CHAPTER 2

SEMI WEAKLY g*- CLOSED SETS IN TOPOLOGICAL SPACE

2.1 INTRODUCTION

In 1963, Levine [53] introduced the concept of a semi open set. Mashhour et.al [66], Njastad [72] and Abd El.Monsef et.al [1] introduced pre - open sets, α - open sets and β - open set respectively. Bhattacharya and Lahiri [11] , Arya and Nour [8], Maki et.al [61] introduced sg - closed sets, gs - closed sets, αg - closed sets respectively.

Stone [97] introduced and investigated strong forms of open sets called regular sets. Andrijevic [2], Bhattacharya and Lahiri [11], Levine ([52], [53]), Mashhour et.al [66] and Njastad [72] defined and discussed semi pre-open sets, generalized semi open sets, semi generalized open sets, generalized open sets, semi open sets, pre-open sets and α-open sets respectively

Devi et.al [20] introduced spaces called ${}_{\alpha}T_d$ spaces and ${}_{\alpha}T_b$ spaces. Also Devi et.al [21] introduced T_d-spaces. Nagaveni [70] introduced T_{wg} - spaces and T_{swg} - spaces in topological spaces.

2.2 SEMI WEAKLY g*- CLOSED SETS

In this section a new class of closed sets called swg*- closed sets is introduced and study further some of their properties.

Definition 2.2.1: A subset A of a topological space (X, τ) is called semi weakly g*-closed (denoted by swg*-closed) set, if $gcl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi open set.

Theorem 2.2.2: Let A be a swg*-closed set in a topological space X. Then gcl(A)-A contains no non-empty semi closed set in X.

Proof: Suppose that F is a semi closed subset of gcl(A)-A. Implies $F \subseteq gcl(A)$ and $F \subseteq A^c$. Since F^c is semi open set and A is swg*-closed set, $gcl(A) \subseteq F^c$. Therefore $F \subseteq gcl(A) \cap (gcl(A))^c = \phi$. Hence gcl(A) - A contains no non-empty semi closed set in X.

Theorem 2.2.3: If a set A in a topological space X is swg*- closed set then $gcl(A) - A = \phi$

Proof: Assume that A is swg*- closed set. Since gcl(A) = A, therefore $gcl(A) - A = \phi$.

Theorem 2.2.4: Suppose that $B \subseteq A \subseteq X$, B is swg*-closed set relative to A and that A is a swg*- closed subset of X, then B is swg*- closed relative to X.

Proof: Let $B \subseteq U$ and suppose that U is semi - open set in X. Then $B \subseteq A \cap U$ and hence $gcl_A(B) \subseteq A \cap U$. It follows that $A \cap gcl(B) \subseteq A \cap U$ and $A \subseteq U \cup (gcl(B))^c$. Since A is swg*-closed set in X, $gcl(A) \subseteq U \cup (gcl(B))^c$. Therefore $gcl(B) \subseteq gcl(A) \subseteq U \cup (gcl(B))^c$ and $gcl(B) \subseteq U$, then B is swg*-closed set relative to X.

Theorem 2.2.5: Let $A \subseteq Y \subseteq X$ and suppose that A is swg*- closed in X, then A is swg*- closed set relative to Y.

Proof: Let A $\subseteq$ Y $\cap$ U and suppose that U is semi open set in X, then A $\subseteq$ U and hence gcl(A) $\subseteq$ U. It follows that Y $\cap$ gcl(A) $\subseteq$ Y $\cap$ U, then A is swg*- closed relative to Y.

Theorem 2.2.6: Every closed set in a topological space X is a swg*- closed set in X.

Proof: Assume that A is closed set in X. Let U be a semi open set in X, such that A $\subseteq$ U and A $\subseteq$ cl(int(A)) $\subseteq$ U. Let gcl(A) $\subseteq$ cl(A) $\subseteq$ U. This implies A $\subseteq$ cl(int(A)) $\subseteq$ cl(A) $\subseteq$ U and U is semi open set. Thus A $\subseteq$ gcl(A) $\subseteq$ U. Therefore A is semi weakly g*- closed set.

Remark 2.2.7: The converse of the above theorem need not be true as seen from the following example.

Example 2.2.8: Let X ={a, b, c} with the topology τ ={X, ϕ, {b}}. In this topological space (X, τ) the subset {c} is swg*- closed set which is not closed set in (X, τ).

Theorem 2.2.9: Every g - closed set is swg*- closed set.

Proof: Let A be g - closed set then cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open set. Now A= gcl(A) as A is g - closed set, follows A is swg*- closed set .

Remark 2.2.10: The converse of the above theorem need be not true as seen from the following example.

Example 2.2.11: Let X = {a, b, c} with the topology τ = {X, ϕ, {a}}. In this topological space (X, τ) the subset {a} is swg*- closed set which is not g - closed set in (X, τ).

Theorem 2.2.12: Every g - open set is swg*- open set.

Proof: Let A be g - open set. Implies A^c is g - closed set. Now A^c = gcl (A^c) as A^c is g - closed set, follows A is swg*- open set.

Remark 2.2.13: The following examples show that swg*- closed set and semi closed set are independent .

Example 2.2.14 : Let X = {a, b, c} with the topology τ = {X, ϕ, {a}, {a, b}}. In this topological space (X, τ) the subset {b} is semi closed set which is not swg*- closed set in (X, τ).

Example 2.2.15: Let X = {a, b, c} with the topology τ = {X, ϕ, {a, c}}. In this topological space (X, τ) the subset {a, b} is swg*- closed set which is not semi - closed set in (X, τ).

Remark 2.2.16: The following examples show that swg*- closed set and pre - closed set are independent .

Example 2.2.17: Let X = {a, b, c} with the topology τ = {X, ϕ, {a, b}}. In this topological space (X, τ) the subset {b} is pre - closed set which is not swg*- closed set in (X, τ).

Example 2.2.18: Let X = {a, b, c} with the topology τ = {X, ϕ, {a}, {a, c}}. In this topological space (X, τ) the subset {a, b} is swg*- closed set which is not pre-closed set in (X, τ).

Remark 2.2.19: The following examples show that swg*- closed set and α -closed set are independent .

Remark 2.2.20: Let X = {a, b, c} with the topology τ = {X, ϕ, {b}}. In this topological space (X, τ) the subset {b} is swg*- closed set which is not α- closed set in (X, τ).

Example 2.2.21: Let X = {a, b, c} with the topology τ = {X, ϕ, {a}, {a, c}}. In this topological space (X, τ) the subset {c} is α - closed set which is not swg*- closed set in (X, τ).

Remark 2.2.22: The following examples show that swg*- closed set and β-closed (or semi - pre closed) set are independent .

Example 2.2.23: Let X = {a, b, c} with the topology τ = {X, ϕ, {c}}. In this topological space (X, τ) the subset {c} is swg*- closed set which is not β-closed set in (X, τ).

Example 2.2.24: Let X = {a, b, c} with the topology τ = {X, ϕ, {a, b}}. In this topological space (X, τ) the subset {b} is β - closed set which is not swg*- closed in (X, τ).

Remark 2.2.25: The following examples show that swg*- closed set and sg - closed set are independent .

Remark 2.2.26: Let X = {a, b, c} with the topology τ = {X, ϕ, {c}}. In this topological space (X, τ) the subset {c} is swg*- closed set which is not sg - closed set in (X, τ).

Example 2.2.27: Let X = {a, b, c} with the topology τ = {X, ϕ, {a}, {a, b}}. In this topological space (X, τ) the subset {b} is sg - closed set which is not swg*- closed set in (X, τ).

Remark 2.2.28: The following examples show that swg*- closed set and gs- closed set are independent.

Example 2.2.29: Let X = {a, b, c} with the topology τ = {X, ϕ, {a}, {a, b}}. In this topological space (X, τ) the subset {b} is gs - closed set which is not swg*- closed set in (X, τ).

Example 2.2.30: Let X = {a, b, c} with the topology τ = {X, ϕ, {a}}. In this topological space (X, τ) the subset {a} is swg*- closed set which is not gs-closed set in (X, τ)

Remark 2.2.31: The following examples show that swg*- closed set and αg - closed set are independent.

Example 2.2.32: Let X = {a, b, c} with the topology τ = {X, ϕ, {a}, {a, b}}. In this topological space (X, τ) the subset {b} is αg - closed set which is not swg*- closed set in (X, τ).

Example 2.2.33: Let X = {a, b, c} with the topology τ = {X, ϕ, {a}}. In this topological space (X, τ) the subset {a} is swg*- closed set which is not αg - closed set in (X, τ).

Remark 2.2.34: The following examples show that swg*- closed set and gα-closed set are independent.

Example 2.2.35: Let X = {a, b, c} with the topology τ = {X, ϕ, {a}, {a, b}}. In this topological space (X, τ) the subset {b} is gα - closed set which is not swg*-closed set in (X, τ).

Example 2.2.36: Let X = {a, b, c} with the topology τ = {X, ϕ, {b}}. In this topological space (X, τ) the subset {b} is swg*- closed set which is not gα-closed set in (X, τ).

Remark 2.2.37: The following examples show that swg*- closed set and gsp-closed set are independent .

Example 2.2.38: Let X = {a, b, c} with the topology τ = {X, ϕ, {a, b}}. In this topological space (X, τ) the subset {b} is gsp- closed set which is not swg*- closed set in (X, τ).

Example 2.2.39: Let X = {a, b, c} with the topology τ = {X, ϕ ,{c}}. In this topological space (X, τ) the subset {c} is swg*- closed set which is not gsp-closed set in (X, τ).

Remark 2.2.40: The following examples show that swg*- closed set and wg- closed set are independent .

Example 2.2.41: Let X = {a, b, c} with the topology τ = {X, ϕ, {a}}. In this topological space (X, τ) the subset {a} is swg*- closed set which is not wg - closed set in (X, τ).

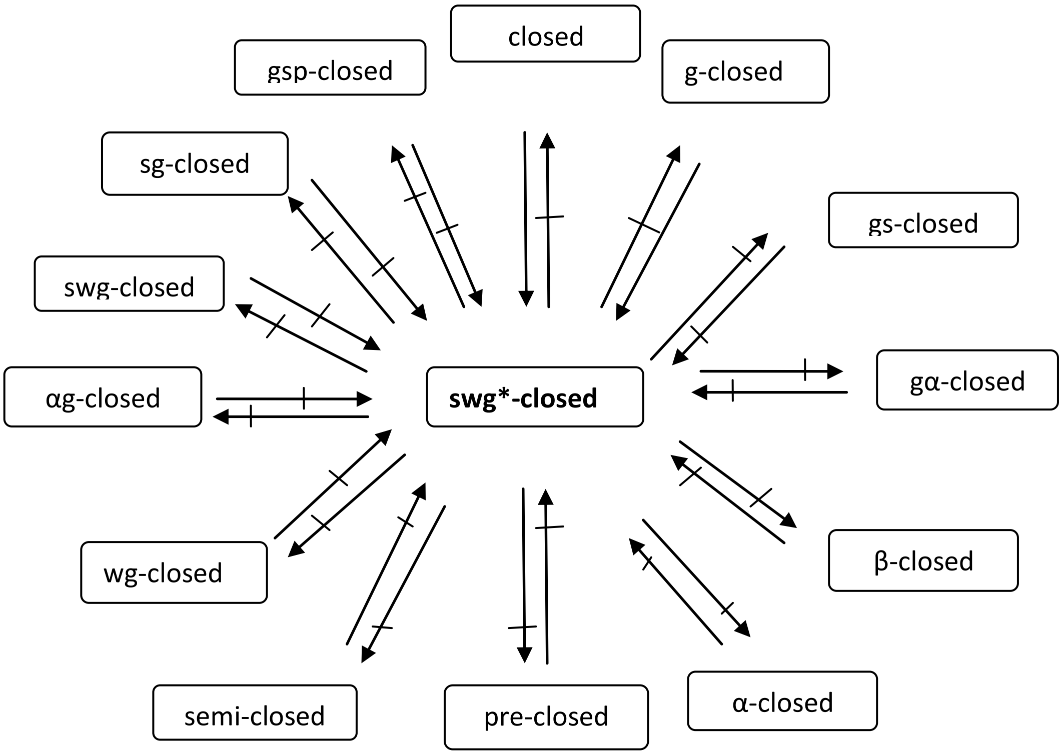
Example 2.2.42: Let X = {a, b, c} with the topology τ = {X, ϕ, {a}, {a, c}}. In this topological space (X, τ) the subset {c} is wg-closed set which is not swg*- closed set in (X, τ).

Remark 2.2.43: The following examples show that swg*-closed set and swg-closed set are independent.

Example 2.2.44: Let X = {a, b, c} with the topology τ ={X, ϕ, {a}}. In this topological space (X, τ) the subset {a} is swg *- closed set which is not swg- closed set in (X, τ).

Example 2.2.45: Let X = {a, b, c} with the topology τ ={X, ϕ, {a, b}}. In this topological space (X, τ) the subset {b} is swg - closed set which is not swg*- closed set in (X, τ).

Remark 2.2.46 : From the above results the following relationship is obtained.



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2.3 SEMI WEAKLY g*- OPEN SETS

In this section swg*- open set is introduced and some its properties are investigated.

Definition 2.3.1: A subset A of a topological space (X, τ) is called semi weakly g*-open (briefly swg*- open) set if and only if A^c is semi weakly g*- closed.

Theorem 2.3.2: Every open set in a topological space X is a swg*- open set in X.

Proof: Let A be open set. This implies A^c is closed set. By theorem 2.2.6 every closed set is swg*-closed set. A^c is swg*- closed set. Therefore A is swg*-open set.

Theorem 2.3.3: A set A in a topological space X is swg*-open set if and only if $F \subseteq$ gint(A), whenever F is semi closed set and $F \subseteq A$.

Proof : Assume that A is swg* open set in X. Let F be semi closed set and $F \subseteq A$. This implies F^c is semi open set and $A^c \subseteq F^c$. Since A^c is swg*-closed set, $gcl(A^c) \subseteq F^c$. Implies $gcl(A^c)$ = (gint(A)), $(g\ int(A))^c \subseteq F^c$. Therefore $F \subseteq g\ int(A)$. Conversely assume that $F \subseteq g\ int(A)$ whenever F is semi closed set and $F \subseteq A$. Let U be a semi open set in X containing A^c . Therefore U^c is a semi closed set contained in A by hypothesis $U^c \subseteq$ g int(A) taking complements $U \supseteq gcl(A^c)$. Therefore A^c is swg*-closed set in X. Hence A is swg* -open set in X.

Theorem 2.3.4: If $A \subseteq B \subseteq X$ where A is swg*- open set relative to B and B is semi swg*- open set relative to X , then A is swg*- open set relative to X.

Proof : Let F be a semi closed set and suppose that $F \subseteq A$. Then F is semi closed set relative to B and hence $F \subseteq g\ int_B(A)$. Therefore there exists a semi open set U such that $F \subseteq U \cap B \subseteq A$. But $F \subseteq U^* \subseteq B$ for semi open set U*, since B is swg*-open set in X. Thus $F \subseteq U^* \cap U \subseteq B \cap U \subseteq A$. It follows that $F \subseteq g\ int(A)$, because set A is swg*- open set . That implies $F \subseteq g\ int\ (A)$, whenever F is semi closed and $F \subseteq A$. Therefore A is semi weakly g*- open set in X.

Theorem 2.3.5: If $g\ int(A) \subseteq B \subseteq A$ and if A is swg*-open set then B is swg*-open set.

Proof: $A^c \subseteq B^c \subseteq gcl(A^c)$ and since A^c is swg*- closed set. It follows that B^c is swg*- closed set because A is swg*- open set and $A \subseteq B \subseteq gcl(A)$. Then B is swg*-open set.

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2.4 T_{swg*}- SPACES

This section introduces T_{swg*}- space also contains the study of separation axioms on this topological space.

Definition 2.4.1: A topological space (X, τ) is said to be T_{swg*} - space if every swg* - closed sets are g - closed set .

Theorem 2.4.2: Every $T_{½}$- space is T_{swg*} - Space

Proof: Let X be $T_{½}$ - space. Then every g - closed set is closed in X. By theorem 2.2.9, every g - closed set is swg*- closed set in X. Hence every swg*-closed set is closed. Then X is T_{swg*} - Space.

Remark 2.4.3: The converse of the above theorem need not be true as seen from the following example.

Example 2.4.4: Let X = {a, b, c} and $\tau = \{X, \phi, \{a\}, \{a, c\}\}$. This topological space (X, τ) is T_{swg*}- space and not a $T_{1/2}$ - space, as the g - closed set {a, b} is not a closed set in (X, τ).

Remark 2.4.5:The following examples show that ${}_{\alpha}T_d$- space and T_{swg*} - space are independent.

Example 2.4.6: Let X = {a, b, c} and τ = {X, ϕ, {b}}. This topological space (X, τ) is not ${}_{\alpha}T_d$ - space and not a T_{swg*} - space, as the swg*- closed set {b} is not g - closed in (X, τ).

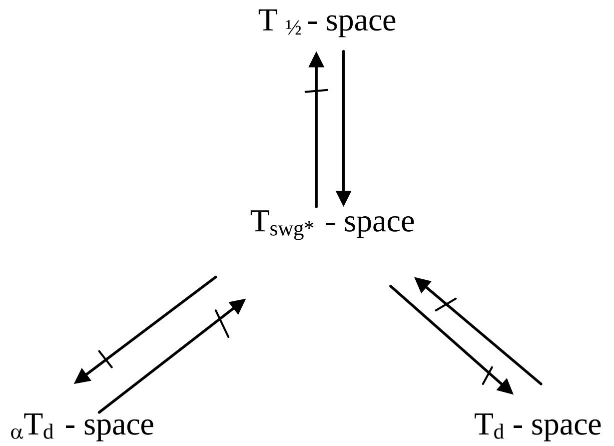
Example 2.4.7: Let X = {a, b, c} and τ = {X, ϕ, {b},{b, c}}. This topological space (X, τ) is T_{swg*} - space and not a ${}_{\alpha}T_d$ - space, as the αg - closed set {c} is not a g- closed set in (X, τ).

Remark 2.4.8: The following examples show that T_d - space and T_{swg*} - space are independent .

Example2.4.9: Let X = {a, b, c} and τ = {X, ϕ, {a}}. This topological space (X, τ) is T_d - space and not a T_{swg*} - space, as the swg*- closed set {a} is not a g - closed in (X, τ).

Example 2.4.10: Let X = {a, b, c} and τ = {X, ϕ, {a}, {a, c}}. This topological space (X, τ) is T_{swg*} - space and not a T_d - space, as the gs - closed set {c} is not a g- closed in (X, τ).

Remark 2.4.11: The above results leads to the following diagram.



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2.5 SWG* -OPEN SETS AND SWG*- NEIGHBOURHOODS

In this section swg*- neighbourhoods (shortly swg*-nbhd) in topological spaces are defined and some of their properties are studied.

Definition 2.5.1: Let x be a point in a topological space X. A subset N of X is said to be a swg*- neighbourhood of x, if and only if there exists a swg*- open set G such that $x \in G \subset N$.

Definition 2.5.2: Let x be a point in a topological space X. The set of all swg* - neighbourhood of a x is called the swg*- neighbourhood system at x which is denoted by swg*- N (x).

Theorem 2.5.3: Every neighbourhood N of $x \in X$ is a swg*- neighbourhood of X.

Proof: Let N be a neighbourhood of point $x \in X$. By definition of neighbourhood, there exists on open set G such that $x \in G \subset N$. As every open set is swg*- open set G such that $x \in G \subset N$. Here N is swg*- neighbourhood of x.

Remark 2.5.4: A swg*- neighbourhood N of $x \in X$ need not be a neighbourhood of x in X as seen from the following example.

Example 2.5.5: Consider the topological space X = {a, b, c} with the topology τ= {X, ϕ, {b}, {b, c}}, then SWG*O (X) = {X, ϕ, {b}, {c}, {b, c}}. The set {a, c} is swg*- neighbourhood of the point c, since the swg*- open set {c} is such that $c \in \{c\} \subset \{a, c\}$. However, the set {a, c} is not a neighbourhood of the point c, since no open set G exists such that $c \in G \subset \{a, c\}$.

Theorem 2.5.6: Let x be a point in a topological space X and swg* - N(x) be the collection of all swg* - neighbourhood of x, then the following results hold.

(i) $\forall x \in X$, swg * - N (x) $\neq \phi$

(ii) N $\in$ swg*-N(x) $\Rightarrow$ x $\in$ N.

(iii) N $\in$ swg*-N(x), M $\supset$ N $\Rightarrow$ M $\in$ swg*- N(x)

(iv) N $\in$ swg*-N (x) $\Rightarrow$ there exists M $\in$ swg*- N(x) such M $\subset$ N and M $\in$ swg*- N(y) for every y $\in$ M.

Proof: (i) Since X is swg* - open set, it is a swg*- nbhd of every $x \in X$. Hence there exists at least one swg*- neighbourhood (namely-X) for each $x \in X$. Hence swg*- N(x) $\neq \phi$ for every $x \in X$.

(ii) If N $\in$ swg*- N(x), then N is a swg*- neighbourhood of x. So by definition of swg*- neighbourhood, x $\in$ N.

(iii) Let N $\in$ swg*- N(x) and M $\supset$ N. Then there is a swg*- open set U such that x$\in$U $\subset$ N. Since N $\subset$ M, x $\in$U $\subset$ M and M is swg*- neighbourhood of x. Hence M$\in$ swg*- N(x).

(iv) If N $\in$ swg* - N(x), then there exists a swg* - open set M such that x $\in$M $\subset$ N. Since M is a swg*- open set, it is swg*- neighbourhood of each of its points. Therefore M $\in$ swg*- N(y) for every y $\in$ M.

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