In this chapter, we have proposed Adomian Decomposition Method (ADM) to find the numerical solutions of the system (2.2.8) of six coupled ODEs. Through this scheme we have obtained the solutions in the form of power series. The constants involve in the power series are determined using the Adomian Polynomials. Through the iterations, degree of Adomian polynomials is improved and solution is approximated to the next higher degree polynomial. Continuing like this, solution is
developed in the form of power series and the convergence is guaranteed [1, 2, 22]. The minimum error in the solution is of the order $10^{-6}$. This error can be reduced by decreasing size of subinterval for the given partition of interval.

The software MATHEMATICA is better than C-Programming and MATLAB for symbolic calculations. Hence MATHEMATICA 9 is used for programming and calculations.

5.1 Introduction

In previous chapters, the numerical solution of system (2.2.8) has been obtained by Modified Euler Method [15] and by Runge-Kutta fourth order Method [16]. Now in this chapter the Adomian Decomposition Method is implemented to find the numerical solution of system (2.2.8) passing through the initial values taken over the invariant surface (2.2.10). The implementation of this method is discussed in the section 5.4.1.

5.2 Adomian Decomposition Method

In the 1980’s, George Adomian (1923-1996)[3] introduced a new powerful method for solving nonlinear functional equations. Since then, this method has been known as the Adomian Decomposition Method (ADM). The technique is based on a decomposition of a solution of a nonlinear operator equation in a series of functions. Each term of the series is obtained by using a polynomial generated from power series expansion
of an analytic function. The more details of this method can be referred from [9, 21, 23, 24, 28, 30, 32, 35]. A systematic use of this ADM to find the solutions of ODEs is given by J. Biazar et. al. [7], Varsha Daftardar-Gejji and H. Jafari [23, 24].

Now we consider a system of ordinary differential equations as:

\[
\begin{align*}
    y_1' &= f_1(x, y_1, \ldots, y_n), \\
    y_2' &= f_2(x, y_1, \ldots, y_n), \\
    &\vdots \\
    y_n' &= f_n(x, y_1, \ldots, y_n).
\end{align*}
\]

(5.2.1)

Since every \(n^{th}\) order ordinary differential equation can be reduced to a system consisting of \(n\) first order ordinary differential equations. Consequently the ADM can be used to find the solution of \(n^{th}\) order differential equation. Because of this J. Biazar et. al. [7] restricted their study to a system of first order differential equations. In the following section ADM is discussed.

## 5.3 Implementing Adomian Decomposition Method

J. Biazar et al. [7], Varsha Daftardar-Gejji and H. Jafari [23, 24] have presented the system (5.2.1), as:

\[
Ly_i = f_i(x, y_1, \ldots, y_n), \quad i = 1, 2, \ldots, n
\]

(5.3.1)
where $L$ is the linear operator $\frac{d}{dx}$ with the inverse $L^{-1} = \int_{0}^{x}(.)dx$. Applying the inverse operator on (5.3.1) the following canonical form is obtained, which is suitable for applying Adomian decomposition method.

$$y_i = y_i(0) + \int_{0}^{x} f_i(x, y_1, ..., y_n)dx, \quad i = 1, 2, ..., n.$$ (5.3.2)

As usual in Adomian Decomposition Method the solution of (5.3.2) is considered to be as the sum of a series.

$$y_i = \sum_{j=0}^{\infty} f_{i,j}, \quad i = 1, 2, ..., n$$ (5.3.3)

and the integral in the equation (5.3.2), as the sum of the following series.

$$f_i(x, y_1, ..., y_n) = \sum_{j=0}^{\infty} A_{i,j}(f_{i,0}, f_{i,1}, ..., f_{i,j}), \quad i = 1, 2, ..., n$$ (5.3.4)

where $A_{i,j}(f_{i,0}, f_{i,1}, ..., f_{i,n})$ are called Adomian polynomials. Substituting (5.3.3) and (5.3.4) into (5.3.2) we get

$$\sum_{j=0}^{\infty} f_{i,j} = y_i(0) + \int_{0}^{x} \sum_{j=0}^{\infty} A_{i,j}(f_{i,0}, f_{i,1}, ..., f_{i,j})dx,$$

$$= y_i(0) + \sum_{j=0}^{\infty} \int_{0}^{x} A_{i,j}(f_{i,0}, f_{i,1}, ..., f_{i,j})dx.$$ (5.3.5)

From which we define

$$f_{i,0} = y_i(0);$$

$$f_{i,n+1} = \int_{0}^{x} A_{i,n}(f_{i,0}, f_{i,1}, ..., f_{i,n})dx, \quad i = 1, 2, ..., n.$$ (5.3.6)

### 5.4 Numerical Solution

In this chapter, we have obtained the numerical solutions of a system (2.2.8) with the initial values taken over the invariant surface (2.2.10) by
using ADM. While implementing this method, we have used alternative algorithm to calculate Adomian Polynomials. More details about this alternative algorithm may be found in [5, 8].

5.4.1 Implementation of ADM

The alternative algorithm [5, 6, 8, 10, 20] is used to calculate the Adomian Polynomials [5, 8] with help of MATHEMATICA 9. Jun-Sheng Duan [20] has obtained the recurrence relations for the simplified index matrices, which provide a convenient algorithm for rapid generation of the multivariable Adomian polynomials. Also E. Babolian and Sh. Javadi [6] have presented a good scheme of calculating Adomian polynomials. Whereas the approach to calculate Adomian Polynomials by J. Biazar et. al. [7] is most suitable to us to find the numerical solution of the system (2.2.8).

Now we proceed to find to find the numerical solution of the system (2.2.8) by using ADM. Let us consider the initial conditions \( b_0(b_{10}, b_{20}, b_{30}) \) and \( w_0(w_{10}, w_{20}, w_{30}) \) at \( t = 0 \). We are aiming to find the solution in the form of following equations.

\[
\begin{align*}
{w_1} & = \sum_{j=0}^{\infty} w_{1j};\quad {w_2} = \sum_{j=0}^{\infty} w_{2j};\quad {w_3} = 1, \\
{b_1} & = \sum_{j=0}^{\infty} b_{1j};\quad {b_2} = \sum_{j=0}^{\infty} b_{2j};\quad {b_3} = \sum_{j=0}^{\infty} b_{3j}.
\end{align*}
\]

(5.4.1)

In above equations, \( w_{1j}, w_{2j}, b_{ij} \) for \( i = 1, 2, 3 \) and \( j = 0, 1, 2, \ldots \) are determined through the successive iterations. According to ADM, operating the \( L^{-1} \) operator on both sides of the system (2.2.8), we have
obtained the first iteration as below,

\[
\begin{align*}
\frac{dw_{11}}{dt} &= w_{10} - \frac{g}{\rho_b} \int_0^t b_{20} dt, \quad w_{21} = w_{20} + \frac{g}{\rho_b} \int_0^t b_{10} dt, \\
\frac{db_{11}}{dt} &= b_{10} + \frac{1}{2} \int_0^t (w_{20} b_{30} - w_{30} b_{20}) dt, \\
\frac{dw_{21}}{dt} &= w_{20} + \frac{g}{\rho_b} \int_0^t (w_{30} b_{10} - w_{10} b_{30}) dt, \\
\frac{db_{21}}{dt} &= b_{20} + \frac{1}{2} \int_0^t (w_{10} b_{20} - w_{20} b_{10}) dt.
\end{align*}
\]

Let us denote

\[
\begin{align*}
\frac{1}{2}(w_{20} b_{30} - w_{30} b_{20}) &= \frac{1}{2}(k_5 k_3 - k_2) = k_6, \\
\frac{1}{2}(w_{30} b_{10} - w_{10} b_{30}) &= \frac{1}{2}(k_1 - k_4 k_3) = k_7, \\
\frac{1}{2}(w_{10} b_{20} - w_{20} b_{10}) &= \frac{1}{2}(k_4 k_2 - k_5 k_1) = k_8.
\end{align*}
\]

Substituting the values from equation (5.4.3) in equation (5.4.2), the first iteration is as follows.

\[
\begin{align*}
\frac{dw_{11}}{dt} &= k_4 - \frac{g}{\rho_b} k_2 t, \quad w_{21} = k_5 + \frac{g}{\rho_b} k_1 t, \\
\frac{db_{11}}{dt} &= k_1 + k_6 t, \quad b_{21} = k_2 + k_7 t, \quad b_{31} = k_3 + k_8 t.
\end{align*}
\]

Again integrating (5.4.4) from 0 to \( t \), 2\textsuperscript{nd} iteration to the system (2.2.8) is as follows

\[
\begin{align*}
\frac{dw_{12}}{dt} &= w_{11} - \frac{g}{\rho_b} \int_0^t b_{21} dt, \quad w_{22} = w_{21} + \frac{g}{\rho_b} \int_0^t b_{11} dt, \\
\frac{db_{12}}{dt} &= b_{11} + \frac{1}{2} \int_0^t (w_{21} b_{31} - b_{21}) dt, \\
\frac{db_{22}}{dt} &= b_{21} + \frac{1}{2} \int_0^t (b_{11} - w_{11} b_{31}) dt, \\
\frac{db_{32}}{dt} &= b_{31} + \frac{1}{2} \int_0^t (w_{11} b_{21} - w_{21} b_{11}) dt.
\end{align*}
\]
In above equations we substitute the values of \( w_{11}, w_{21}, b_{11}, b_{21} \) and \( b_{31} \) from equations (5.4.4) and integrating, we get

\[
\begin{align*}
   w_{12} &= k_4 - \frac{2g}{\rho_b} k_2 t - \frac{k_7 t^2}{2}, \\
   w_{22} &= k_5 + \frac{2g}{\rho_b} k_1 t + \frac{g b_1 t^2}{2\rho_b}, \\
   b_{12} &= k_1 + \frac{1}{2} (k_3 k_5 - k_2 + 2k_6) t \\
   &\quad + \frac{1}{4} \left(\frac{g}{\rho_b} k_1 k_3 + k_5 k_8 - k_7\right) t^2 + \frac{g}{6\rho_b} k_1 k_8 t^3, \\
   b_{22} &= k_2 + \frac{1}{2} (k_1 + 2k_7) t \\
   &\quad + \frac{1}{4} \left(k_6 - k_3 k_4 - k_4 k_8 + \frac{g}{\rho_b} k_2 k_3\right) t^2 - \frac{g}{6\rho_b} k_2 k_8 t^3, \\
   b_{32} &= k_3 + \frac{1}{2} (2k_8 + k_2 k_4 - k_1 k_5) t \\
   &\quad + \frac{1}{2} \left(k_2 k_7 - k_5 k_6 - \frac{2g}{\rho_b} k_1^2 - \frac{g}{\rho_b} k_2 k_7\right) t^2 - \frac{g}{6\rho_b} k_1 k_6 t^3.
\end{align*}
\]

(5.4.6)

Continuing like this, we compute the \((n + 1)\)th iteration and these are given in the form of following recursion relations.

\[
\begin{align*}
   w_{1n+1} &= w_{1n} - \frac{g}{\rho_b} \int_0^t b_{2n} dt, \\
   w_{2n+1} &= w_{2n} + \frac{g}{\rho_b} \int_0^t b_{1n} dt, \\
   b_{1n+1} &= b_{1n} + \frac{1}{2} \int_0^t (w_{2n} b_{3n} - b_{2n}) dt, \\
   b_{2n+1} &= b_{2n} + \frac{1}{2} \int_0^t (b_{1n} - w_{1n} b_{3n}) dt, \\
   b_{3n+1} &= b_{3n} + \frac{1}{2} \int_0^t (w_{1n} b_{2n} - w_{2n} b_{1n}) dt.
\end{align*}
\]

(5.4.7)

In the above recurrence relations, the integrand part in all the integrals are in terms of pre-determined .
5.5 Experimental Results

MATHEMATICA 9 has been used for calculating polynomials and solutions. After calculations, the results has been verified with exact solutions. We have considered the initial conditions $b_{10} = 0.001, b_{20} = 0.0, b_{30} = 1.0, w_{10} = 0.0005, w_{20} = 0.00309, w_{30} = 1.00$ and $g = 9.8, \rho_b = 2$. We have performed six iterations by ADM and obtained following solutions.

\begin{align*}
w_1 &= 0.0005 + 0.1 - 0.0018375t^2 - 0.00378526t^3 - 0.000847282t^4 \\
&+ 0.0000630942t^5 + 1.9495 \times 10^{-10}t^6 + 7.80998 \times 10^{-11}t^7 \\
&- 2.02055 \times 10^{-11}t^8 - 2.61528 \times 10^{-11}t^9 - 4.76506 \times 10^{-12}t^{10} \\
&+ 3.72678 \times 10^{-17}t^{11}.
\end{align*}

\begin{align*}
w_2 &= 0.00309 + 0.0098t + 0.0113558t^2 + 0.00338913t^3 \\
&- 0.000946346t^4 - 0.000189875t^5 + 5.99331 \times 10^{-14}t^6 \\
&- 4.27014 \times 10^{-10}t^7 - 2.31585 \times 10^{-10}t^8 - 9.70443 \times 10^{-11}t^9 \\
&- 1.43407 \times 10^{-11}t^{10} + 2.07682 \times 10^{-16}t^{11} - 7.6088 \times 10^{-22}t^{12}.
\end{align*}

\begin{align*}
w_3 &= 1.0
\end{align*}
\[ b_1 = 0.001 + 0.00309t + 0.00103749t^2 - 0.000386264t^3 \\
- 0.0000864582t^4 + 6.43756 \times 10^{-6}t^5 - 2.7517 \times 10^{-10}t^6 \\
- 1.77324 \times 10^{-10}t^7 - 9.11841 \times 10^{-11}t^8 - 1.7302 \times 10^{-11}t^9 \\
- 1.48059 \times 10^{-12}t^{10} - 1.42617 \times 10^{-12}t^{11} - 6.24261 \times 10^{-13}t^{12} \\
- 8.49761 \times 10^{-14}t^{13} - 9.16258 \times 10^{-19}t^{14} - 8.2046 \times 10^{-19}t^{15} \\
- 2.52515 \times 10^{-19}t^{16} - 5.01867 \times 10^{-20}t^{17} - 4.63536 \times 10^{-21}t^{18} \\
- 5.78547 \times 10^{-26}t^{19} - 2.54018 \times 10^{-26}t^{20} - 1.8486 \times 10^{-27}t^{21} \\
+ 3.79518 \times 10^{-32}t^{22} - 2.61256 \times 10^{-37}t^{23} + 6.02809 \times 10^{-43}t^{24}. \]

\[ b_2 = 0. + 0.0005t + 0.00115875t^2 + 0.000345829t^3 \\
- 0.0000965659t^4 - 0.0000193749t^5 + 4.77509 \times 10^{-11}t^6 \\
- 3.87138 \times 10^{-12}t^7 + 3.86758 \times 10^{-13}t^8 - 9.46058 \times 10^{-12}t^9 \\
- 8.03788 \times 10^{-12}t^{10} - 3.07135 \times 10^{-12}t^{11} - 4.60284 \times 10^{-13}t^{12} \\
- 8.96157 \times 10^{-19}t^{13} - 1.74133 \times 10^{-18}t^{14} - 1.85618 \times 10^{-19}t^{15} \\
+ 2.05084 \times 10^{-24}t^{16} - 5.25597 \times 10^{-26}t^{17} - 5.53518 \times 10^{-27}t^{18} \\
+ 3.68283 \times 10^{-32}t^{19}. \]

\[ b_3 = 1. - 0.000016995t - 2.38726 \times 10^{-6}t^2 - 1.77675 \times 10^{-6}t^3 \\
- 5.81188 \times 10^{-7}t^4 - 9.26956 \times 10^{-8}t^5 + 9.85997 \times 10^{-8}t^6 \\
+ 2.23779 \times 10^{-8}t^7 - 6.16238 \times 10^{-9}t^8 - 1.38577 \times 10^{-9}t^9 \\
+ 3.49743 \times 10^{-15}t^{10} - 5.93659 \times 10^{-15}t^{11} - 7.30831 \times 10^{-16}t^{12} \\
+ 1.54358 \times 10^{-17}t^{13} - 5.58945 \times 10^{-21}t^{14} - 2.06705 \times 10^{-21}t^{15} \\
- 4.30585 \times 10^{-22}t^{16} - 3.78019 \times 10^{-23}t^{17} + 5.83633 \times 10^{-28}t^{18} \\
- 2.27599 \times 10^{-33}t^{19}. \]

The following graphs show results obtained by ADM and we have used MATHEMATICA 9 to plot these graphs.
5.5 Experimental Results

Figure 5.1: Graphs for $b_1$ on unstable manifold by ADM

Figure 5.2: Graphs for $b_2$ on unstable manifold by ADM

Figure 5.3: Graphs for $b_3$ on unstable manifold by ADM
5.5 Experimental Results

The numerical solutions is calculated with the step size of $h = 0.0001$. We have taken $t_0 = 0$ and we have considered the intervals $[0, h]$, $[h, 2h]$, $[2h, 3h]$ and so on. We have calculated the numerical solution with this step size. Regarding these numerical solutions, we had a discussion with Prof. Varsha Daftardar-Gejji, Department of Mathematics, University of Pune and Dr. Sachin Bhalekar, Department of Mathematics, Shivaji university, Kolhapur. in this discussion, we came to know that we can obtain better solutions if we divide the above said intervals into smaller
subintervals. Consequently, we have refined each interval into 10 subintervals of equal size 0.00001 and obtained numerical solutions on these subintervals. Finally we have compared the these solutions with exact solutions. The results are then presented in the form of tables as below.

<table>
<thead>
<tr>
<th>Table 5.1: Values of $b_1$ by ADM</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t$</td>
</tr>
<tr>
<td>-------</td>
</tr>
<tr>
<td>0</td>
</tr>
<tr>
<td>0.0001</td>
</tr>
<tr>
<td>0.0002</td>
</tr>
<tr>
<td>0.0003</td>
</tr>
<tr>
<td>0.0004</td>
</tr>
<tr>
<td>0.0005</td>
</tr>
<tr>
<td>0.0006</td>
</tr>
<tr>
<td>0.0007</td>
</tr>
<tr>
<td>0.0008</td>
</tr>
<tr>
<td>0.0009</td>
</tr>
</tbody>
</table>

Here it can be noticed that the error in $b_1$ remains almost same before and after refinement. Whereas the error in $b_2$ increases after refinement. This increase in error of $b_2$ gets nullified by the decrease of error in $b_3$. Since after each iteration, the value of $|b|^2 = 1$ is calculated.
### 5.5 Experimental Results

#### Table 5.2: Values of $b_2$ by ADM

<table>
<thead>
<tr>
<th>$t$</th>
<th>Exact</th>
<th>Before refinement</th>
<th>Error</th>
<th>After refinement</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0.0001</td>
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<td>$5.00 \times 10^{-08}$</td>
<td>$-7.95882 \times 10^{-08}$</td>
<td>$7.96 \times 10^{-08}$</td>
</tr>
<tr>
<td>0.0002</td>
<td>0</td>
<td>$1.00046 \times 10^{-07}$</td>
<td>$1.00046 \times 10^{-07}$</td>
<td>$-1.59153 \times 10^{-07}$</td>
<td>$1.59 \times 10^{-07}$</td>
</tr>
<tr>
<td>0.0003</td>
<td>0</td>
<td>$1.50104 \times 10^{-07}$</td>
<td>$1.50104 \times 10^{-07}$</td>
<td>$-2.38695 \times 10^{-07}$</td>
<td>$2.39 \times 10^{-07}$</td>
</tr>
<tr>
<td>0.0004</td>
<td>0</td>
<td>$2.00185 \times 10^{-07}$</td>
<td>$2.00185 \times 10^{-07}$</td>
<td>$-3.18214 \times 10^{-07}$</td>
<td>$3.18 \times 10^{-07}$</td>
</tr>
<tr>
<td>0.0005</td>
<td>0</td>
<td>$2.5029 \times 10^{-07}$</td>
<td>$2.5029 \times 10^{-07}$</td>
<td>$-3.97709 \times 10^{-07}$</td>
<td>$3.98 \times 10^{-07}$</td>
</tr>
<tr>
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<td>0</td>
<td>$3.00417 \times 10^{-07}$</td>
<td>$3.00417 \times 10^{-07}$</td>
<td>$-4.77181 \times 10^{-07}$</td>
<td>$4.77 \times 10^{-07}$</td>
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<tr>
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<td>$4.00742 \times 10^{-07}$</td>
<td>$-6.36056 \times 10^{-07}$</td>
<td>$6.36 \times 10^{-07}$</td>
</tr>
<tr>
<td>0.0009</td>
<td>0</td>
<td>$4.50939 \times 10^{-07}$</td>
<td>$4.50939 \times 10^{-07}$</td>
<td>$-7.15459 \times 10^{-07}$</td>
<td>$7.15 \times 10^{-07}$</td>
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#### Table 5.3: Values of $b_3$ by ADM

<table>
<thead>
<tr>
<th>$t$</th>
<th>Exact</th>
<th>Before refinement</th>
<th>Error</th>
<th>After refinement</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
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<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>0.0001</td>
<td>0.99999999</td>
<td>0.9999999998</td>
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<td>$3.09226 \times 10^{-10}$</td>
</tr>
<tr>
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<td>$9.94001 \times 10^{-07}$</td>
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</tr>
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<tr>
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</tr>
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</table>
### Table 5.4: Values of $w_1$ by ADM

<table>
<thead>
<tr>
<th>$w_1$</th>
<th>ADM</th>
<th>Exact</th>
<th>Before refinement</th>
<th>Error</th>
<th>After refinement</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
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<td>0.0005</td>
<td>0.0005</td>
<td>0.0005</td>
<td>0</td>
<td>0.0005</td>
<td>$1.33732 \times 10^{-11}$</td>
</tr>
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<td>0.0005</td>
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<td>0.0005</td>
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### Table 5.5: Values of $w_2$ by ADM

<table>
<thead>
<tr>
<th>$w_2$</th>
<th>ADM</th>
<th>Exact</th>
<th>Before refinement</th>
<th>Error</th>
<th>After refinement</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t$</td>
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<td></td>
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<td>0.003093922</td>
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<td>0.003094</td>
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<td>$3.84727 \times 10^{-06}$</td>
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<td>$5.82921 \times 10^{-06}$</td>
</tr>
</tbody>
</table>
5.6 Critical Time Interval

The value \( t_c \) of time variable \( t \) is said to be critical if the value of \( k \) given by (2.2.12) becomes complex at \( t = t_c \). Because of the limitation of software, it is highly difficult to find the critical value \( t_c \) but we determine here the interval in which the critical value falls.

We find the numerical solutions \( w_1, w_2, b_1, b_2 \) and \( b_3 \) in terms of polynomials, which are approximated at the sixth iteration using ADM.

Further more we find the polynomial for \( k \) as below.

\[
k = (\sqrt{38.2 - 0.000333102t} - 0.0000467903t^2 - 0.0000348243t^3 \\
- 0.000011391t^4 - 0.0000018168t^5 + 0.0000019325t^6 \\
+ 4.386 \times 10^{-7}t^7 - 1.2078 \times 10^{-7}t^8 - 2.7161 \times 10^{-8}t^9 \\
+ 6.8549 \times 10^{-14}t^{10} - 1.1635 \times 10^{-13}t^{11} - 1.4324 \times 10^{-14}t^{12} \\
+ 3.0254 \times 10^{-16}t^{13} - 1.0955 \times 10^{-19}t^{14} - 4.0514 \times 10^{-20}t^{15} \\
- 8.4394 \times 10^{-21}t^{16} - 7.4091 \times 10^{-22}t^{17} + 1.1439 \times 10^{-26}t^{18} \\
- 4.4609 \times 10^{-32}t^{19})(0 + 0.000016995t + 0.0000023872t^2 \\
+ 0.00000177675t^3 + 5.8118 \times 10^{-7}t^4 + 9.2695 \times 10^{-8}t^5 \\
- 9.8599 \times 10^{-8}t^6 - 2.2377 \times 10^{-8}t^7 + 6.1623 \times 10^{-9}t^8 \\
+ 1.3857 \times 10^{-9}t^9 - 3.4974 \times 10^{-15}t^{10} + 5.9365 \times 10^{-15}t^{11} \\
+ 7.3083 \times 10^{-16}t^{12} - 1.5435 \times 10^{-17}t^{13} + 5.5894 \times 10^{-21}t^{14} \\
+ 2.0670 \times 10^{-21}t^{15} + 4.3058 \times 10^{-22}t^{16} + 3.7801 \times 10^{-23}t^{17} \\
- 5.8363 \times 10^{-28}t^{18} + 2.2759 \times 10^{-33}t^{19})/(2 - 0.000016995t \\
- 0.0000023872t^2 - 0.00000177675t^3 - 5.8118 \times 10^{-7}t^4 \\
- 9.2695 \times 10^{-8}t^5 + 9.8599 \times 10^{-8}t^6 + 2.2377 \times 10^{-8}t^7 \\
- 6.1623 \times 10^{-9}t^8 - 1.3857 \times 10^{-9}t^9 + 3.4974 \times 10^{-15}t^{10} \\
- 5.9365 \times 10^{-15}t^{11} - 7.3083 \times 10^{-16}t^{12} + 1.5435 \times 10^{-17}t^{13} \\
- 5.5894 \times 10^{-21}t^{14} - 2.0670 \times 10^{-21}t^{15} - 4.3058 \times 10^{-22}t^{16} \\
- 3.7801 \times 10^{-23}t^{17} + 5.8363 \times 10^{-28}t^{18} - 2.2759 \times 10^{-33}t^{19})
\]

From this equation, we determined the interval \((10.15387168, 10.15387169)\)
in which the critical value $t_c$ lies. MATHEMATICA 9 is used for investigating this interval.

5.7 Conclusion

The scheme of Adomian Decomposition Method has been implemented to obtain the numerical solution of the system (2.2.8). In the calculations, we found the errors is of order $10^{-11}$. During the implementation of this method, we conclude that this method is very effective to obtain the numerical solutions on intervals of small size. The convergence of this method is discussed by G. Adomian in his book ([3], Page No.9). Hence, the numerical solutions obtained are convergent to the exact solutions and the error is bounded. The error occurred in the calculations can be reduced by reducing step size. The interval in which the critical value $t_c$ falls is $(10.15387168, 10.15387169)$.

Note 5.7.1. This work has been published in the International Journal as, ‘Numerical Solution to System of Six Coupled Nonlinear ODEs by Adomian Decomposition Method’, International Journal of Mathematics and Computer Research, ISSN:2320 7167, Vol-(3), No.2, Pages 876-887, (2015), (cf. [17]).