CHAPTER 1

INTRODUCTION

The term topology was introduced in 1847 at Germany by Johann Benedict Listing. Topology as a science was formed through the works of the great French Mathematician Hennpoincare at the end of 19^{th} century. Since then, many concepts of topology have been generalized in topological spaces. Topology is applied in Differential Equations, Functional Analysis, Classical Mechanics, Theoretical Physics, General Theory of Relativity, Econometrical, Quantum machanics , Sociology etc. Topology is an indispensable subject of study with open-sets as well as closed sets being the most fundamental concept in topological spaces.

This thesis mainly deals with the study of a new type of sets in a topological spaces called semi weakly g*- closed (briefly swg*-closed) sets, its respective separation axioms, continuous mappings, closed mappings, homeomorphisms, irresolute mappings, contra semi weakly g*-continuous functions, (τ_i, τ_j)-semi weakly g*-closed sets, (τ_i, τ_j)- Quasi semi weakly g*- closed functions and also (τ_i, τ_j)- semi weakly g*- continuous functions.

This chapter begins with, section 1.1 having the discussion of strong and weak forms of open sets and closed sets. Section 1.2 presents some separation axioms. Section 1.3 deals with strong and weak forms of continuous mappings. Section 1.4 is devoted to irresolute mappings. Section 1.5 gives closed mappings and open mappings. Section 1.6 presents the generalized homeomorphisms. Section 1.7 explains with the contra continuous functions. Section 1.8 presents bitopological spaces. Section 1.9 deals continuous functions in bitopological spaces. The last section contains the contributions of the author. Throughout the thesis, X, Y and Z denote the topological spaces (X, τ), (Y, σ) and (Z, η) respectively.

1.1 STRONG AND WEAK FORMS OF OPEN AND CLOSED SETS

The first step of generalizing closed sets introduced by Levine [52] in 1970. Ever since general topologists extend the study of generalized closed sets on the basis of generalized open sets, regular open sets, α-open sets, semi open sets, semi pre-open sets, pre-open sets etc. Dontcheve [26], Veerakumar [102] and Nagaveni [70] introduced generalized semi-pre closed sets, g*-closed sets and weakly generalized closed sets respectively.

Definition 1.1.1: A subset A of a topological space (X, τ) is called semi-open set [53], if A $\subseteq$ cl(int(A)) and semi closed set if int(cl(A)) $\subseteq$ A.

Definition 1.1.2: A subset A of a topological space (X, τ) is called pre-open set [66], if A $\subseteq$ int(cl(A)) and pre-closed set if cl(int(A)) $\subseteq$ A.

Definition 1.1.3: A subset A of a topological space (X, τ) is called α-open set [72], if A $\subseteq$ int(cl(int(A))) and α-closed set if cl(int(cl(A))) $\subseteq$ A.

Definition 1.1.4: A subset A of a topological space (X, τ) is called semi pre-open set [2] or β-open set, if A $\subseteq$ cl(int(cl(A))) and semi pre-closed set or β-closed set, if int(cl(int(A))) $\subseteq$ A.

Definition 1.1.5: A subset A of a topological space (X, τ) is called regular open set [97]**,** if A = int(cl(A)) and regular closed set if cl(int(A)) = A.

Definition 1.1.6: A subset A of a topological space (X, τ) is called semi regular set [23], if both semi open set and semi closed set in (X, τ).

Definition 1.1.7: A subset A of a topological space (X, τ) is called generalized closed (briefly g-closed) set [52], if cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open set in (X, τ).

Definition 1.1.8: A subset A of a topological space (X, τ) is called semi-generalized closed (briefly sg-closed) set [11], if scl(A) $\subseteq$ U whenever A $\subseteq$ U and U is semi open set in (X, τ).

Definition 1.1.9: A subset A of a topological space (X, τ) is called generalized semi closed (briefly gs-closed) set [8], if scl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open set in (X, τ).

Definition 1.1.10: A subset A of a topological space (X, τ) is called α-generalized closed (briefly αg-closed) set [61], if αcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open set in (X, τ).

Definition 1.1.11: A subset A of a topological space (X, τ) is called generalized α-closed (briefly gα-closed) set [62], if αcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is α-open set in (X, τ).

Definition 1.1.12: A subset A of a topological space (X, τ) is called generalized semi- pre-closed (briefly gsp-closed) set [26], if spcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open set in (X, τ).

Definition 1.1.13: A subset A of a topological space (X, τ) is called weakly generalized closed (briefly wg-closed) set [70], if cl(int(A)) $\subseteq$ U whenever A $\subseteq$ U and U is open set in (X, τ).

Definition 1.1.14: A subset A of a topological space (X, τ) is called semi weakly generalized closed (briefly swg-closed) set [70], if cl(int(A)) $\subseteq$ U whenever A $\subseteq$ U and U is semi open set in (X, τ).

Definition 1.1.15: A subset A of a topological space (X, τ) is called regular weakly generalized closed (briefly rwg-closed) set [70], if cl(int(A)) $\subseteq$ U whenever A $\subseteq$ U and U is regular open set in (X, τ).

Definition 1.1.16: A subset A of a topological space (X, τ) is called g- closed [101] set, if cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is g-open set in (X, τ).

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1.2 SEPARATION AXIOMS

Levine [54] introduced $T_{1/2}$- spaces. Dunham [30], Maheswari and Prasad ([58], [59]), Jankovie and Reilly [45] investigated some separation axioms. Bhattacharyya and Lahiri [11], Dontchev [26] and Devi et.al [19] defined and investigated semi $T_{1/2}$- space, semi pre-$T_{½}$ spaces, T_b and T_d spaces respectively.

Definition 1.2.1: A topological space (X, τ) is called $T_{1/2}$-space [52], if every g-closed subset of (X, τ) is closed set in (X, τ).

Definition 1.2.2: A topological space (X, τ) is called semi-$T_{1/2}$-space [11], if every semi generalized-closed subset of (X, τ) is semi-closed set in (X,τ).

Definition 1.2.3: A topological space (X, τ) is called semi-pre-$T_{1/2}$-space [26], if every generalized semi pre closed subset of (X, τ) is semi-pre-closed set in (X, τ).

Definition 1.2.4: A topological space (X, τ) is called T_b-space [21], if every generalized semi closed subset of (X, τ) is closed set in (X, τ)

Definition 1.2.5: A topological space (X, τ) is called T_d-space [21], if every-generalized semi closed subset of (X, τ) is generalized closed set in (X, τ).

Definition 1.2.6: A topological space (X, τ) is called ${}_{\alpha}T_d$-space [20], if every α-generalized closed subset of (X, τ) is generalized closed set in (X, τ).

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1.3 STRONG AND WEAK FORMS OF CONTINUOUS FUNCTIONS

Various forms of continuous maps were introduced and investigated by Baker [10] , Husain [40] , Jankovic [44] , Levine [54] , Noiri ([74], [75], [82]). Balachandran et.al [9] introduced and studied generalized semi continous mappings. Various forms of semi continuous functions have been investigated by several topologist like Arockairani [3] , Arya and Nour [8] , Arya and Deb [5] , Biswas [12], Crossley and Hildebrand [17], Dorsett [29] , Espelie and joseph [33] , Hamlett [39] , Jankovic [41], Levine [53], Lin [55] , Neubrunnova [71] , Noiri [76] , Noiri [77] , Noiri and Ahmad [78], Noiri [79] , Noiri [81] , Papic [85] , Popa [90].

Definition 1.3.1: Let (X, τ) and (Y, σ) be any two topological spaces. A map $f : (X, \tau) \rightarrow (Y, \sigma)$ is said to be continuous function [53], if the inverse image of every open set in (Y, σ) is open set in (X, τ) .

Definition 1.3.2: Let (X, τ) and (Y, σ) be any two topological spaces. A map $f : (X, \tau) \rightarrow (Y, \sigma)$ is said to be semi-continuous function [53], if the inverse image of every open set in (Y, σ) is semi open set in (X, τ) .

Definition 1.3.3: Let (X, τ) and (Y, σ) be any two topological spaces. A map $f : (X, \tau) \rightarrow (Y, \sigma)$ is said to be pre-continuous function [66], if the inverse image of every open set in (Y, σ) is pre-open set in (X, τ).

Definition 1.3.4 : Let (X, τ) and (Y, σ) be any two topological spaces. A map $f : (X, \tau) \rightarrow (Y, \sigma)$ is said to be α-continuous function [67], if the inverse image of every open set in (Y, σ) is α-open set in (X, τ) .

Definition 1.3.5: Let (X, τ) and (Y, σ) be any two topological spaces. A map $f : (X, \tau) \rightarrow (Y, \sigma)$ is said to be β-continuous function [1], if the inverse image of every open set in (Y, σ) is β-open set in (X, τ) .

Definition 1.3.6: Let (X, τ) and (Y, σ) be any two topological spaces. A map $f : (X, \tau) \to (Y, \sigma)$ is said to be g-continuous function [9], if the inverse image of every open set in (Y, σ) is g - open set in (X, τ) .

Definition 1.3.7: Let (X, τ) and (Y, σ) be any two topological spaces. A map $f : (X, \tau) \to (Y, \sigma)$ is said to be sg-continuous function [98], if the inverse image of every open set in (Y, σ) is sg-open set in (X, τ) .

Definition 1.3.8: Let (X, τ) and (Y, σ) be any two topological spaces. A map $f : (X, \tau) \to (Y, \sigma)$ is said to be gs-continuous function [19], if the inverse image of every open set in (Y, σ) is gs-open set in (X, τ).

Definition 1.3.9: Let (X, τ) and (Y, σ) be any two topological spaces. A map $f : (X, \tau) \to (Y, \sigma)$ is said to be gα-continuous function [67], if the inverse image of every open set in (Y, σ) is gα-open set in (X, τ).

Definition 1.3.10: Let (X, τ) and (Y, σ) be any two topological spaces. A map $f : (X, \tau) \to (Y, \sigma)$ is said to be αg-continuous function [61], if the inverse image of every open set in (Y, σ) is αg-open set in (X, τ).

Definition 1.3.11: Let (X, τ) and (Y, σ) be any two topological spaces. A map $f : (X, \tau) \to (Y, \sigma)$ is said to be gsp-continuous function [25], if the inverse image of every open set in (Y, σ) is gsp-open set in (X, τ).

Definition 1.3.12: Let (X, τ) and (Y, σ) be any two topological spaces. A map f : (X, τ) $\rightarrow$ (Y, σ) is said to be rwg-continuous function [70], if the inverse image of every open set in (Y, σ) is rwg-open set in (X, τ).

Definition 1.3.13: Let (X, τ) and (Y, σ) be any two topological spaces. A map f : (X, τ) $\rightarrow$ (Y, σ) is said to be perfectly continuous function [83], if the inverse image of both open and closed set in (Y, σ) is open set in (X, τ).

Definition 1.3.14: Let (X, τ) and (Y, σ) be any two topological spaces. A map f : (X, τ) $\rightarrow$ (Y, σ) is said to be completely continuous function [84], if the inverse image of every regular open set in (Y, σ) is open set in (X, τ).

Definition 1.3.15: Let (X, τ) and (Y, σ) be any two topological spaces. A map f : (X,τ) $\rightarrow$ (Y,σ) is said to be swg-continuous function [70], if the inverse image of every swg-open set in (Y, σ) is open set in (X, τ).

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1.4 IRRESOLUTE FUNCTIONS

Balachandran et.al [9] introduced and studied the concept of a class of maps, namely g - continuous maps which includes the class of continuous maps and a class of gc - irresolute maps. Cammaroto [14] introduced and investigated almost irresolute functions. Faro [34] studied strongly α - irresolute mappings in topological spaces. Nagaveni [70] introduced a class of weakly generalized irresolute and semi weakly generalized irresolute maps in topological spaces. Dimaio and Noiri [25], Faro [34], Commaroto and Noiri [14], Maheshwari and Prasad [57] and Sundaram [99] introduced various irresolute mappings.

Definition 1.4.1: Let (X, τ) and (Y, σ) be any two topological spaces. A map f : (X, τ) → (Y, σ) is called irresolute function [17], if the inverse image of every semi open set in (Y, σ) is semi-open set in (X, τ).

Definition 1.4.2: Let (X, τ) and (Y, σ) be any two topological spaces. A map f : (X, τ) → (Y, σ) is called gc-irresolute function [99], if the inverse image of every g-open set in (Y, σ) is g-open set in (X, τ).

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1.5 CLOSED MAPPINGS AND OPEN MAPPINGS

Malghan [65] introduced and investigated some properties of generalized closed mappings. Noiri [73], Biswas [13], Mashhour [66], Sundaram [99], Devi [22] defined and studied semi - closed and semi- open mappings, α- open mappings, α - closed mappings, pre - open mappings and generalized open mappings, αg-closed mappings respectively. Gnanambal [38] defined gpr- closed mappings. Mashhour et.al [66] defined a class of mappings called pre - closed mappings which contain the class of closed mappings. Sen and Bhattacharya [93] studied the properties of pre-closed mappings.

Definition 1.5.1: Let (X, τ) and (Y, σ) be any two topological spaces. A map $f : (X, \tau) \rightarrow (Y, \sigma)$ is called generalized-closed map [99], if the image of the every closed set in (X, τ) is generalized-closed set in (Y, σ).

Definition 1.5.2: Let (X, τ) and (Y, σ) be any two topological spaces. A map $f : (X, \tau) \rightarrow (Y, \sigma)$ is called semi-closed map [99], if the image of the every closed set in (X, τ) is semi-closed set in (Y, σ).

Definition 1.5.3: Let (X, τ) and (Y, σ) be any two topological spaces. A map $f : (X, \tau) \rightarrow (Y, \sigma)$ is called α-closed map [80], if the image of the every closed set in (X, τ) is α-closed set in (Y, σ).

Definition 1.5.4: Let (X, τ) and (Y, σ) be any two topological spaces. A map $f : (X, \tau) \rightarrow (Y, \sigma)$ is called pre-closed map [31], if the image of the every closed set in (X, τ) is pre-closed set in (Y, σ).

Definition 1.5.5: Let (X, τ) and (Y, σ) be any two topological spaces. A map $f : (X, \tau) \rightarrow (Y, \sigma)$ is called regular closed map [56] , if the image of the every closed set in (X, τ) is regular closed set in (Y, σ).

Definition 1.5.6: Let (X, τ) and (Y, σ) be any two topological spaces. A map $f : (X, \tau) \rightarrow (Y, \sigma)$ is called generalized open map [19], if the image of the every open set in (X, τ) is generalized open set in (Y, σ).

Definition 1.5.7: Let (X, τ) and (Y, σ) be any two topological spaces. A map f : (X, τ) $\rightarrow$ (Y, σ) is called semi-open map [80], if the image of the every open set in (X, τ) is semi-open set in (Y, σ).

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1.6 GENERALIZATIONS OF HOMEOMORPHISM IN TOPOLOGICAL SPACES

Biswass [13], Crossley and Hildebrand [17], Gentry and Hoyle [37], Tadros [100], Umehara and Maki [101] and Maki et.al [64] introduced and investigated various types of homeomorphisms.

Definition 1.6.1: Let (X, τ) and (Y, σ) be any two topological spaces. A map f : (X, τ) $\rightarrow$ (Y, σ) is called generalized continuous [9], if the inverse image of the every open set in (Y, σ) is g - open set in (X, τ).

Definition 1.6.2: Let (X, τ) and (Y, σ) be any two topological spaces. A bijective function f : (X, τ) $\rightarrow$ (Y, σ) is called a homeomorphism [4] , if f is both continuous and open.

Definition 1.6.3: Let (X, τ) and (Y, σ) be any two topological spaces. A bijective function f : (X, τ) $\rightarrow$ (Y, σ) is called generalized - homeomorphism (briefly g- homeomorphisms) [64], if f is both g - continuous and g - open.

Definition 1.6.4: Let (X, τ) and (Y, σ) be any two topological spaces. A bijective function f : (X, τ) $\rightarrow$ (Y, σ) is called gc-homeomorphism [64], if f is gc-irresolute and f^{-1} is also gc-irresolute.

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1.7 CONTRA CONTINUOUS FUNCTIONS IN TOPOLOGICAL SPACES

Contra semi continuous functions were introduced and investigated by Dontchev and Noiri [28]. Jafari and Noiri ([42], [43]) introduced contra pre - continuous functions and α - continuous functions in topological spaces. Dontchev [27] defined contra continuity in topological spaces.

Definition 1.7.1: Let (X, τ) and (Y, σ) be any two topological spaces. A function f : (X, τ) $\rightarrow$ (Y, σ) is called a contra continuous [27], if the inverse image of the every open set in (Y, σ) is closed set in (X, τ).

Definition 1.7.2: Let (X, τ) and (Y, σ) be any two topological spaces. A function f : (X, τ) $\rightarrow$ (Y, σ) is called a contra semi - continuous [28], if the inverse image of the every open set in (Y, σ) is semi-closed set in (X, τ).

Definition 1.7.3: Let (X, τ) and (Y, σ) be any two topological spaces. A function f : (X, τ) $\rightarrow$ (Y, σ) is called a contra pre-continuous [42], if the inverse image of the every open set in (Y, σ) is pre-closed set in (X, τ).

Definition 1.7.4: Let (X, τ) and (Y, σ) be any two topological spaces. A function f : (X, τ) → (Y, σ) is called a contra α-continuous [43], if the inverse image of the every open set in (Y, σ) is α-closed set in (X, τ).

Definition 1.7.5: Let (X, τ) and (Y, σ) be any two topological spaces. A function f : (X, τ) → (Y, σ) is called a contra β-continuous [27], if the inverse image of the every open set in (Y, σ) is β-closed set in (X, τ).

Definition 1.7.6: A function f: X → Y is called almost continuous function [95] at a point x∈X if for every open set V in Y containing f(x), there is an open set U in X containing x such that f(U) ⊂ V. If f is almost continuous function at every point of X, then it is called almost continuous function.

Definition 1.7.7: The Kernel of a set A [88] denoted by $A^{\wedge}$ is the intersection of all super sets of A i,e. $A^{\wedge} = \cap \{ U: U \supset A , U \in \tau \}$.

Result 1.7.8: The following properties hold for sub sets A and B of a topological space X [88].

(i) X ∈ ker (A), if and only if A ∩ F ≠ ϕ for any F∈ C (X,x)

(ii) A ⊆ ker (A) and A= Ker (A) if A is open in X.

(iii) If A ⊆ B, then ker (A) ⊆ Ker (B)

Definition 1.7.9: A topological space X is said to be Urysohn [60] if for any two distinct points x and y of X, there exist open sets U and V containing x and y respectively such that $cl(U) \cap cl(V) = \phi$.

Definition 1.7.10 : For function $f : X \to Y$, the subset $\{(X, f(x)),\ x \in X)\} \subseteq XxY$ is called the graph [56] of f and is denoted by Gr(f).

Definition 1.7.11: Let $\pi_1 : XxY \to X$ be defined by the equation $\pi_1(x,y) = x$; let $\pi_2 : XxY \to Y$ be defined by the equation $\pi_2 (x,y) = y$. The maps π_1 and π_2 are called the projections [56] of XxY onto its first and second factors respectively.

Definition 1.7.12: The set scl(A) - sint(A) is called the semi frontier [18] of A is denoted by A_s^f.

Definition 1.7.13: A topological space X is regular [56] if F is a closed subset of X and $p \in X$ does not belong to F, then there exist disjoint open sets G and H such that $F \subset G$ and $p \in H$.

Defintion 1.7.14: A space X is said to be weakly hausdorff [78], if each element of X is an intersection of closed sets.

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1.8 BITOPOLOGICAL SPACES

Kelly [47] defined a bitopological space as a set equipped with two topologies on the set and initiated a systematic study of bitopological spaces in 1963. Following the work of Kelly on bitopological spaces, various author Arya and

Nour [7], Dimaio and Noiri [24], Fukutake ([35], [36]). Maki et.al [63], Maheswari and Prasad [59], Jelic [46], Mrsevic Mukerjee et.al [69] , Patti [86], Singal and Jain [94] , Popa [89], Reilly [91], Devi [22], Sampthkumar [92], Arockiarani [4] and Gnanambal [38] turned their attention to the various concept of topology by considering bitopological spaces instead of topological spaces.

Definition 1.8.1: A subset A of a bitopological space (X, τ_i, τ_j) is called (τ_i, τ_j) - semi - open set [59], if $A \subseteq (\tau_j\text{-cl}(\tau_i\text{- int}(A)))$ and (τ_i, τ_j)-semi - closed set, if $(\tau_i\text{-int}(\tau_j\text{-cl}(A))) \subseteq A$.

Definition 1.8.2: A subset A of a bitopological space (X, τ_i, τ_j) is called (τ_i, τ_j)-pre - open set [46], if $A \subseteq (\tau_i\text{-int}(\tau_j\text{-cl}(A)))$ and (τ_i, τ_j)- pre - closed set, if $(\tau_j\text{-cl}(\tau_i\text{-int}(A))) \subseteq A$.

Definition 1.8.3: A subset A of a bitopological space (X, τ_i, τ_j) is called (τ_i, τ_j)-α - open set [31], if $A \subseteq (\tau_i\text{-int}(\tau_j\text{-cl}(\tau_i\text{-int}(A))))$ and (τ_i, τ_j)-α - closed set, if $(\tau_j\text{-cl}(\tau_i\text{- int}(\tau_j\text{-cl}(A)))) \subseteq A$.

Definition 1.8.4: A subset A of a bitopological space (X, τ_i, τ_j) is called (τ_i, τ_j)-generalized closed (briefly (τ_i, τ_j)-g-closed) set [35], if $\tau_j\text{-cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is τ_i - open in (X, τ_i, τ_j).

Definition 1.8.5: A subset A of a bitopological space (X, τ_i, τ_j) is called (τ_i, τ_j)-semi generalized closed (briefly (τ_i, τ_j)-sg-closed) set [51], if $\tau_j\text{-scl}(A) \subseteq U$ whenever $A \subseteq U$ and U is τ_i-semi open in (X, τ_i, τ_j)

Definition 1.8.6: A subset A of a bitopological space (X, τ_i, τ_j) is called (τ_i, τ_j) - generalized semi-closed (briefly (τ_i, τ_j)-gs-closed) set [51], if τ_j-scl(A) $\subseteq$ U whenever A $\subseteq$ U and U is τ_i-open in (X, τ_i, τ_j) .

Definition 1.8.7: A subset A of a bitopological space (X, τ_i, τ_j) is called (τ_i, τ_j)-α - generalized closed (briefly (τ_i, τ_j)-αg-closed) set [51], if τ_j- αcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is τ_i-open in (X, τ_i, τ_j).

Definition 1.8.8: A subset A of a bitopological space (X, τ_i, τ_j) is called (τ_i, τ_j) - generalized-α-closed (briefly (τ_i, τ_j)-gα-closed) set [51], if τ_j-αcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is τ_i-α-open in (X, τ_i, τ_j)

Definition 1.8.9: A bitopological space (X, τ_i, τ_j) is said to be (τ_i, τ_j)-$T_{1/2}$-space [35], if every (τ_i, τ_j)- g-closed set is τ_j-closed .

Definition 1.8.10: A space (X, τ_i, τ_j) is said to be pairwise normal [91], if for each τ_i-closed set A and τ_j-closed set B disjoint from A, there is a τ_i-open set U containing A and a τ_j-open set V containing B such that U $\cap$ V = ϕ.

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1.9 (τ_i, τ_j)-CONTINUOUS FUNCTIONS IN BITOPOLOGICAL SPACES

Fukutake [35] extended the concept of generalized closed sets of Levine [52] to bitopological spaces. Maki et.al [62] introduced and investigated the concept of generalized continuous maps. Bitopological spaces have been studied by various authors ([6], [7], [24], [68] and [92]).

Definition 1.9.1: Let (X, τ_i, τ_j) and (Y, σ_i, σ_j) be bitopological spaces. A function $f : (X, \tau_i, \tau_j) \to (Y, \sigma_i, \sigma_j)$ is said to be (τ_i, τ_j)-σ_i-semi continuous function [59], if the inverse image of every σ_i- closed set in Y is (τ_i, τ_j)- semi closed set in X.

Definition 1.9.2: Let (X, τ_i, τ_j) and (Y, σ_i, σ_j) be bitopological spaces. A function $f : (X, \tau_i, \tau_j) \to (Y, \sigma_i, \sigma_j)$ is said to be (τ_i, τ_j) -σ_i-pre-continuous function [46], if the inverse image of every σ_i -closed set in Y is (τ_i, τ_j) - pre - closed set in X.

Definition 1.9.3: Let (X, τ_i, τ_j) and (Y, σ_i, σ_j) be bitopological spaces. A function $f : (X, \tau_i, \tau_j) \to (Y, \sigma_i, \sigma_j)$ is said to be (τ_i,τ_j) -σ_i-α-continuous function [48], if the inverse image of every σ_i- closed set in Y is (τ_i, τ_j) - α- closed set in X.

Definition 1.9.4: Let (X, τ_i, τ_j) and (Y, σ_i, σ_j) be bitopological spaces. A function $f : (X, \tau_i, \tau_j) \to (Y, \sigma_i, \sigma_j)$ is said to be (τ_i, τ_j) -σ_i-semi generalized continuous function [51], if the inverse image of every σ_i-closed set in Y is (τ_i, τ_j) - semi generalized closed set in X.

Definition 1.9.5 : Let (X, τ_i, τ_j) and (Y, σ_i, σ_j) be bitopological spaces. A function $f : (X, \tau_i, \tau_j) \to (Y, \sigma_i, \sigma_j)$ is said to be (τ_i, τ_j) -σ_i-generalized semi continuous function [49], if the inverse image of every σ_i- closed set in Y is (τ_i,τ_j) -generalized semi - closed set in X.

Definition 1.9.6 : Let (X, τ_i, τ_j) and (Y, σ_i, σ_j) be bitopological spaces. A function $f : (X, \tau_i, \tau_j) \to (Y, \sigma_i, \sigma_j)$ is said to be (τ_i, τ_j)- σ_i-α-generalized continuous function

[48], if the inverse image of every σ_i –closed set in Y is (τ_i, τ_j) -α- generalized closed set in X.

Definition 1.9.7: Let (X, τ_i, τ_j) and (Y, σ_i, σ_j) be bitopological spaces. A function $f : (X, \tau_i, \tau_j) \rightarrow (Y, \sigma_i, \sigma_j)$ is said to be (τ_i, τ_j)-σ_i-generalized α-continuous function [48], if the inverse image of every σ_i-closed set in Y is (τ_i, τ_j)- generalized α-closed set in X.

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1.10 CONTRIBUTIONS OF THE AUTHOR

In the light of the above work, the researcher has obtained several interesting generalizations as the following topics.

(i) Semi weakly g*-closed Set in Topological Spaces.

(ii) On semi weakly g* -continuous Function in Topological Spaces

(iii) On contra semi weakly g*-continuous Functions in Topological Spaces

(iv) (τ_i, τ_j) -Semi weakly g*-closed Sets in Bitopological Spaces

(v) (τ_i, τ_j) -Quasi semi weakly g*-closed Functions in Bitopological Spaces

(vi) (τ_i, τ_j) -Semi weakly g*- continuous Functions in Bitopological Spaces

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1.11NOTATIONS

(i) ϕ = Null set or empty set

(ii) int(A) = interior of A

(iii) cl(A)= Closure of A

(iv) A^c = The complement of A

(v) g int(A) = generalized interior of A

(vi) gcl(A) = generalized closure of A

(vii) scl(A) = semi closure of A

(viii) pcl(A) = pre-closure of A

(ix) αcl(A) = α-closure of A

(x) spcl(A) = semi pre-closure of A

(xi) swg*cl(A) = semi weakly g*-closure of A

In the diagrams,

(xii) A $\longrightarrow$ B means, A implies B

(xiii) A $\not\longrightarrow$ B means, A does not implies B.

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