{b, c\}\} \) and \( \sigma_i = \{Y, \phi, \{a, b\}\}, \sigma_j = \{Y, \phi, \{b\}\} \). Let \( f : (X, \tau_i, \tau_j) \to (Y, \sigma_i, \sigma_j) \) be defined by identity map. A function \( f \) is \((\tau_i, \tau_j)\)-\( \sigma_i \)-semi continuous which is not \((\tau_i, \tau_j)\)-\( \sigma_i \)-
swg*-continuous function, since the inverse image of \( \sigma_i \)-closed set \( \{c\} \) in \( Y \) is not \((\tau_i, \tau_j)\)-swg*-closed in \( X \).

**Remark 5.5.14:** \((\tau_i, \tau_j)\)-\(\sigma_i\)-pre-continuous function and \((\tau_i, \tau_j)\)-\(\sigma_i\)-swg*-continuous function are independent.

**Example 5.5.15:** Let \((X, \tau_i, \tau_j)\) and \((Y, \sigma_i, \sigma_j)\) be two bitopological spaces. Let \( X = Y = \{a, b, c\} \) with the bitopologies \( \tau_i = \{X, \phi, \{b\}\}, \tau_j = \{X, \phi, \{c\}\} \) and \( \sigma_i = \{Y, \phi, \{a, c\}\}, \sigma_j = \{Y, \phi, \{b\}, \{b, c\}\} \). Define \( f : (X, \tau_i, \tau_j) \to (Y, \sigma_i, \sigma_j) \) by \( f(a) = c, f(b) = b, f(c) = a \). A function \( f \) is \((\tau_i, \tau_j)\)-\(\sigma_i\)-swg*-continuous which is not \((\tau_i, \tau_j)\)-\(\sigma_i\)-pre-continuous function, since the inverse image of \( \sigma_i \)-closed set \( \{b\} \) in \( Y \) is not \((\tau_i, \tau_j)\)-swg*-closed in \( X \).

**Example 5.5.16:** Let \((X, \tau_i, \tau_j)\) and \((Y, \sigma_i, \sigma_j)\) be two bitopological spaces. Let \( X = Y = \{a, b, c\} \) with the bitopologies \( \tau_i = \{X, \phi, \{b, c\}\}, \tau_j = \{X, \phi, \{b\}, \{b, c\}\} \) and \( \sigma_i = \{Y, \phi, \{a, b\}\}, \sigma_j = \{Y, \phi, \{a\}\} \). Let \( f : (X, \tau_i, \tau_j) \to (Y, \sigma_i, \sigma_j) \) be defined by identity map. A function \( f \) is \((\tau_i, \tau_j)\)-\(\sigma_i\)-pre-continuous which is not \((\tau_i, \tau_j)\)-\(\sigma_i\)-swg*-continuous function, since the inverse image of \( \sigma_i \)-closed \( \{c\} \) in \( Y \) is not \((\tau_i, \tau_j)\) swg*-closed in \( X \).

**Remark 5.5.17:** \((\tau_i, \tau_j)\)-\(\sigma_i\)-\(\alpha\)-continuous function and \((\tau_i, \tau_j)\)-\(\sigma_i\)-swg*-continuous function are independent.
Example 5.5.18: Let \((X, \tau_i, \tau_j)\) and \((Y, \sigma_i, \sigma_j)\) be two bitopological spaces. Let 
\(X = Y = \{a, b, c\}\) with the bitopologies \(\tau_i = \{X, \phi, \{a\}\}, \tau_j = \{X, \phi, \{b\}\}\) and 
\(\sigma_i = \{Y, \phi, \{c\}\}, \sigma_j = \{Y, \phi, \{a, b\}\}\). Define \(f : (X, \tau_i, \tau_j) \rightarrow (Y, \sigma_i, \sigma_j)\) by \(f(a) = b, f(b) = a, f(c) = c\). A function \(f\) is \((\tau_i, \tau_j)\)-\(\sigma_i\)-\swg*-continuous which is not \((\tau_i, \tau_j)\)-\(\sigma_i\)-\(\alpha\)-continuous function, since the inverse image of \(\sigma_i\)-closed set \(\{a, b\}\) in \(Y\) is not \((\tau_i, \tau_j)\)-\(\alpha\)-closed in \(X\).

Example 5.5.19: Let \((X, \tau_i, \tau_j)\) and \((Y, \sigma_i, \sigma_j)\) be two bitopological spaces. Let 
\(X = Y = \{a, b, c\}\) with the bitopological \(\tau_i = \{X, \phi, \{b, c\}\}, \tau_j = \{X, \phi, \{b\}, \{b, c\}\}\) and 
\(\sigma_i = \{Y, \phi, \{a, b\}\}, \sigma_j = \{Y, \phi, \{a\}, \{a, b\}\}\). Define \(f : (X, \tau_i, \tau_j) \rightarrow (Y, \sigma_i, \sigma_j)\) by 
f(a) = b, f(b) = a, f(c) = c. A function \(f\) is \((\tau_i, \tau_j)\)-\(\sigma_i\)-\(\alpha\)-continuous which is not 
\((\tau_i, \tau_j)\)-\(\sigma_i\)-\swg*-continuous function, since the inverse image of \(\sigma_i\)-closed set \(\{c\}\) in \(Y\) is not 
\((\tau_i, \tau_j)\)-\swg*-closed in \(X\).

Remark 5.5.20: \((\tau_i, \tau_j)\)-\(\sigma_i\)-semi generalized continuous function and \((\tau_i, \tau_j)\)-\(\sigma_i\)-
\swg*-continuous function are independent.

Example 5.5.21: Let \((X, \tau_i, \tau_j)\) and \((Y, \sigma_i, \sigma_j)\) be two bitopological spaces. Let 
\(X = Y = \{a, b, c\}\) with the bitopologies \(\tau_i = \{X, \phi, \{a\}\}, \tau_j = \{X, \phi, \{b\}\}\) and 
\(\sigma_i = \{Y, \phi, \{c\}\}, \sigma_j = \{Y, \phi, \{a, b\}\}\). Define \(f : (X, \tau_i, \tau_j) \rightarrow (Y, \sigma_i, \sigma_j)\) by 
f(a) = b, f(b) = a, f(c) = c. A function \(f\) is \((\tau_i, \tau_j)\)-\(\sigma_i\)-\swg*-continuous which is not 
\((\tau_i, \tau_j)\)-\(\sigma_i\)-semi generalized continuous function, since the inverse image of 
\(\sigma_i\)-closed set \(\{a, b\}\) in \(Y\) is not \((\tau_i, \tau_j)\)-semi generalized closed in \(X\).
Example 5.5.22: Let \((X, \tau_i, \tau_j)\) and \((Y, \sigma_i, \sigma_j)\) be two bitopological spaces. Let

\[ X = Y = \{a, b, c\} \]

with the bitopologies \(\tau_i = \{X, \phi, \{b, c\}\}, \tau_j = \{X, \phi, \{b\}, \{b, c\}\}\) and \(\sigma_i = \{Y, \phi, \{a, b\}\}, \sigma_j = \{Y, \phi, \{a\}, \{a, b\}\}\). Define \(f : (X, \tau_i, \tau_j) \rightarrow (Y, \sigma_i, \sigma_j)\) by \(f(a) = b, f(b) = a, f(c) = c\). A function \(f\) is \((\tau_i, \tau_j)\)-\(\sigma_i\)-semi generalized continuous which is not \((\tau_i, \tau_j)\)-\(\sigma_i\)-swg*-continuous function, since the inverse image of \(\sigma_i\)-closed set \(\{c\}\) in \(Y\) is not \((\tau_i, \tau_j)\)-swg*-closed in \(X\).

Remark 5.5.23: \((\tau_i, \tau_j)\)-\(\sigma_i\)-generalized semi continuous function and \((\tau_i, \tau_j)\)-\(\sigma_i\)-swg*-continuous function are independent.

Example 5.5.24: Let \((X, \tau_i, \tau_j)\) and \((Y, \sigma_i, \sigma_j)\) be two bitopological spaces. Let

\[ X = Y = \{a, b, c\} \]

with the bitopologies \(\tau_i = \{X, \phi, \{b, c\}\}, \tau_j = \{X, \phi, \{b\}, \{b, c\}\}\) and \(\sigma_i = \{Y, \phi, \{a, b\}\}, \sigma_j = \{Y, \phi, \{a\}, \{a, b\}\}\). Define \(f : (X, \tau_i, \tau_j) \rightarrow (Y, \sigma_i, \sigma_j)\) by \(f(a) = b, f(b) = a, f(c) = c\). A function \(f\) is \((\tau_i, \tau_j)\)-\(\sigma_i\)-generalized semi continuous which is not \((\tau_i, \tau_j)\)-\(\sigma_i\)-swg*-continuous function, since the inverse image of \(\sigma_i\)-closed set \(\{c\}\) in \(Y\) is not \((\tau_i, \tau_j)\)-swg*-closed in \(X\).

Example 5.5.25: Let \((X, \tau_i, \tau_j)\) and \((Y, \sigma_i, \sigma_j)\) be two bitopological spaces. Let

\[ X = Y = \{a, b, c\} \]

with the bitopologies \(\tau_i = \{X, \phi, \{a, b\}\}, \tau_j = \{X, \phi, \{b, c\}\}\) and \(\sigma_i = \{Y, \phi, \{c\}\}, \sigma_j = \{Y, \phi, \{b\}, \{a, c\}\}\). Define \(f : (X, \tau_i, \tau_j) \rightarrow (Y, \sigma_i, \sigma_j)\) by \(f(a) = b, f(b) = a, f(c) = c\). A function \(f\) is \((\tau_i, \tau_j)\)-\(\sigma_i\)-swg*-continuous which is not \((\tau_i, \tau_j)\)-\(\sigma_i\)-generalized semi continuous function, since the inverse image of \(\sigma_i\)-closed set \(\{a, b\}\) in \(Y\) is not \((\tau_i, \tau_j)\)-generalized semi closed in \(X\).
Remark 5.5.26: $(\tau_i, \tau_j)$- $\sigma_i$-$\alpha$- generalized continuous function and $(\tau_i, \tau_j)$- $\sigma_i$-$\text{swg}^*$-continuous function are independent.

Example 5.5.27: Let $(X, \tau_i, \tau_j)$ and $(Y, \sigma_i, \sigma_j)$ be two bitopological spaces. Let $X = Y = \{a, b, c\}$ with the bitopologies $\tau_i = \{X, \phi, \{a, b\}\}$, $\tau_j = \{X, \phi, \{a, c\}\}$ and $\sigma_i = \{Y, \phi, \{c\}\}$, $\sigma_j = \{Y, \phi, \{b\}\}$. Define $f : (X, \tau_i, \tau_j) \to (Y, \sigma_i, \sigma_j)$ by $f(a) = b$, $f(b) = a$, $f(c) = c$. A function $f$ is $(\tau_i, \tau_j)$- $\sigma_i$-$\text{swg}^*$ continuous which is not $(\tau_i, \tau_j)$- $\sigma_i$-$\alpha$- generalized continuous function, since the inverse image of $\sigma_i$-closed set $\{a, b\}$ in $Y$ is not $(\tau_i, \tau_j)$-$\alpha$-generalized closed in $X$.

Example 5.5.28: Let $(X, \tau_i, \tau_j)$ and $(Y, \sigma_i, \sigma_j)$ be two bitopological spaces. Let $X = Y = \{a, b, c\}$ with the bitopologies $\tau_i = \{X, \phi, \{a\}\}$, $\tau_j = \{X, \phi, \{a, b\}\}$ and $\sigma_i = \{Y, \phi, \{a, c\}\}$, $\sigma_j = \{Y, \phi, \{c, \{a, c\}\}\}$. Define $f : (X, \tau_i, \tau_j) \to (Y, \sigma_i, \sigma_j)$ by $f(a) = c$, $f(b) = b$, $f(c) = a$. A function $f$ is $(\tau_i, \tau_j)$- $\sigma_i$-$\alpha$- generalized continuous which is not $(\tau_i, \tau_j)$- $\sigma_i$-$\text{swg}^*$-continuous function, since the inverse image of $\sigma_i$-closed set $\{b\}$ in $Y$ is not $(\tau_i, \tau_j)$-$\text{swg}^*$-closed in $X$.

Remark 5.5.29: $(\tau_i, \tau_j)$- $\sigma_i$- generalized $\alpha$- continuous function and $(\tau_i, \tau_j)$- $\sigma_i$-$\text{swg}^*$-continuous function are independent.
Example 5.5.30: Let \((X, \tau_i, \tau_j)\) and \((Y, \sigma_i, \sigma_j)\) be two bitopological spaces. Let \(X=Y= \{a, b, c\}\) with the bitopologies \(\tau_i=\{X, \phi, \{a\}\}\), \(\tau_j=\{X, \phi, \{c\}\}\) and \(\sigma_i=\{Y, \phi, \{b\}\}\), \(\sigma_j=\{Y, \phi, \{a, \{b, c\}\}\}\)\. Define \(f: (X, \tau_i, \tau_j) \rightarrow (Y, \sigma_i, \sigma_j)\) by \(f(a)=c, f(b)=b, f(c)=a\). A function \(f\) is \((\tau_i, \tau_j)\)-\(\sigma_i\)-\(\text{swg}^*\)-continuous which is not \((\tau_i, \tau_j)\)-\(\sigma_i\)-generalized \(\alpha\)-continuous function, since the inverse image of \(\sigma_i\)-closed set \(\{a, c\}\) in \(Y\) is not \((\tau_i, \tau_j)\)-generalized \(\alpha\)-closed in \(X\).

Example 5.5.31: Let \((X, \tau_i, \tau_j)\) and \((Y, \sigma_i, \sigma_j)\) be two bitopological spaces. Let \(X=Y= \{a, b, c\}\) with the bitopologies \(\tau_i=\{X, \phi, \{b, c\}\}\), \(\tau_j=\{X, \phi, \{b, c\}\}\) and \(\sigma_i=\{Y, \phi, \{a, b\}\}\), \(\sigma_j=\{Y, \phi, \{b, c\}, \{c\}\}\)\. Let \(f: (X, \tau_i, \tau_j) \rightarrow (Y, \sigma_i, \sigma_j)\) be defined by identity map. A function \(f\) is \((\tau_i, \tau_j)\)-\(\sigma_i\)-generalized \(\alpha\)-continuous which is not \((\tau_i, \tau_j)\)-\(\sigma_i\)-\(\text{swg}^*\)-continuous function, since the inverse image of \(\sigma_i\)-closed set \(\{c\}\) in \(Y\) is not \((\tau_i, \tau_j)\)-\(\text{swg}^*\)-closed in \(X\).
Remark 5.5.32: From the above results the following relation is obtained.

\[(\tau_i, \tau_j) \rightarrow \sigma_i\text{-semi continuous}\]

\[(\tau_i, \tau_j) \rightarrow \sigma_i\text{-semi generalized continuous}\]

\[(\tau_i, \tau_j) \rightarrow \sigma_i\text{-generalized semi continuous}\]

\[(\tau_i, \tau_j) \rightarrow \sigma_i\text{-semi weakly }g^*\text{-continuous}\]

\[(\tau_i, \tau_j) \rightarrow \sigma_i\text{-pre-continuous}\]

\[(\tau_i, \tau_j) \rightarrow \sigma_i\text{-}\alpha\text{-continuous}\]

\[(\tau_i, \tau_j) \rightarrow \sigma_i\text{-}\alpha\text{-generalized continuous}\]

\[(\tau_i, \tau_j) \rightarrow \sigma_i\text{-generalized }\alpha\text{-continuous}\]
5.6 (τ_i, τ_j)-SEMI WEAKLY g*-STRONGLY CONTINUOUS FUNCTIONS AND (τ_i, τ_j)-SEMI WEAKLY g*-IRRESOLUTE FUNCTIONS

In this section (τ_i, τ_j)-semi weakly g*-strongly continuous functions and (τ_i, τ_j)-semi weakly g*-irresolute functions are introduced and some of their properties are investigated.

Definition 5.6.1: A function f : (X, τ_i, τ_j) → (Y, σ_i, σ_j) is called (τ_i, τ_j)-semi weakly g*-strongly-continuous (briefly (τ_i, τ_j)-swg*-s-continuous) if f⁻¹ (V) is (τ_i, τ_j)-semi weakly g*-strongly-closed in X for every σ_j-closed set V of Y.

Definition 5.6.2: A function f : (X, τ_i, τ_j) → (Y, σ_i, σ_j) is called (τ_i, τ_j)-semi weakly g*-strongly-irresolute (briefly (τ_i, τ_j)-swg*-s-irresolute) if f⁻¹ (V) is (τ_i, τ_j)-semi weakly g*-strongly-closed in X for every semi weakly g*-strongly-closed set V of Y.

Definition 5.6.3: A subset A of a bitopological space X is called (τ_i, τ_j)-semi weakly g*-closed set (briefly (τ_i, τ_j)-swg*-closed) if τ_j-gcl (A) ⊂ U whenever A ⊂ U and U is τ_i-semi open in X. If A ⊂ X is (τ_i, τ_j)-swg*-closed and (τ_j, τ_i)-swg*-closed, then it is said to be (τ_i, τ_j)-semi weakly g*-strongly closed (briefly (τ_i, τ_j)-swg*-s-closed) set.

Lemma 5.6.4: If a function f : (X, τ_i, τ_j) → (Y, σ_i, σ_j) is an τ_i-closed, then for each subset S ⊂ Y and each τ_i-open set U containing f⁻¹ (S), there is a σ_i-open set V containing S such that f⁻¹ (V) ⊂ U.
Proof: Let $S \subseteq Y$ and $U$ is $\tau_i$-open containing $f^{-1}(S)$, Put $V = Y - f(X - U)$. Then $U$ is $\sigma_i$-open set in $Y$ containing $S$. It follows that $f^{-1}(V) \subset U$.

Lemma 5.6.5: If a function $f : (X, \tau_i, \tau_j) \to (Y, \sigma_i, \sigma_j)$ is surjective $\tau_j$-continuous, then for every subset $B$ of $Y$, $\tau_j$-cl $(f^{-1}(B)) \subset f^{-1}(\tau_j$-cl$(B))$.

Proof: Let $x \in \tau_j$-cl$(f^{-1}(B))$. Suppose that $V$ be $\tau_j$-open set of $Y$ containing $f(x)$, i.e. $f(x) \in V$, then $x \in f^{-1}(V)$. Since $f^{-1}(V)$ is $\tau_j$-open of $X$, then $f^{-1}(V) \cap f^{-1}(B) \neq \emptyset$. This implies that $f^{-1}(V \cap B) \neq \emptyset$ and $V \cap B \neq \emptyset$. Thus $f(x) \in \tau_j$-cl$(B)$ and $x \in f^{-1}(\tau_j$-cl$(B))$. This means $x \in f^{-1}(\tau_j$-cl$(B))$. Hence $\tau_j$-cl$(f^{-1}(B)) \subset f^{-1}(\tau_j$-cl$(B))$.

Theorem 5.6.6: Let $f : (X, \tau_i, \tau_j) \to (Y, \sigma_i, \sigma_j)$ be a function and $f$ is $(\tau_i, \tau_j)$-semi weakly $g^*$-strongly continuous, then for each $x \in X$. Also $\sigma_j$-open set $V$ containing $f(x)$, there is an $(\tau_i, \tau_j)$-swg*-strongly open set $U$ containing $x$ such that $f(U) \subset V$.

Proof: Let $x \in X$ and $V$ be $\sigma_j$-open set containing $f(x)$. Then $f$ is $(\tau_i, \tau_j)$-semi weakly $g^*$-strongly-continuous, so $f^{-1}(V)$ is $(\tau_i, \tau_j)$-semi weakly $g^*$-strongly-open set of $X$ which containing $X$. If $U = f^{-1}(V)$ then $f(U) \subset V$.

Theorem 5.6.7: Let $f : (X, \tau_i, \tau_j) \to (Y, \sigma_i, \sigma_j)$ be a function and for each $x \in X$. Also $\sigma_i$-open set $V$ containing $f(x)$, there is an $(\tau_i, \tau_j)$-swg*-strongly open set $U$ containing $x$ such that $f(U) \subset V$, then $f((\tau_i, \tau_j)$-swg*-s-cl$(A)) \subset \tau_j$-cl$(f(A))$ for each subset $A$ of $X$. 

98
Proof: Let $A$ be a subset of a bitopological space $X$ and $f(x) \not\in \tau_j\text{-cl}(f(A))$, then there exists $\sigma_j$-open set $V$ of $Y$ containing $f(x)$ such that $V \cap f(A) = \emptyset$. Then each $x \in X$ and for each $\sigma_j$-open set $V$ containing $f(x)$. There is an $(\tau_i, \tau_j)$-swg*-strongly open set $U$ containing $x$ such that $f(U) \subseteq V$. Hence $f(U) \cap f(A) = \emptyset$. Implies $U \cap A = \emptyset$. Consequently, $x \not\in (\tau_i, \tau_j)$-semi weakly g*-strongly cl(A) and $f(x) \not\in f((\tau_i, \tau_j)$-semi weakly g*-strongly -cl(A)). Implies $f ((\tau_i, \tau_j)$-swg*-s cl(A)) $\subset \tau_j$-cl(f(A)) for each subset $A$ of $X$.

**Theorem 5.6.8:** Let $f : (X, \tau_i, \tau_j) \to (Y, \sigma_i, \sigma_j)$ be a function and $f ((\tau_i, \tau_j)$-swg*-s-cl(A)) $\subset \tau_j$-cl(f(A)) for each subset $A$ of $X$, then $((\tau_i, \tau_j)$-swg*-s-cl(f(A))) $\subset f^{-1}(\tau_j$-cl(B)) for each subset $B$ of $Y$.

**Proof:** Let $B$ be a subset of $Y$ and $A=f^{-1}(B).$ Then $f(\tau_i, \tau_j)$ swg*-s cl(A)) $\subset \tau_j$-cl(f(A)) for each subset $A$ of $X$. Therefore $((\tau_i, \tau_j)$-semi weakly g*-strongly-cl (f $^{-1}(B)))$ $\subset \tau_j$-cl (f(f $^{-1}(B))).$ Thus $(\tau_i, \tau_j)$-strongly semi weakly g*-cl(f $^{-1}(B))$ $\subset f^{-1}(\tau_j$-cl(B)).

**Theorem 5.6.9:** If $f : (X, \tau_i, \tau_j) \to (Y, \sigma_i, \sigma_j)$ is bijectives $(\tau_i, \tau_j)$-semi weakly g*-open and $(\tau_i, \tau_j)$-semi weakly g*-strongly -continuous, then $f$ is $(\tau_i, \tau_j)$-semi weakly g*-strongly -irresolute.
**Proof:** Let $V$ be $(\tau_i, \tau_j)$-semi weakly $g^*$-strongly closed set of $Y$ and let $f^{-1}(V) \subseteq U$, where $U$ be $(\tau_i, \tau_j)$-semi weakly $g^*$-open set. Clearly $V \subseteq f(U)$. Since $f(U)$ is $(\tau_i, \tau_j)$-semi weakly $g^*$-open and $V$ is $(\tau_i, \tau_j)$-semi weakly $g^*$-strongly closed set in $Y$. Then $\tau_j\text{-cl}(V) \subseteq f(U)$ and $f^{-1}(\tau_j\text{-cl}(V)) \subseteq U$. Since $f$ is $(\tau_i, \tau_j)$-semi weakly $g^*$-continuous and $\tau_j\text{-cl}(V)$ is $\sigma_j$-closed in $Y$, then $\tau_j\text{-cl}(f^{-1}(\tau_j\text{-cl}(V))) \subseteq U$. Hence $\tau_j\text{cl}(f^{-1}(V)) \subseteq U$. Therefore $f^{-1}(V)$ is $(\tau_i, \tau_j)$-semi weakly $g^*$-strongly closed in $X$. Hence $f$ is $(\tau_i, \tau_j)$-semi weakly $g^*$-strongly irresolute.

**Theorem 5.6.10:** If $f : (X, \tau_i, \tau_j) \rightarrow (Y, \sigma_i, \sigma_j)$ is $(\tau_i, \tau_j)$-semi weakly $g^*$-irresolute and $\tau_j$-closed, then every $(\tau_i, \tau_j)$-semi weakly $g^*$-strongly closed set $A$ of $X$, $f(A)$ is $(\tau_i, \tau_j)$-semi weakly $g^*$-strongly closed set of $Y$.

**Proof:** Let $A$ be an $(\tau_i, \tau_j)$-semi weakly $g^*$-strongly closed set. Suppose that $f(A) \subseteq U$, where $U$ is an $(\tau_i, \tau_j)$-semi weakly $g^*$-open in $Y$. Then $A \subseteq f^{-1}(U)$ and $f^{-1}(U)$ is $(\tau_i, \tau_j)$-semi weakly $g^*$-open and $f$ is $(\tau_i, \tau_j)$-semi weakly $g^*$-irresolute function. Since $A$ is semi weakly $g^*$-strongly closed, $\tau_j\text{-cl}(A) \subseteq f^{-1}(U)$ and hence $f(\tau_j\text{-cl}(A)) \subseteq U$. Therefore $\tau_j\text{-cl}(f(A)) \subseteq \tau_j\text{-cl}(f(\tau_j\text{-cl}(A))) = f(\tau_j\text{-cl}(A)) \subseteq U$ as $f$ is $\tau_j$-closed. Hence $f(A)$ is $(\tau_i, \tau_j)$-semi weakly $g^*$-strongly closed set in $Y$.

**Theorem 5.6.11:** If $f : (X, \tau_i, \tau_j) \rightarrow (Y, \sigma_i, \sigma_j)$ is surjective, $\tau_j$-closed and $\tau_j$-continuous, then for every $(\tau_i, \tau_j)$-semi weakly $g^*$-strongly closed set $B$ of $Y$, $f^{-1}(B)$ is $(\tau_i, \tau_j)$-semi weakly $g^*$-strongly closed set in $X$. 

100
Proof: Let $B$ be an $(\tau_i, \tau_j)$-semi weakly $g^*$-strongly closed subset of $Y$ and $f^{-1}(B) \subset U$, where $U$ is a $\tau_j$-open set of $X$. As $f$ is $\tau_j$-closed and by lemma 5.6.4, there is a $\sigma_j$-open set $V$ such that $B \subset V$ and $f^{-1}(V) \subset U$. Since $B$ is $(\tau_i, \tau_j)$-semi weakly $g^*$-strongly closed set and $B \subset V$. Then $\tau_j$-cl($B$) $\subset V$. Hence $f^{-1}(\tau_j$-cl($B$)) $\subset f^{-1}(V) \subset U$. By Lemma 5.6.5, $\tau_j$-cl($f^{-1}(B)$) $\subset U$ and hence $f^{-1}(B)$ is $(\tau_i, \tau_j)$-semi weakly $g^*$ - strongly closed set in $X$, since every $\tau_j$-open set is $(\tau_i, \tau_j)$-semi weakly $g^*$-open set.

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