CHAPTER 4

CONTRA SEMI WEAKLY g*-CONTINUOUS FUNCTIONS IN TOPOLOGICAL SPACES

4.1 INTRODUCTION

Dontchev [27] introduced contra continuous function in topological spaces. Contra semi continuous functions introduced and investigated by Dontchev and Noiri [28]. Jafari and Noiri [42,43] introduced contra α-continuous functions and pre continuous functions in topological spaces. Balachandran et.al [9] introduced and studied generalized continuous functions and sg-continuous functions. Mashhour [66] introduced the notions of pre-open set and pre- continuous function in topological spaces. Abd. El. Monsef [1] introduced and discussed β-continuous functions. Devi et.al [19] defined and investigated gs-continuous funtions. Dontchev [26] defined gsp- continuous functions. In this chapter contra swg*- continuous functions is introduced and some of their properties are investigated.

4.2 CONTRA SEMI WEAKLY g*- CONTINUOUS FUNCTIONS

In this section contra semi weakly g*-continuous functions is introduced and study some of their properties.

Definition 4.2.1: Let (X, τ) and (Y, σ) be any two topological spaces. A map $f : (X, \tau) \rightarrow (Y, \sigma)$ is called a contra semi weakly g*-continuous function, if the inverse image of every open set in (Y, σ) is swg*- closed set in (X, τ).

Theorem 4.2.2 : If a function $f : X \to Y$ is contra swg*-continuous function and Y is regular space then f is swg*-continuous function.

Proof: Let X be an arbitrary point of x and let V be an open set of Y containing f(x). Since Y is regular space, there exist an open set W in Y containing f(x), such that $cl(W) \subseteq V$. Since f is contra swg*-continuous function, there exists $U \in SWG^*O(X,x)$ such that $f(U) \subseteq cl(W)$. Then $f(U) \subseteq cl(W) \subseteq V$. Hence f is swg*- continuous function.

Theorem 4.2.3 : If a function $f : X \to Y$ is contra continuous function then f is contra swg*- continuous function.

Proof : Let V be an open set in Y. Since f is contra continuous function,$f^{-1}(V)$ is closed set in X. By theorem 2.2.6, $f^{-1}(V)$ is swg*- closed set in X. Hence f is contra swg*- continuous function.

Definition 4.2.4: Let A be a sub set of a topological space (X, τ).

(i) The set $\cap \{F \subset X, A \subseteq F$, F is swg* - closed set$\}$ is called the swg* closure of A and it is denoted by $cl_{swg*}(A)$.

(ii)The set $\cup \{ F \subset X, F \subseteq A$, F is swg*- open set$\}$ is called swg*- interior of A and it is denoted by $int_{swg*}(A)$.

Theorem 4.2.5: Let A be a subset of (X, τ).

(a) If A is swg*- closed set then gcl(A)-A does not contain any non-empty semi-closed set.

(b) If A is swg*-closed set and $A \subseteq B \subseteq gcl(A)$, then B is swg*- closed set.

Proof:

(a) Suppose that A is swg*- closed set and let F be a non - empty semi -closed set with $F \subseteq gcl(A)-A$. Then $A \subseteq X - F$ and $gcl(A) \subseteq X - F$. Hence $F \subseteq X-gcl(A)$. Thus it is a contradiction. Hence gcl(A) - A does not contain any non-empty semi - closed set.

(b) Let U be a semi - open set of (X, τ) such that $B \subseteq U$. Then $A \subseteq U$ since A is swg*- closed set and $gcl(A) \subseteq U$. Since A is swg*- closed set $gcl(A) \subseteq U$. Now $gcl(B) \subseteq gcl(gcl(A)) \subseteq U$.Therefore B is also a swg*- closed set of (X, τ).

Theorem 4.2.6: For a function $f : (X, \tau) \rightarrow (Y, \sigma)$ the following conditions are equivalent :

(a) f is contra swg* - continuous function;

(b) For every closed subset F of Y, $f^{-1}(F) \in SWG^*O(X)$;

(c) For each $x \in X$ and each $F \in C(Y,f(x))$ there exists $U \in SWG^*O(X,x)$, such that $f(U) \subseteq F$;

(d) $F(cl_{swg^*}(A)) \subseteq ker(f(A))$ for every sub set A of X;

(e) $cl_{swg^*}(f^{-1}(B)) \in f^{-1}(ker(B))$ for every subset B of Y.

Proof :

(a) $\Rightarrow$ (b)

Let f be a contra swg*-continuous function. By definition 4.2.1, every open subset F of Y is $f^{-1}(F) \in SWG^*C(X)$. Also every closed subset F of Y is $f^{-1}(F) \in SWG^*O(X)$.

(b) $\Rightarrow$(c)

Let F be any closed set of Y and X $\in$ f $^{-1}$(F). Then f(x) $\in$ F and there exists $U_x \in$ swg*O (X,x) such that f (U_x) $\subseteq$ F. Therefore f $^{-1}$(F)= $\cup\{U_x$: X$\in$f $^{-1}$(F)$\}$ and f $^{-1}$(F) is swg*- open set.

(c) $\Rightarrow$(d)

Let A be any subset of X. Suppose that Y $\notin$ ker (f (A)). Then by result 1.7.8, there exists F $\in$ C (Y, f(x)) such that f(A) $\cap$ F = ϕ. Therefore A $\cap$ f $^{-1}$(F) = ϕ and since f $^{-1}$ (F) is swg*-open set then cl_{swg*}(A) $\cap$ f $^{-1}$(F) = ϕ. Therefore f(cl_{swg*}(A) $\cap$ F = ϕ and Y$\notin$f (cl_{swg*}(A)). This implies f (cl_{swg*}(A)) $\subseteq$ ker(f (A)).

(d) $\Rightarrow$ (e)

Let B be any subset of Y. Then by result 1.7.8, f(cl_{swg*}(f $^{-1}$(B)) $\subseteq$ker(f(f $^{-1}$(B)) $\subseteq$ kerB. Thus cl_{swg*} (f $^{-1}$(B) $\subseteq$ f^{-1}(ker (B)).

(e) $\Rightarrow$ (a)

Let B be any open set of Y.Then by result 1.7.8, cl_{swg*} (f $^{-1}$(B)) $\subseteq$f^{-1}(ker (B))=f $^{-1}$(B). This implies f $^{-1}$(B) is swg*- closed in X. Hence f is contra swg*- continuous function.

Theorem 4.2.7:Let f : X $\rightarrow$ Y be a function then the following are equivalent.

(a)The function f is swg*-continuous function ;

(b)For each point x$\in$X and each open set V of Y with f(x) $\in$ V, there exists a swg*-open set U of X, such that x$\in$ U and f(U) $\subset$ V.

Proof : (a) $\Rightarrow$(b)

Let $f(x) \in V$. Then $x \in f^{-1}(V) \in SWG^*O(X)$, since f is swg*-continuous function. Let $U = f^{-1}(V)$, then $x \in X$ and $f(U) \subset V$.

(b)$\Rightarrow$(a)

Let V be an open set of Y and let $x \in f^{-1}(V)$. Implies $f(x) \in V$. Then $f(x) \in V$ and thus there exists swg*- open set U_x of X such that $x \in U_x$ and $f(U) \subset V$. Now $x \in U_x \subset f^{-1}(V)$ and $f^{-1}(V) = \cup U_x$. Then $f^{-1}(V)$ is swg*-open set in X. Therefore f is swg*-continuous function.

Definition 4.2.8 : Let (X, τ) and (Y, σ) be the topological spaces. A function $f : X \rightarrow Y$ is called almost swg*- continuous function if for each point $x \in X$ and each open set V of Y containing f(x) there exists $U \in SWG^*O(X,x)$ such that $f(U) \subseteq int_{swg^*}(cl(V))$.

Theorem 4.2.9: A function $f : X \rightarrow Y$ is almost swg*- continuous function if and only if for each point $x \in X$ and each open set V of Y containing f(x), there exists $U \in SWG^*O(X,x)$ such that $f(U) \subseteq V$.

Proof: Let V be open set of Y containing f(x) for each $x \in X$. This implies V is an open set of Y containing f(x) for each $x \in X$. Since f is almost swg*-continuous function there exist $U \in SWG^*O(X,x)$ such that $f(U) \subseteq int_{swg^*}(cl(V))$.

Conversely, if for each $x \in X$ and each open set V of Y containing f(x), there exists $U \in$ SWG* O (X,x) such that $f(U) \subseteq V$. This implies V is an open set of Y containing f(x), there exists $U \in$ SWG*O (X,x) such that $f(U) \subseteq V = int_{swg*}(cl(V))$. Therefore f is almost swg*- continuous function.

Definition 4.2.10: A function $f : X \rightarrow Y$ is said to be pre swg*- open set, if the image of each swg*- open set in Y is swg*- open set in X.

Theorem 4.2.11: If a function $f : X \rightarrow Y$ is a pre swg* - open set and contra swg*- continuous function then f is almost swg*- continuous function.

Proof: Let x be any arbitrary point of X and V be an open set containing f (x). Since f is contra swg*- continuous function then by theorem 4.2.6(c), there exists $V \in$ SWG*O (X,x) such that $f(U) \subseteq cl(v)$. Since f is pre swg*- open set, f (U) is swg*- open set in Y. Therefore, $f(U) = int_{swg*}f(U) \subseteq int_{swg*}(cl(f(U)) \subseteq int_{swg*}(cl(V))$. This shows that f is almost swg*- continuous function.

Definition 4.2.12 : The swg*- frontier of A of a topological space (X, τ) denoted by $Fr_{swg*}(A)$ is defined by $Fr_{swg*}(A) = cl_{swg*}(A) \cap cl_{swg*}(X - A)$.

Theorem 4.2.13: If $K=\{x : X, V \cap U \neq \phi, U \subseteq X\}$ for every swg*-open set V containing x, then $cl_{swg*}(U) = K$.

Proof : Let $x \in K \Leftrightarrow V \cap U \neq \phi$, $x \in V$, V is a swg*-open set,

$\Leftrightarrow x \in V$ or every swg*-open set containing x contains a pointof U other then X,

$\Leftrightarrow x \in cl_{swg*}(U)$.

Theorem 4.2.14 : The set of all points x of X at which $f : X \to Y$ is not contra swg*- continuous function is identical with the union of the swg*- frontier of the inverse image of closed sets of Y containing f (x).

Proof: Suppose f is contra swg*- continuous function at $x \in X$. There exists $F \in C(Y, f(x))$ such that $f(U) \cap (Y - F) \neq \phi$ for every $U \in SWG^*O (X,x)$. Implies $U \cap f^{-1}(Y-F) \neq \phi$. Therefore $x \in cl_{swg^*} (f^{-1}(Y - F)) = cl_{swg^*}(X- f^{-1}(F))$. However $x \in cl_{swg^*} (f^{-1} (F)) \cap cl_{swg^*}(f^{-1}(Y-F))$. Therefore $x \in Fr_{swg^*} (f^{-1} (F))$. Suppose that $x \in Fr_{swg^*}(f^{-1}(F))$ for some $F \in C (Y, f (x))$ and f is contra swg*-continuous function at x, then there exists $U \in SWG^*O (X,x)$ such that $f(U) \subseteq F$. Therefore $x \in U \subseteq f^{-1}(F)$ and hence $x \in int_{swg^*}(f^{-1}(F)) \subseteq X - Fr_{swg^*}(f^{-1}(F))$. This is a contradiction. Hence f is not contra swg*-continuous function.

Theorem 4.2.15: Let $(X_{\lambda}, \lambda \in \Lambda)$ be any family of topological space. If $f: X \to \pi X_{\lambda}$ is a contra swg*-continuous function then $P_{\lambda 0} f : X \to X_{\lambda}$ is a contra swg*-continuous function for each $\lambda \in \Lambda$, where P_{λ} is the projection of πX_{λ} on to X_{λ}.

Proof : Consider a fixed $\lambda \in \Lambda$. Suppose U_{λ} is an arbitrary open set in X_{λ}. Then $P_{\lambda}^{-1}(U_{\lambda})$ is open set in πX_{λ}. Since f is contra swg*- continuous function, (by definition 4.2.1) $f^{-1}(P_{\lambda}^{-1} (U_{\lambda})) =(P_{\lambda 0}f)^{-1} (U_{\lambda})$ is swg*- closed set in X. Therefore $P_{\lambda 0} f$ is contra swg*- continuous function.

Theorem 4.2.16: If $f : X \to Y$ be surjective swg*- irresolute function and pre swg*-open set and $g : Y \to Z$ be any function and if $g \circ f : X \to Z$ is contra swg*-continuous function, then g is contra swg*- continuous function.

Proof: Let $g \circ f : X \to Z$ is contra swg*- continuous function and let F be a closed subset of Z. Then $(g \circ f)^{-1}(F)$ is a swg*- open set of X. Implies $f^{-1}(g^{-1}(F))$ is an swg*-open subset of X. Since f is pre swg*- open set $f(f^{-1}(g^{-1}(F))$ is swg*-open subset of Y. Thus $g^{-1}(F)$ is an swg*-open set in Y. Hence g is contra swg* continuous function.

Definition 4.2.17 : The graph Gr (f) of a function $f : X \to Y$ is said to be contra swg*-closed set if for each $(x,y) \in (X,Y)$ - Gr (f), there exists $U \in SWG^*O(X,x)$ and $V \in C(Y,y)$ such that $(UxV) \cap Gr(f) = \phi$. Symbolically f is C swg*- closed set in the product space X xY.

Definition 4.2.18: The Graph Gr(f) of a function $f : X \to Y$ is Cswg*- closed in XxY if and only if for each $(x,y) \in (XxY)$ - Gr(f), there exists $U \in SWG^*O(X,x)$ and $V \in (Y,y)$ such that $f(U) \cap V = \phi$.

Theorem 4.2.19: If $f : X \to Y$ is contra swg*- continuous function and Y is Urysohn, then f is Cswg*-closed set in the product space XxY.

Proof : Let $(x, y) \in (XxY)$ - Gr(f), then $y \neq f(x)$ and there exists open sets H_1 and H_2 such that $f(x) \in H_1$, $y \in H_2$ and $cl(H_1) \cap cl(H_2) = \phi$. From definition 4.2.18, there exists $V \in SWG^*O(X,x)$ such that $f(V) \subseteq cl(H_1)$. Therefore $f(V) \cap cl(H_2) = \phi$. This implies f is Cswg*- closed set.

Theorem 4.2.20: If $f : X \to Y$ and $g : X \to Y$ are contra swg*-continuous function and Y is Urysohn then $K = \{x \in X , f(x) = g(x)\}$ is swg*- closed set in X.

Proof : Let $x \in X - K$, then $f(x) \neq g(x)$. Since Y is Urysohn, there exists open sets U and V such that $f(x) \in U$, $g(x) \subset V$ and $cl(U) \cap cl(V) = \phi$. Since f and g are contra swg*- continuous function, $f^{-1}(cl(U)) \in SWG^*O(X)$ and $g^{-1}(cl(V)) \in SWG^*O(X)$. Let $A = f^{-1}(cl(U))$ and $B = f^{-1}(cl(V))$, then A and B contains X. Let $C = A \cap B$, then C is swg*- open set in X. Hence $f(C) \cap g(C) = \phi$ and $x \notin cl_{swg^*}(K)$. Thus K is swg*- closed set in X.

Theorem 4.2.21 : Let $f : X \rightarrow Y$ be a function and let $g : X \rightarrow XxY$ be the graph function of f, defined by $g(x)= (x, f(x))$ for every $x \in X$. If g is contra swg*- continuous function, then f is contra swg*-continuous function.

Proof : Let U be an open set in Y then XxU is an open set in XxY. Since g is contra swg*- continuous function, it follows that $f^{-1}(U) = g^{-1}(XxU)$ is an swg* closed set in X. Thus f is contra swg*- continuous function.

Theorem 4.2.22 : If $f : X \rightarrow Y$ is swg*- continuous function and Y is T_1 -space, then f is Cswg*- closed set in XxY.

Proof: Let $(x,y) \in (XxY- Gr(f)$. Then $f(x) \neq y$ and there exists an open set V of Y such that $f(x) \in V$ and $y \notin V$. Since f is swg* - continuous function there exists $U \in SWG^*O(X,x)$ such that $f(U) \subseteq V$. Therefore $f(U) \cap (Y-V) = \phi$ and $Y-U \in C(Y,y)$. Hence f is Cswg*- closed set in XxY.

Definition 4.2.23:

(a) A topological space X is said to be swg*- T_1 if for each pair of distinct points x and y in X, there exists swg*- open sets U and V containing x and y respectively such that $y \notin U$ and $x \notin V$.

(b) A topological space X is said to be swg*-T_2 if for each pair of distinct points x and y in X, there exists swg*-open sets U and V containing x and y respectively such that $U \cap V = \phi$.

Theorem 4.2.24: Let X be a topological space and for each pair of distinct points x and y in X, there exists a map f of X in to a urysohn topological space Y such that $f(x) \neq f(y)$ and f is contra swg*-continuous function at x and y, then X is swg*- T_2.

Proof: Let x and y be any two distinct points in X. Then there exists urysohn space Y and a function $f : X \to Y$ such that $f(x) \neq f(y)$. Also f is contra swg*- continuous function at x and y. Let $a = f(x)$ and $b = f(y)$, such that $a \neq b$. Since Y is urysohn, there exists open sets V and W containing a and b, respectively such that $cl(V) \cap cl(W) = \phi$. Since f is contra swg*- continuous function at x and y, there exists swg*-open sets A and B containing a and b respectively such that $f(A) \subseteq cl(V)$ and $f(B) \subseteq cl(W)$. Then $f(A) \cap f(B) = \phi$. Implies $A \cap B = \phi$. Hence X is swg*- T_2.

Theorem 4.2.25: If $f : X \to Y$ is a contra swg*-continuous injection and Y is weakly hausdorff space then X is swg* - T_1.

Proof: Suppose that Y is weakly hausdorff space. For any distinct points x_1 and x_2 in X, there exists closed sets U and V in Y, such that $f(x_1) \in U$, $f(x_2) \notin U$, $f(x_1) \notin V$ and $f(x_2) \in V$. Since f is contra swg*- continuous function $f^{-1}(U)$ and $f^{-1}(V)$ are swg*-open set, subsets of X, such that $x_1 \in f^{-1}(U)$, $x_2 \notin f^{-1}(U)$, $x_1 \notin f^{-1}(V)$ and $x_2 \in f^{-1}(V)$. Hence X is swg*- T_1.

Theorem 4.2.26: Let $f : X \rightarrow Y$ be a swg*-closed graph. If f is injective then X is swg*- T_1.

Proof: Let x_1 and x_2 be any two distinct points of X. Then $(x_1,f(x_2)) \in (XxY)-G(f)$, there exist a swg*- open set U in X containing x_1 and $F \in C(Y, f(x_2))$ such that $f(U) \cap F = \phi$. Hence $U \cap f^{-1}(F) = \phi$. Therefore $x_2 \notin U$. This implies that x is swg*-T_1

Definition 4.2.27: A topological space X is said to be ultra hausdorff, if for each pair of distinct points x and y in X, there exists clopen sets A and B containing x and y respectively such that $A \cap B = \phi$.

Theorem 4.2.28 : Let $f : X \rightarrow Y$ be a contra swg*- continuous function injection. If Y is ultra hausdorff space, then X is swg*- T_2.

Proof: Let x_1 and x_2 be any two distinct points of X , then $f(x_1) \neq f(x_2)$ and there exist clopen sets U and V containing $f(x_1)$ and $f(x_2)$ respectively such that $U \cap V = \phi$. Since f is contra swg*- continuous function, then $f^{-1}(U) \in SWG^* O(X)$ and $f^{-1}(V) \in SWG^* O(X)$ such that $f^{-1}(U) \cap f^{-1}(V) = \phi$. Hence X is swg*-$T_2$.

Remark 4.2.29 : The following examples show that the contra swg*-continuous function and the contra α-continuous function are independent.

Example 4.2.30: Let $X = Y = \{a, b, c\}$ with the topologies $\tau = \{X, \phi, \{b\}\}$, $\sigma = \{Y, \phi, \{a\}, \{b\}, \{a, b\}\}$ and $f : X \to Y$ be the identity map. This function f is contra swg*- continuous function which is not contra α-continuous function , since the inverse image of the open set {b} in Y is not α - closed set in X.

Example 4.2.31: Let $X = Y = \{a, b, c\}$ with the topologies $\tau = \{X, \phi, \{a\}, \{a, c\}\}$ and $\sigma = \{Y, \phi, \{c\}\}$. The map $f : X \to Y$ is defined by f(a) = b, f(b) = a, f(c) = c. This function f is contra α- continuous function which is not contra swg*- continuous function, since the inverse image of the open set {c} in Y is not swg*- closed set in X.

Remark 4.2.32: The following examples show that contra swg*- continuous function and contra semi continuous function are independent.

Example 4.2.33: Let $X = Y = \{a, b, c\}$ with the topologies $\tau = \{X, \phi, \{a\}\}$ and $\sigma = \{Y, \phi, \{a, c\}\}$. The map $f : X \to Y$ is defined by f(a)= c, f(b) = b, f(c) = a . This f is contra swg*- continuous function which is not contra semi continuous function , since the inverse image of the open set {a, c} in Y is not semi closed set in X.

Example 4.2.34: Let $X = Y = \{a, b, c\}$ with the topologies $\tau = \{X, \phi, \{a\}, \{a, b\}\}$ and $\sigma = \{Y, \phi, \{b\}\}$. The map $f : X \to Y$ is defined by f(a)= c, f(b) =b, f(c) = a. This function f is contra semi continuous function which is not contra swg*-

continuous function, since the inverse image of the open set {b} in Y is not swg*-closed set in X.

Remark 4.2.35 : The following examples show that contra swg*- continuous function and contra pre-continuous function are independent.

Example 4.2.36: Let X = Y = {a, b, c} with the topologies τ = {X, ϕ,{a},{a, c} } and σ= {Y, ϕ, {a, b}}. The map f : X$\rightarrow$ Y is defined by f(a) = b, f(b) = a, f(c) = c. The function f is contra swg* - continuous function which is not contra pre-continuous function, since the inverse image of the open set {a, b} in Y is not pre-closed set in X.

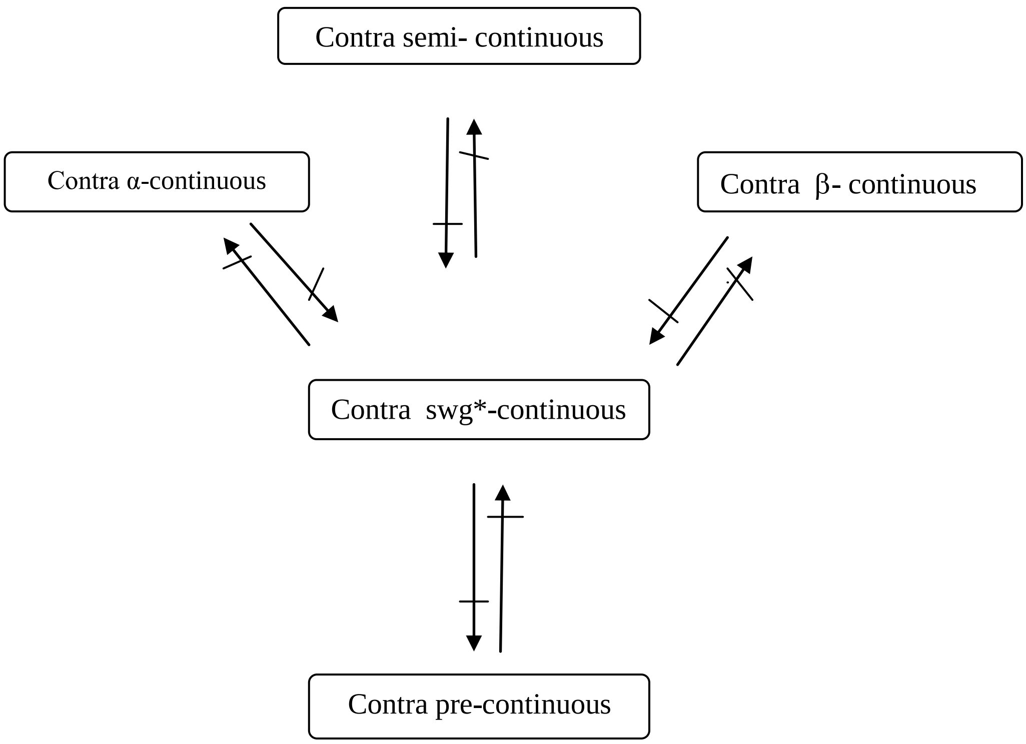
Example 4.2.37: Let X = Y ={a, b, c} with the topologies τ = {X, ϕ, {a}, {a, b}} and σ = {Y, ϕ, {b, c}, {b}}. The map f : X$\rightarrow$ Y is defined by f(a) = c, f(b) = b, f(c) = a. This function f is contra pre-continuous function which is not contra swg*-continuous function, since the inverse image of the open set {b} in Y is not swg*-closed set in X .

Remark 4.2.38: The following examples show that contra swg*- continuous function and contra β-continuous function are independent.

Example 4.2.39: Let X = Y ={a, b, c} with the topologies τ = {X, ϕ, {c}} and σ = {Y, ϕ, {b, c}}. The map f : X $\rightarrow$ Y is defined by f(a) = a, f(b)=c, f(c) = b. This function f is contra swg*- continuous function which is not contra β-continuous function, since the inverse image of the open set {b, c} in Y is not β-closed set in X.

Example 4.2.40 : Let X = Y ={a, b, c} with the topologies τ = {X, ϕ, {a}, {a, c}} and σ = {Y, ϕ, {c}}. The map f : X → Y is defined by f(a) = b, f(b) = a, f(c) = c. This function f is contra β-continuous function which is not contra swg*-continuous function, since the inverse image of the open set {c} in Y is not swg*-open set in X.

Remark 4.2.41: From the above results the following diagram is obtained.



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