WAVE MOTION OF SMALL AMPLITUDE IN A HOT GAS.
This part deals with another linear problem viz. the small amplitude wave propagation in a compressible medium. This when formulated and solved in its complete generality is an investigation of considerable physical interest. It deals with predicting all conceivable modes of small disturbances, that the system under consideration is capable of generating or supporting. When studied in its completeness this not only tells about the dispersive properties of the medium, an information of great import in the theory of communication, it also helps to understand the phenomena of shock structure, aerodynamic noise, stability and turbulence. Mathematically it is formulated as an eigen-value problem. It is well known that in a compressible medium, two types of waves can propagate viz transverse and longitudinal. A considerable amount of literature exists on this topic specially works of Laplace, Stokes & Rayleigh (See Lamb 1932). Recently Synge (1957) examined this problem in a general way by including the effect of viscosity and heat conduction and also showed that there can exist amplitude waves which have no frequency and wave length like the other type called phase waves. It is also shown, though less explicitly, that the transverse waves (a class of phase waves) are damped because of viscosity and are not effected by the energetics of the system while thermal conduction and viscosity both damp the longitudinal waves.

Because of his interest in the study of sun spots
and other astrophysical problems, Alfven (1950) found that in an electrically conducting incompressible fluid in the presence of a uniform magnetic field, the small plane disturbances, which are essentially transverse waves (because $\nabla \cdot \mathbf{u} = 0$, $\mathbf{u}$ being the velocity vector) propagate along the magnetic lines of force. The velocity of propagation of such waves depends upon the strength of the magnetic field and fluid density. The experimental verification of the existence of such waves has been done by Lundquist (1949) and Linheart (1954) who used mercury and liquid sodium as the working fluids. In the plasma generated from hydrogen or inert gases the propagation of these waves have been observed experimentally by Allen (1959), Jephcott (1959), Wilcox (1960) and others.

Although Alfven waves (which are also called magneto-hydrodynamic waves) also represent one of the possible oscillations of a compressible fluid, yet there exist other compressible waves which are also affected by magnetic field. Depending upon the orientation of the impressed magnetic field, there are two types of magneto-acoustic waves (oblique magnetic field) - fast and slow. The former propagates with a velocity greater than the velocity of sound in that medium while the later travels with a velocity which is smaller than the sound speed. For a plane wave propagating normal to the impressed magnetic field, there is only one such compressive mode called magnetosonic and travels with a velocity which is the
resultant of sound and Alfven wave velocity. Such waves, for the first time, have been considered by Van de Hulst (1951) and Herlofson (1950). They have been further investigated by Friedricks (1957), Banos (1955), Lighthill (1960), Astrom (1950,51), Ludford (1959) and others. Banos (1958) examined the effect of finite electrical conductivity along with viscosity and heat conduction. Nearly all the authors mentioned above, confined themselves to the study of forced oscillations but Ludford (1959) seems to be the first to examine the problem of free oscillations in this context by taking the electrical conductivity of the fluid to be finite. The later problem as distinguished from that of forced oscillations differs in the study of eigen-values. For example in the case of free oscillation problem one investigates the time behaviour of disturbances of given wave length (stability aspects ) while in the other case the eigen-values of wave number (or wave length) are explored assuming the frequency of the disturbances to be a known real quantity associated with the disturbing force. This distinction, however, holds for an infinite dissipative system; for a conservative one these two problems merge into each other.

The need to extend this to high temperature fluid mechanics where the role played by radiative transfer may become significant, has also been felt and we have contributions which embody this refinement within its scope, as applied to the problem under consideration. The high
temperature effect of dissociation on the wave motion is studied by Moore (1958). Stokes is the first to examine the effect of radiative transfer on sound propagation. He assumed the air to be highly transparent. A general treatment of the problem including viscosity, heat conduction along with radiative transfer has been given by Prokofyev (1957, 61), Vincenti and Baldwin (1962) have also examined this problem for a radiating medium and also investigated the effect of a radiating wall. Pai & Speth (1959) have studied the effect of radiation and magnetic field in this context. They have included the effect of radiation pressure and energy in their analysis and used diffusion approximation of the radiative transfer equation. Their formulation is characterized by a single radiation parameter \( \gamma \) which is the ratio of the radiation to gas pressure, and is such that as \( \gamma \to 0 \) the system of equations reduce to those of no radiative transfer. This would represent a state (i.e. \( \gamma \to 0 \)) when radiation pressure is negligible as compared to gas pressure which may not necessarily be the same when radiative transfer is unimportant. For example the radiation pressure and energy become important only at very high temperatures \( \sim 10^7 \) or so while radiation as a mode of energy transfer may have to be considered at much lower temperatures say \( \sim 300^\circ \) K or so. Radiation pressure and energy give rise to only a modified acoustic velocity in these problems and the diffusion approximation of radiative transfer to a modified conductivity,
which in turn give rise to a mode similar to the one obtainable for an ordinary non-radiating thermally conducting gas. The latter is not a radiation induced mode (in the sense of Vincenti & Baldwin 1962) in the real sense of the word. This is so because in the presence of both thermal conduction and radiative-transfer and for a general value of the absorption coefficient one gets three independent modes - ordinary acoustic mode, one due to thermal conduction and the third due to radiation. For a diffusion approximation the last two modes become degenerate. In this sense, therefore the analysis of Pai & Speth (1959) though more tractable is rather restricted. The recent works of Vincenti and Baldwin (1962) and Baldwin (1963) for an ordinary gas are quite general including the effect of change of the absorption coefficient with frequency. However, their analysis is only true for one-dimensional gas flow and does not easily admit itself to two or three dimensional problems.

This part is devoted to the study of some aspects of this problem and is divided into two chapters. In the first chapter we have examined the problem of small amplitude wave propagation for a radiating ionized fluid in the absence as well as presence of magnetic field. The medium is considered to be infinite in extent and the effect of presence of solid boundaries is not taken into account. Radiation pressure and energy have been neglected. Milne-Eddington approximation has been used to reduce the integro-differential equations of radiative transfer to a second order equation
and has already been discussed in the chapter on fundamental equations. The present approximation is also useful in the study of two or three dimensional problems but has been used for the study of wave propagation in one direction only. The second chapter deals with problem of gravitational instability and the effect of radiative transfer on Jean's criteria. This problem falls under the category of free oscillation problems when the effect of self gravitation is also included. Such a problem in the absence of radiation has been studied by Jeans (see Chandrasekhar 1961). Its extension to hydromagnetics in the absence as well as presence of rotation has been carried out by Chandrasekhar (1961). The effects of viscosity, thermal conduction and finite electrical conductivity have been examined by Kumar (1960,61) and Pacholczyk & Stodolkiewicz (1960) respectively. In this connection it may be pointed out that looked from the point of view of wave propagation, the subject matter of part one, broadly speaking, also falls under the category of free oscillation problem. The only difference being that there the time behaviour of transverse waves is examined while here the time behaviour of longitudinal waves is considered.
CHAPTER I: EFFECT OF RADIATIVE TRANSFER ON WAVE MOTION OF SMALL AMPLITUDE IN A HOT FLUID.

In this chapter we shall investigate the problem of small amplitude wave motion in a radiating medium. The basic equations have already been given in the chapter on fundamental equations. Now we proceed to linearize these equations. The initial equilibrium state of plasma is characterized by

\[ \mathbf{u}^0 = \mathbf{0}; E^0 = 0; \mathbf{H}^0 = \mathbf{H}^0; \rho = \rho_0; \ \rho = \rho_0; \ \mathbf{T} = \mathbf{T}_0; \ \Phi = \Phi_0 = 0. \]

When this is perturbed the resultant motion is assumed to be a small, such that it can be represented by the linearized form of the equations which are

\[
\begin{align*}
\frac{\partial \rho}{\partial t} + \text{div} (\rho \mathbf{u}) &= 0 \\
\rho \frac{\partial \mathbf{u}}{\partial t} &= -\nabla p + \mu \nabla \times \mathbf{H}^0 + \rho \gamma \nabla \mathbf{u} \\
\rho \frac{\partial \mathbf{\Theta}}{\partial t} &= \frac{\partial \mathbf{E}}{\partial t} + \mathbf{\Theta} + \rho \mathbf{\Theta} \\
\text{curl} \mathbf{E} &= -\mu \frac{\partial \mathbf{\Theta}}{\partial t} \\
\text{curl} \mathbf{H} &= 4\pi \frac{\partial \mathbf{J}}{\partial t} \\
\mathbf{J} &= \sigma \left[ \mathbf{E} + \mu \nabla \times \mathbf{H}^0 \right] \\
\frac{\partial \Phi}{\partial t} &= \frac{\partial \mathbf{F}}{\partial t} + \mathbf{\Theta} + \Theta \mathbf{T}_0; \ \Phi = -\frac{df}{dx} \\
\frac{\partial^2 F}{\partial x^2} &= 4\pi \kappa \mathbf{S} \frac{\partial \mathbf{\Theta}}{\partial x} + 3\kappa^2 F
\end{align*}
\]

where \( S = 4\sigma_0 \mathbf{F}_0 \), \( \sigma_0 \) being the Stefan's constant. The suffix \( x \) from \( F \) has been removed. \( \mathbf{u}, \rho, \rho, \mathbf{\Theta}, \mathbf{F}, \mathbf{H} \) and \( \Phi \) represent the disturbed state of the medium. Equations (1) are the basic equations of the problem.
Dispersion relation.

The axis of \( x \) has been taken along the direction of propagation and \( xy \)-plane contains the uniform magnetic field \( \vec{H}_0 = (H_x, H_y, 0) \). Since we are interested in the periodic solutions of (1) we take all the variables proportional to \( \exp[i(\omega t - \lambda x)] \) where \( \omega \) is the angular frequency and \( \lambda \) the wave number which is equal to \( \lambda \pi / (\text{wave length}) \). The perturbed quantities may, then be divided into two groups involving \( (E_x, E_y, h_z, \omega) \) and \( (E_z, h_x, u, v, p, p, F) \) respectively. \( E_i, E_x, E_z; h_x, h_y, h_z; u, v, w \) being the components of \( \vec{E}, \vec{H} \) and \( \vec{U} \) along the coordinate axes. The first group may be said to represent a transverse wave but the second group does not characterize a purely longitudinal wave as Pai and Speth (1959) put it. On the other hand it represents the interaction of a longitudinal wave with a transverse wave whose plane of polarization is the plane containing the magnetic field. It can easily be seen that the first group remains unaffected by the compressibility and thus by radiative transfer too. It is because of the fact that in the case of transverse wave motion the energy equation gets uncoupled from the momentum equation and so any process contributing to the energetics of the system will leave the transverse wave unaffected while it will have a definite influence on the longitudinal waves. On the other hand, a non-conservative system of forces like rotation, magnetic field etc. will definitely contribute to the transverse...
wave motion. In this chapter, since we are interested in the study of the influence of radiative transfer, we shall keep our discussion confined to the second group which is affected by this mode of energy transfer. The first group has already been studied extensively in literature.

In the case of forced oscillations $\omega$ is supposed to be a given real quantity associated with the disturbing force and the eigen-values of $\lambda$ (which in general are complex) given by the determinantal equation of (1). Since the disturbed quantities have been assumed to vary like $\exp i(\omega t - \lambda x)$ the dispersion relation for these two cases remains the same. However, it leads to different conclusions depending on whether $\lambda$ or $\omega$ is complex. The determinantal equation of (1) can easily be written as

$$
\begin{align*}
\left[ \left( \frac{1}{R} + \frac{4}{3} i \frac{\omega}{p} \frac{1}{\bar{\rho}} \right) (1 + \frac{3 \alpha}{\bar{\rho}}) \frac{\bar{\lambda}}{\bar{\rho}} - \lambda^2 \right] \frac{i \omega}{p} + 3 \frac{\bar{\omega}}{\bar{\rho}} & \left( \gamma \lambda^2 + i \omega (i \omega + \nu \lambda^2 + \nu^2 \lambda^3) \right) \\
+ \frac{4}{3} \left( \frac{\omega^3 \rho_0}{\bar{\rho}} \right) & \left[ \left( \gamma \lambda^2 + i \omega (i \omega + \nu \lambda^2 + \nu^2 \lambda^3) \right) \frac{i \omega \bar{\rho}}{p} \right] = 0
\end{align*}
$$

The first square bracket on the left hand side of (2) is nothing but the expression for the dispersion relation of a real gas when radiative transfer effects are also taken into account. It may again be noticed that in this general case the initial temperature level of the medium is represented by a parameter $\lambda$ which has already appeared in
our previous investigations on stability. The equation (2) differs considerably from the corresponding relation obtained by Pai and Speth (1959). Their dispersion relation is a quartic in $\lambda^2$ as compared to the fifth order in our case and, therefore, gives only four modes of wave propagation. The extra mode which appears in our case is due to the different behaviour of molecular diffusion and radiative transfer not present in their analysis and in the limit of $k \to 0$ disappears from the above dispersion relation. In the next section we shall discuss the problem of forced oscillation in some detail.

FORCED OSCILLATIONS

Equation (2) is a fifth order equation in $\lambda^2$ and thus gives five different modes of wave propagation. It seems difficult to decipher the dispersion relation as given by equation (2). The study of some particular cases, however, seems useful.

Ordinary radiating gas:

For this we have $\nu = \kappa = \gamma = 0 \quad \frac{\kappa}{\rho} = \nu = 0$. This case corresponds to the one already studied by Vincenti and Baldwin (1962). A comparison of the results of the present section with those obtained by those authors will indicate the usefulness of the Milne-Eddington approximation employed here. The dispersion relation for this reduces to

$$\lambda^4 \frac{4\pi S}{k} \frac{1}{\rho + \frac{\omega^2 + \frac{\rho}{\kappa}}{\lambda^2 + 3k^2}} - \lambda^3 \left[ -i\omega + \frac{\rho}{k} \frac{\omega^2 + \frac{4\pi S}{\lambda^2 + 3k^2}}{\lambda^2 + 3k^2} \right] \frac{i\omega^3 \rho}{k} = 0 $$

(3)
This being a quadratic in shows that there are two
different modes of longitudinal wave propagation (the
interacting transverse wave being of hydromagnetic origin
is absent in the present case), a conclusion already
reached by Vincenti and Baldwin (1962).

Let us define

\[ N = \frac{4\pi}{5} \frac{S}{\beta c_0 a_0} \quad \beta = \frac{k a_0}{\omega} \quad C = a_0^2 \frac{\lambda^2}{\omega^2} \quad (4) \]

where \( a_0 \) is the adiabatic sound velocity equal to \( \sqrt{\gamma p_0} \)
and other symbols have their usual meaning. Equation (3)
now becomes

\[ (1 - \nu N \beta) \lambda^2 - C (1 - 3 \beta^2 i \nu N \beta) - 3\beta^2 = 0 \quad (5) \]

The solution of (5) is the same as obtained by Vincenti
and Baldwin (1962) except for the parameters \( N \) and \( \beta \).
These can be correlated with their \( N \) and \( \beta \) if we
take \( m = 1 \) and \( n^2 = 3 \) in the expressions for the exponential
integrals. A comparison with the values of \( m \) and \( n^2 \)
used in their calculations will show that they are quite
near to 1 and 3 respectively. So the use of Milne-
Eddington approximation gives results without any serious
loss in their physical content, with much less labour
than involved in the integro-differential equation of
transfer. As also pointed out by Vincenti and Baldwin,
the general solution of equation (5) is quite laborious.
We have obtained some approximate solutions of (5); but
not by series expansion in powers of \( \gamma^{-1} \) as done
by these authors but by defining some suitable parameters.
and looking for the solution for their limiting values. This approach much simpler and logical gives results easily in the asymptotic cases. Speaking of parameters, it may be mentioned that in general any problem embodying radiative transfer within its scope will be characterised by two parameters – one representative of initial temperature and other that of absorption coefficient $\mathcal{N}$ and $\beta$ above (as also in the works of Vincenti and Baldwin) are two such parameters. In asymptotic cases, however, namely transparent and opaque approximation they can always be suitably combined into one. If we define $\Delta = \mathcal{N} \beta / (1 + 3 \beta^2)$ it is such that it reduces to the single parameter definable in the two asymptotic cases (viz transparent and opaque). This parameter happens to be the one that occurs in the study of the free oscillation problem latter where it characterises the radiation phenomena irrespective of the mean freepath of the medium.

Coming back to the solution of equation (5), we have

$$C_1, C_2 = \left( \frac{1 - 3 \beta^2}{1 + 3 \beta^2} - i \gamma \Delta \right) \pm \left[ \left( 1 - 2 i \gamma \Delta \right) - 2 i \gamma \Delta \left( \frac{1 - 3 \beta^2}{1 + 3 \beta^2} + \frac{6 \beta^2}{\gamma(1 + 3 \beta^2)} \right) \right]^{1/2}$$

(6)

Now if $\Delta \ll 1$ or $\Delta \gg 1$

$$C_1, C_2 = \left( \frac{1 - 3 \beta^2}{1 + 3 \beta^2} - i \gamma \Delta \right) \pm \left( 1 - 2 i \gamma \Delta \right) \left[ 1 - i \gamma \Delta \left( \frac{1 - 3 \beta^2}{1 + 3 \beta^2} + \frac{6 \beta^2}{\gamma(1 + 3 \beta^2)} \right) \right]^{1/2}$$

$$= 2 \left( \frac{1}{1 + 3 \beta^2} - i \Delta \right)$$

(7)
for all \( \beta \) and when \( \Delta \to 1 \), we have

\[
C_1 C_2 = \left( \frac{1-3\beta^2}{1+3\beta^2} - i\gamma\Delta \right) \pm \left( \gamma\Delta \right) \frac{\gamma}{(1+i) \left[ \frac{1-3\beta^2}{1+3\beta^2} + \frac{6\beta^4}{(\alpha+3\beta^2)^2} \right] \left[ 1 + \frac{i(1-\gamma^2)\Delta}{4\Delta(1-3\beta^2) + 6\beta^4} \right]}
\]

\[
= 2 \left( \frac{1}{1+3\beta^2} - i\Delta \right)
\]

(8)

for all \( \beta \) except when \( \beta \) is zero or infinity. Calculations based on these expressions were performed and it was found that the boundary layer phenomena as discussed by Vincenti and Baldwin is also present in our equations and the general agreement with these results was quite good.

It may be pointed out that the transition from adiabatic sound (\( \Delta \ll 1 \)) to isothermal sound wave (for \( \Delta \gg 1 \)) takes place in a thin boundary layer of thickness \( \frac{1}{\Delta} \) and the transition in this region is governed by the corresponding equation for \( \Delta \ll 1 \) not \( \Delta \ll 1 \) or \( \Delta \gg 1 \).

The solution between \( \chi = 0 \) and \( \chi = \frac{1}{\Delta} \) where \( \chi = 3\beta^2/(1+3\beta^2) \) had to be joined by a continuous curve the major part of which is to be calculated from the expression for \( \Delta \ll 1 \).

In such problems one can always define a characteristic length in which the wave will decay to \( \frac{1}{\xi} \) of its value. If

\[
\frac{\omega}{\lambda} = \xi^* + i\gamma^*
\]

then such a length may be defined as

\[
\lambda_a = \frac{\xi^*}{\omega \gamma^*}
\]

(9)

This length comes out to be proportional to \( \lambda_a \), the radiation mean free path and the constant of proportionality
depends upon $\beta$ and the solution of equations (6) to (8).

**Ideal plasma:**

In this case $\gamma = \kappa = \gamma = 0$ and the only non-adiabatic process present is the radiative transfer. The dispersion relation (2) in the non-dimensional form can be written as

$$
\left[ \frac{N\beta}{\lambda^2 + 3\beta \omega_0^2} \right] \lambda^4 - \lambda^3 \left[ \frac{N\beta \omega}{\lambda^2 + 3\beta \omega_0^2} - \frac{\omega^2 P_0}{P_0} - i\omega \right] - \frac{i\omega P_0}{\gamma P_0} \right] (N \lambda - \omega)
$$

$$
+ i\omega \nu_0 \lambda \left[ i \frac{\lambda^2 \omega_0^2 P_0}{P_0} - \frac{N\beta}{\lambda^2 + 3\beta \omega_0^2} - \frac{\omega P_0}{P_0} \right] = 0 \quad (10)
$$

The last equation differs from that obtainable from Pai and Speth's (1959) general dispersion relation and in the absence of radiation pressure and energy form a particular case of the above equation.

To start with we assume $\nu_y = 0$; $\nu_x \neq 0$ i.e. there is no transverse applied magnetic field. The dispersion relation then degenerates into two factors one giving the propagation of ordinary damped acoustic wave as well as the radiation induced wave already discussed. The second factor gives the Alfvén wave which is not affected by the radiative transfer. Clearly these two basic modes are uncoupled. This latter conclusion has also been arrived at by Pai & Speth (1959) for the diffusion approximation of radiative transfer equation.

Next we assume $\nu_x = 0$ and $\nu_y \neq 0$ i.e. the longitudinal component of the magnetic field is absent.
In this case the dispersion relation becomes

\[ \frac{N \beta}{\lambda^2 + 3 \beta \omega^2 a^2} \lambda^4 - \lambda^2 \left\{ \frac{N \beta}{\lambda^2 + 3 \beta \omega^2 a^2} \frac{\omega^2 \gamma}{a^2} - i \right\} - i \frac{\omega^2}{a^2} \]

\[ - i V_f^2 a^2 \left[ \frac{i N \beta}{\lambda^2 + 3 \beta \omega^2 a^2} \frac{\gamma}{a^2} \lambda^2 - \frac{1}{a^2} \right] = 0 \]  

(11)

This being a quadratic in \( \lambda^2 \) gives us two different modes of wave propagation. In the limiting case of completely cold \( (N \rightarrow 0) \), transparent \( (\beta \rightarrow 0) \) or opaque plasma \( (\beta \rightarrow \infty) \), the dispersion relation reduces to

\[ \lambda^4 (1 + V_f^2 a^2) = \omega^2 a^2 \]  

(12)

Thus the velocity of propagation of the disturbances then becomes \( (a_0^2 + V_f^2)^{1/2} \). This is known as the effective sound speed.

When \( N \rightarrow \infty \) \( (\beta \text{ finite}) \), the dispersion relation reduces to

\[ \frac{\lambda^2}{\lambda^2 + 3 \beta \omega^2 a^2} \left[ \lambda^2 - \frac{\gamma \omega^2}{a^2} + \frac{\gamma V_f^2}{a^2} \lambda^2 \right] = 0 \]  

(13)

Obviously, the two roots of this equation are zero and the other two correspond to a modified acoustic wave whose velocity of propagation is given by \( (a_0^2 + V_f^2)^{1/2} \). We call this as the modified or the effective isothermal sound speed. Thus, as in the non-magnetic case, in the various limit of the parameters discussed above the non-equilibrium problem with radiative transfer reduces to
the corresponding case of equilibrium problem of acoustic wave propagation.

For a general magnetic field configuration i.e. \( V_x, V_y \neq 0 \), the dispersion relation (2) is a cubic in \( \lambda^2 \). In the limit of \( N \to 0 \) or \( \beta \to 0 \) or \( \beta \to \infty \) the results obtained for the magneto-gas-dynamic case hold and have been discussed by Friedricks (1957). Now, when \( N \to \infty \), the dispersion relation becomes

\[
\frac{\lambda^2}{\lambda^2 + 3\beta^2 \frac{\omega^2}{a^2}} \left[ (\lambda^2 - \gamma \frac{\omega^2}{a^2})(V_x^2 + \lambda^2 - \omega^2) - V_y^2 \lambda^2 \frac{\omega^2}{a^2} \right] = 0
\]  

(14)

The two roots of the equation (4) are zero, the other two are given by

\[
(1 - \gamma \frac{V_{\gamma}^2}{a^2})(V_x^2 - V_y^2) - \gamma V_x^2 V_y^2 \frac{a^2}{V^2} = 0
\]

(15)

where \( V = \omega/\lambda \)

This equation has the same form as that for the ordinary magneto-gas-dynamic case, except that the adiabatic sound velocity is replaced by the isothermal sound speed, and being a quadratic in \( V^2 \) possesses two roots viz \( V^{\text{fast}} \) and \( V^{\text{slow}} \). Thus as in the adiabatic case we have

\[
V_{\text{slow}} < a^2/\gamma < V_{\text{fast}}.
\]

This has no counterpart in the equilibrium magneto-acoustic case.

Another case which is of interest, in this context, arises when the gas pressure is less than the magnetic pres-
sure. This is characterised, in terms of our symbols, by the statement \( a_0^* \ll V_y^* \). In the absence of radiative transfer it has been discussed by Lighthill (1960). The dispersion relation, in this case, for a transverse magnetic field reduces to

\[
(V_y^2 - \omega^2 \Lambda^2) \left[ \gamma N \beta - i \left( 1 + \frac{3}{2} \beta^2 \omega^2 a_0^* \Lambda^2 \right) \right] = 0
\]  

(16)

Thus the velocity of propagation is independent of \( \Lambda^2 \) and \( \beta \). In fact it is just the Alfvén wave velocity. This, however, would not be the case if there were a longitudinal magnetic field also present.

The table 1 gives, in a consolidated way, the various excitable modes, under diverse physical conditions discussed above.

**Damped Modes:**

We now proceed to discuss some of the damped modes in the case of ideal plasma on the same lines as done for a ordinary radiating gas above. The dispersion relation (11) for a transverse magnetic field in terms of the variable \( C' \) already defined, becomes

\[
C^2 \left[ (1 + V_y^2 a_0^2) - i \gamma N (1 + \gamma V_y^2 a_0^2) \right] - C \left[ 1 - 3 \beta^2 (1 + V_y^2 a_0^2) - i \gamma N \beta \right] - 3 \beta^2 = 0
\]

Now if we put

\[
\gamma = \beta (1 + V_y^2 a_0^2)^{\frac{1}{2}}, \quad M = N (1 + V_y^2 a_0^2)^{\frac{1}{2}} \text{ and } C' = C (1 + V_y^2 a_0^2),
\]

it reduces to

\[
C^2 \left[ 1 - M \right] - C' \left[ 1 - 3 \gamma^2 - i \gamma M \gamma \right] - 3 \gamma^2 = 0
\]

(17)
The table gives the various excitable modes for a forced oscillation problem under diverse physical conditions, and are the non-dimensional parameters characterising the radiation phenomena and represent the temperature level and the absorption properties of the medium respectively. Is their suitable combination are the Alfvén wave speeds corresponding to horizontal and transverse magnetic fields respectively and is the adiabatic velocity of sound, all dimensional quantities.

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Suppressing the dash over $C'$, we get

$$C_1, C_2 = \frac{(1 - 3\gamma - i\gamma\Delta) \pm \left[(1 - 3\gamma - i\gamma\Delta)^2 + 12\gamma(1 - \gamma \Delta^2) \right]^{1/2}}{2 \left[1 - i\gamma \Delta^2 \right]}$$

Again let

$$\frac{1 + \gamma V_y/a_0}{1 + \gamma V_y/a_0} = \gamma \frac{a_0^2 + V_y}{a_0^2 + V_y} = \gamma Q (say),$$

where $Q$ represents the ratio of the isothermal magneto-sound speed to the adiabatic magneto-sound speed and for real gases it is always small (lies between $1/\gamma$ and $1$).

Thus

$$C_1, C_2 = \frac{(1 - 3\gamma - i\gamma\Delta^*) \pm \left[(1 - \gamma \Delta^*)^2 - 2i\gamma \Delta^* \left(\frac{1 - 3\gamma}{1 + 3\gamma^*} + \frac{6Q\gamma^*}{1 + 3\gamma^*}\right) \right]}{2 \left[1 + i\gamma \Delta^* \right]}$$

$\Delta^*$ in this $= M \gamma / (1 + 3\gamma^*)$ and plays the role of $\Delta'$ in an ordinary gas discussed earlier and is related to it by the following relation

$$\Delta^* = \Delta /\left(1 + 3\gamma^2 \right)$$

In the approximation $a_0^2 << V_y^2$ discussed already $Q$ is unity and we obtain $C = 1$, a result already discussed. For $\gamma \Delta^* << 1$ and $\gamma \Delta^* \gg 1$ and for all $\gamma$

$$C_1, C_2 = \frac{(1 - 3\gamma^2 - i\gamma \Delta^*) \pm \left[(1 - \gamma \Delta^*)^2 - 2i\gamma \Delta^* \left(\frac{1 - 3\gamma^2}{1 + 3\gamma^2} + \frac{6Q\gamma^*}{1 + 3\gamma^2}\right) \right]}{2 \left[1/(1 + 3\gamma^2) - i\gamma Q \Delta^* \right]}$$

(19)
and for $\Delta^* - 1$ and for all $f$ except $f = 0 \text{ or } \infty$

$$
\sqrt{\frac{(1-3y^2 - i\gamma\Delta^*) \pm (\gamma\Delta^*)^{1/2}}{(1+i3y^2 + i3y^2)^{1/2}}} \left[ 1 + \frac{i(1-i\gamma\Delta^*)}{\frac{1}{2}y\Delta^*(1-3y^2 + 3y^2)} \right]
$$

$$
C_1C_2 = \frac{2}{\sqrt{\frac{1}{(1+3y^2)} - i\gamma Q_0\Delta^*}}
$$

(20)

In this case $Q_0$ is the measure of the strength of the applied magnetic field. These approximate forms can help us to obtain the characteristics of the different waves. Only the first set for $\Delta^* \ll 1$ and $\Delta^* \gg 1$ goes over to the results of the equilibrium theory and corresponds to the four cases studied above. The set corresponding to $\Delta^* - 1$ does not fall under these approximations and so does not give results of the equilibrium theory.

For a general magnetic field, the dispersion relation which is a cubic in $\lambda^2$, may be written as

$$
\frac{INBc}{C + 3\beta^2} = \frac{V_{\lambda_o}^2}{V_{\lambda_o}^2} \frac{C^2 - C (1 + A^2_{\lambda_o}) + 1}{V_{\lambda_o}^2 C^2 - C (1 + \gamma A_{\lambda_o}^2) + \gamma}
$$

(21)

$$
= \frac{A_{\lambda_o}^2 \cos^2 \gamma C^2 - C (1 + A_{\lambda_o}^2) + 1}{A_{\lambda_o}^2 \cos^2 \gamma C^2 - C (1 + \gamma A_{\lambda_o}^2) + \gamma}
$$

where $A^2 = (V_{\lambda_o}^2 + V_0^2)$. $\gamma$ is the angle which $H_0$ makes with the direction of wave propagation. It may be pointed out that the numerator and denominator on the right hand side of the above equation correspond to the propagation of slow and fast waves in the adiabatic and
isothermal cases respectively. As it is, it seems difficult to extend a discussion of the above equation for arbitrary values of the different parameters. However, an insight into the problem can be achieved for $A/\alpha \ll 1$ and $A/\alpha \gg 1$. These are the two cases which we are going to discuss in some detail.

(1) $A/\alpha \ll 1$. From our past experience for a similar problem in non-radiating medium, the dispersion relation can easily be written as

$$\frac{i N \beta C}{C + 3 \beta^2} = \frac{(A/\alpha)^2 \cos^2 \gamma (c-1)}{(A/\alpha)^2 \cos^2 \gamma (c-1)(c-\gamma)}$$

which in turn gives

$$C = \frac{\alpha^2}{A^2 \cos^2 \gamma} \quad (\text{ii}) \quad \text{and} \quad \frac{i N \beta C}{C + 3 \beta^2} = \frac{c-1}{c-\gamma} \quad (\text{ii})$$

The first value of $C$ is nothing but the one corresponding to a slow wave. This shows that to this order of approximation the slow wave is unaffected by radiation. The second equation (22-ii) is nothing but equation (5). This implies that the fast wave under this approximation propagates like an ordinary acoustic wave. From this it may also be seen that the radiation induced wave is also not affected by the magnitude as well as the orientation of the magnetic field.

(ii) $A/\alpha \gg 1$. In this case again inspired by the previous studies for no radiative transfer the dispersion relation splits as

$$C = \frac{\alpha^2}{A^2} \quad (23-i)$$
Equation (23-i) shows that the fast wave is not influenced by radiation to this order of approximation. On the other hand the slow as well as the radiation induced waves, although independent of the strength of the magnetic field yet are influenced by the orientation of the field. In the absence of radiation this result is already known but the manner in which the radiation induced wave is affected by the direction of magnetic field seems to be quite interesting.

**FREE OSCILLATIONS**

In this case one studies as to how the waves of particular wave length behave with respect to time and as such the wave number of the disturbances viz. \( \lambda \) is a known real quantity and one is required to study the eigen-values of \( \omega \). The dispersion relation for the problem is the same as given by equation (2). In what follows, we shall study some of the particular cases of (2). Before we do so it seems advisable to re-examine the parameters characterising the radiation problem. The same parameters (especially \( \beta \) ) as were used to study the problem of forced oscillations, cannot be used in the present case as \( \omega \) is a complex quantity. In the present case a characteristic length is available in the problem. This is the wave length of the disturbances, the time behaviour of which we are required to study. Thus, we put

\[
\beta = \frac{\omega}{\lambda}
\]
Now if small and large would mean that the mean free path
of radiation field is larger or smaller than the wave length
of the disturbances respectively.

Ordinary radiating gas:

The dispersion relation (3) in terms of the above
definition of \( \beta \) becomes

\[
\frac{\omega^3}{a_0^3} - i \gamma \lambda \frac{\omega^2}{a_0^2} \frac{N \beta}{1 + 3 \beta^2} - \frac{\omega \lambda^2}{a_0} + i \frac{N \beta}{1 + 3 \beta^2} \lambda^3 = 0
\]

It may be noticed that the two parameters characterising
the radiation phenomena combine into a single one given by

\[ \Delta = \frac{N \beta}{(1 + 3 \beta^2)} \]

We now have

\[
\frac{\omega^3}{a_0^3} - i \gamma \Delta \lambda \frac{\omega^2}{a_0^2} - \frac{\omega \lambda^2}{a_0} + i \Delta \lambda^3 = 0 \tag{24}
\]

The last equation being a cubic in \( \omega \) gives three values
of \( \omega \) for each \( \lambda \) and \( \Delta \). If we separate the real
and imaginary parts of the dispersion relations (assuming
\( \omega \) to be complex), it will be seen that the real part of
one root is always zero which means pure decay. It will be
seen that this corresponds to the radiation induced wave.

In the limit of \( \Delta \to 0 \), the \( \omega, \lambda \) equation reduces to
that for an isentropic acoustic wave, while in the limit of
\( \Delta \to \infty \) to that of an isothermal sound wave. In order to
know the effect of finite ‘\( \Delta \)’ on these waves, we study
the small deviations from the limiting states discussed above,
by a perturbation method. Thus
On the other hand the damping of the third wave, which
vanishes in these limits for \( \Delta \) depends differently on it. This may be seen by putting \( V^* = i \delta \) in the equation (24), where \( V^* = \omega/\lambda a_0 \). Thus
\[
\frac{\gamma \delta^2 + 1}{\delta^2 + 1} = \delta/\Delta
\]
The last equation being a cubic in \( \delta \) shows that there are three roots which correspond to pure decay. Two of them do not reflect in our analysis due to the order of approximation used in the problem. However, even one of them is sufficient to tell us the behaviour of the wave. For all values of \( \delta \) the left hand side of this equation is of the order of unity or at the most \( \gamma \) which shows that \( \delta \) is proportional to \( \Delta \). This conclusion is independent of the value of \( \Delta \). However for \( \Delta \ll 1 \), \( \delta^2 \) being of the order of \( \Delta^2 \) is negligible as compared to unity while for \( \Delta \gg 1 \) it is sufficiently large so that \( \delta^2 \) is very much greater than unity. Thus for
\[
\Delta \ll 1 \quad ; \quad \delta = \Delta,
\]
and
\[
\Delta \gg 1 \quad ; \quad \delta = \gamma \Delta.
\]
Thus in the case of radiation induced wave
\[
\omega = \begin{cases}
    i \Delta \lambda a_0 & \text{for } \Delta \ll 1 \\
    i \gamma \Delta \lambda a_0 & \text{for } \Delta \gg 1
\end{cases}
\]
These expressions for the damping of the radiation induced wave reflect on its actual behaviour. In the limit of $\Delta \to 0$ one may say that the radiation induced wave is not at all present while in the case of $\Delta \to \infty$ the wave is present but is infinitely damped. In the former case the wave is absent by virtue of not being excited while in the later case the wave is there but does not affect the stability of the system because it gets damped out due to the large temperature level of the medium. It seems that somewhere in between these two limits of $\Delta$ the radiation induced wave gets switched on and then finally damps out practically.

In such problems one can define a characteristic decay time i.e. the time in which the wave of given wave length will decay to $\frac{1}{e}$ th of its value. If we put

$$\omega/\lambda = \xi^* + i\gamma^*$$

then the time of decay is given by

$$T_{\text{decay}} = \frac{1}{\lambda \gamma^*}$$

In the present case it comes out to be as follows:

(i) $\Delta << 1$

$$T_d = \frac{2}{(\gamma-1)\Delta} \frac{1}{\lambda a_0} = \frac{2\beta}{(\gamma-1)\Delta} \frac{\lambda_s}{a_0}$$

This shows that the decay time is proportional to the time taken by a sound wave to travel a distance of radiation
mean free path or the wave length of the disturbance. For

\[ \Delta = 0.1, \quad \gamma = 1.4, \]

this constant of proportionality comes out to be \(2.5/\lambda\) or \(50 \beta\) depending upon the relation used.

(ii) \(\Delta \gg 1\). In this case the characteristic time comes out to be

\[ T_d = \frac{2\gamma^{3/2}\Delta}{(\gamma - 1) \lambda \alpha_{\text{isoth}}}, \]

\[ = \frac{2\beta \Delta \gamma^{3/2}}{(\gamma - 1)} \frac{\lambda_r}{\alpha_{\text{isoth}}} \]

\[ = \frac{\gamma^{3/2} \Delta}{\pi (\gamma - 1)} \left[ \frac{\text{Wave length of the disturbance}}{\text{Isothermal velocity of sound}} \right] \]

\[ = \frac{2\beta \gamma^{3/2} \Delta}{(\gamma - 1)} \left[ \frac{\text{Radiation mean free path}}{\text{Velocity of the isothermal sound wave}} \right] \]

which shows that the decay time of an isothermal sound wave of given wave length is proportional to the time taken by it to travel a distance of wave length or the radiation mean free path. For \(\gamma = 1.4\) and \(\Delta = 10\), the constant of proportionality comes out to be \(41.412/\lambda\) or \((82.324 \beta)\) depending upon the relation used.

The time of decay of the radiation induced wave is also proportional to the times described above in the two limits of \(\Delta\) but the constant of proportionality is different. For example, for \(\Delta \ll 1\) it is \(10 \beta\) and for \(\Delta \gg 1\); \(\beta/14 \gamma^{1/2}\). This shows that the life time of the adiabatic sound wave is about five times the life time of the corresponding or accompanying radiation induced wave. On the other hand the life time of isothermal acoustic
wave is even more and is about 1372 times the life time of the accompanying radiation induced wave.

In the next section we study the effect of a transverse magnetic field on the frequency of the disturbances of given wavelength. Longitudinal magnetic field, since does not interact with the medium, leaves unaffected the different modes.

**Ideal Plasma:**

For a transverse magnetic field the dispersion relation reduces to, in terms of $V^*$ and $\Delta$

$$V^3 - i\gamma \Delta V^2 - V^*(1 + \nu^2 a^2) + i\Delta (1 + \gamma \nu^2 a^2) = 0$$

where $V^*$ and $\Delta$ are already defined in last section.

It may easily be seen that one root of this cubic corresponds to a pure decay. The other two roots correspond to magneto-acoustic and isothermal magneto-acoustic waves which are recovered in the limits of $\Delta \to 0$ and $\Delta \to \infty$ respectively. In order to study the effect of finite on these waves, we proceed as in the last section and obtain

$$\omega = \begin{cases} 
 b_0 \lambda \left[ t + \frac{i(\gamma - 1)}{2} \Delta \left( \frac{a_0}{b_0} \right)^3 \right] & \Delta \ll 1 \\
 \lambda \left\{ \pm b_{\text{on}} + \frac{a_0(\gamma - 1)}{2} \right\} & \Delta \gg 1 
\end{cases}$$

where

$$b_0^- = (a_0^2 + \nu^2) \text{ and } b_{\text{on}}^- = (a_0^2 + \nu^2).$$

It is interesting to note that the damping of isothermal magneto-acoustic wave ($\Delta \gg 1$) is not affected by magnetic field and is the same as obtained in the non-magnetic case while magnetic field reduces the
damping of magneto-acoustic wave ($\Delta \ll 1$). The effect of magnetic field on the radiation induced wave can similarly be studied as done in the last section. We get after putting $\nu^* = i \delta$,

$$\omega = \begin{cases} \frac{i \lambda \Delta \alpha_{\infty}}{\alpha_{\infty} + \nu_y^2} \nu_y^2 & \text{for } \Delta \ll 1 \\ \frac{i \lambda \gamma \Delta \alpha_{\infty}}{\alpha_{\infty} + \nu_y^2} & \text{for } \Delta \gg 1 \end{cases}$$

(29)

Again the result for the case of $\Delta \gg 1$ is independent of the value of the magnetic field. Thus the damping of the different waves in the case of $\Delta \gg 1$ is unaffected by the presence of a transverse magnetic field. However, the time of the decay is not independent of the transverse magnetic field and is proportional to the time taken by a magnetosonic wave (adiabatic for $\Delta \ll 1$ and isothermal for $\Delta \gg 1$) to travel a distance of the wave length of the disturbance or the radiation mean free path. For example, for $\gamma = 1.4$, $\Delta = 0.1$ and $\nu_y^2/\alpha_{\infty} = 1$, the life time of the adiabatic magneto-sonic wave is about 12 times that of the accompanying radiation induced wave and for the ratio of the life time of isothermal magneto-acoustic wave to that of the accompanying radiation induced wave is independent of the magnetic field strength and is same as if magnetic field is not present. This is because of the fact that the damping factor in the limit of $\Delta \gg 1$, for these waves, are independent of the magnetic field strength.

$V_x$, $V_y \neq 0$.

In this case the dispersion relation corresponding
to (10) is given by
\[
\ddot{\psi} - i\gamma \dot{\psi} - \ddot{\psi} (1 + A^2 A^2_{\Delta}) + \frac{\nu^2}{\Delta} \dot{\psi} - i \frac{\Delta V^2}{\Delta} = 0
\]
(30)

where the different symbols have already been defined.

Again it can be seen that out of the five roots of the above equation one corresponds to a pure decay and other four give the damping characteristics of the slow and fast waves. In the limit of \( \Delta \to 0 \), we get
\[
\ddot{\psi} - \ddot{\psi} (1 + A^2 A^2_{\Delta}) + \frac{\nu^2}{\Delta} \dot{\psi} = 0
\]

Then \( \psi_{\text{low}} \) and \( \psi_{\text{fast}} \) are the roots of this biquadratic.

Then the effect of finite but small \( \Delta \) is given by
\[
\omega = \begin{cases} 
\pm \psi_{\text{low}} \lambda \omega_0 + \frac{i \Delta (Y-1) \lambda \omega_0}{2 (1 + \frac{1 - \psi_{\text{low}}^2}{A^2 \omega_0 - \psi_{\text{low}}^2})} & \Delta \ll 1 \\
\pm \psi_{\text{fast}} \lambda \omega_0 + \frac{i \Delta (Y-1) \lambda \omega_0}{2 (1 + \frac{1 - \psi_{\text{fast}}^2}{(\psi_{\text{fast}} - \psi_{\text{low}})^2})} & \Delta \gg 1 
\end{cases}
\]
(31)

Again in the limit of \( \Delta \to \infty \), the dispersion relation reduces to
\[
\ddot{\psi} - \ddot{\psi} (1 + Y A^2 A^2_{\Delta}) + \frac{\nu^2}{\Delta} \dot{\psi} = 0
\]
The corresponding roots of this equation represent the isothermal slow and fast waves. We shall designate them as \( \psi_{\text{fast}} \) and \( \psi_{\text{low}} \). Then the effect of large but finite \( \Delta \) is given by
\[
\omega = \begin{cases} 
\pm \psi_{\text{low}} \lambda \omega_0 + \frac{i \Delta (Y-1) \lambda \omega_0}{2 \Delta \left( Y + \frac{(1 - \psi_{\text{low}}^2)}{A^2 \omega_0 - \psi_{\text{low}}^2} \right)} & \Delta \gg 1 \\
\pm \psi_{\text{fast}} \lambda \omega_0 + \frac{i \Delta (Y-1) \lambda \omega_0}{2 \Delta \left( Y + \frac{(1 - \psi_{\text{fast}}^2)}{(\psi_{\text{fast}} - \psi_{\text{low}})^2} \right)} & \Delta \ll 1 
\end{cases}
\]
(32)
It can easily be seen that in the various limits, these general expressions for \( \omega \) reduce to the cases of transverse and non-magnetic field cases. Another interesting observation about the case \( \Delta \ll 1 \) is that the effect of magnetic field is to reduce the damping. If \( \frac{A^2}{\alpha} \ll 1 \), it is seen that the fast wave becomes an ordinary acoustic wave and its damping factor also gets rid of the strength of the magnetic field. On the other hand if \( \frac{A^2}{\alpha} \gg 1 \), the damping of the fast wave is zero i.e. in the presence of a very strong magnetic field the fast wave is not affected by \( \Delta \) to the orders of approximation employed in the derivation of the expression for \( \omega \). This result is true even for the isothermal fast wave. Slow waves are affected by the strength as well as the direction of the magnetic field. About the radiation induced wave, it may be seen that damping is not affected by magnetic field and is nearly the same as in the non-magnetic case. This result is not quite general as far as the magnetic field strength is concerned but covers some important limiting cases viz. \( \frac{A^2}{\alpha} \ll 1 \) and \( \gg 1 \).

**Discussion:**

The present study on the effect of radiative transfer on the wave motion of small amplitude in an electrically conducting and non-conducting medium, infinite in extent, in the presence of a uniform magnetic field reveals many interesting features. First is the applicability and the usefulness of the Milne-Eddington approximation. Second is the importance of the characteristic length in such
problems. There are two such lengths in our case, namely the radiation mean free path and the other which we use to non-dimensionalize the former, $\frac{ac}{\omega}$ in the case of forced oscillations and wave length in the case of free oscillation problems. It is found that the skin depth is proportional to $\frac{ac}{\omega}$ or $\lambda_n$, which is the representative length of the system. It also shows that the disturbances of high frequency damp in a smaller distance than that of low frequency. Similarly in the case of free oscillation problem the characteristic time of decay of the disturbances is proportional to ($\lambda$ is the wave number) the time taken by a sound wave (or hydromagnetic wave in case of magneto-gas-dynamics) to travel a distance equal to the wave length of the disturbance or the radiation mean free path. Third conclusion is that the sound wave travels a longer distance than the accompanying radiation induced wave before it actually damps. This is true even in the presence of the magnetic field. For example for $\alpha = 3\beta^2/(1 + 3\beta^2) = 0.5$, $\Delta \ll 1$ the ratio of these lengths in the non-magnetic as well as magnetic cases comes out to be approximately $3.5 \times 10^3$, $3.8 \times 10^4$ respectively which decreases with $\Delta$. Some calculations for $\alpha = 0.2, 0.5, 0.8$ and $\frac{\nu^2}{\omega_0^2} = 1$ as given in the accompanying tables were performed to study the effect of high temperatures and magnetic fields. It is found that while the characteristic length for damping of the radiation induced wave increases as we go from $\Delta = 0.1$ to $\Delta = 10.0$.
through $\Delta = 1.0$, the characteristic length for the sound wave decreases for $\Delta = 1$ and then further increases for $\Delta = 10$. This conclusion is even true when there is an impressed uniform transverse magnetic field. Lastly these calculations also reveal the dual role played by the magnetic field in this problem specially for the case when the magnetic pressure is of the order of gas-dynamic pressure. For sound waves (in the different limits of $'\Delta'$ considered here) the effect of magnetic field is to increase the characteristic length for the damping of waves while for radiation induced waves this characteristic length gets decreased because of its presence. So while the magnetic field supports the propagation of one type of waves it tends to oppose the propagation of the other type, viz. radiation induced waves. In other words, the waves which owe their origin to magnetic field are damped by radiation, while those (of course, already damped because of radiation itself) which owe their origin solely to radiation are damped by the magnetic field.

The study of free oscillation problem shows that the radiation induced waves are quite short lived as compared to the life time of accompanying sound waves.
TABLE - 2.
The table showing the constants of proportionality for the skin depth in the case of forced oscillations. is a non-dimensional parameter characterizing the temperature level of the medium. is a measure of the absorption coefficient and lies between zero and one.

<table>
<thead>
<tr>
<th>Acoustic Wave</th>
<th>Radiation Induced Wave</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>50 : 11.23 : 62.77</td>
</tr>
<tr>
<td></td>
<td>9.013 : 0.76 : 5.46</td>
</tr>
<tr>
<td>0.5</td>
<td>50 : 10.47 : 69.01</td>
</tr>
<tr>
<td></td>
<td>0.014 : 0.73 : 3.60</td>
</tr>
<tr>
<td>0.8</td>
<td>50 : 9.38 : 76.72</td>
</tr>
<tr>
<td></td>
<td>0.036 : 0.81 : 2.95</td>
</tr>
</tbody>
</table>

TABLE - 3.
The table showing the constants of proportionality for the skin depth in the case of forced oscillations. is a non-dimensional parameter characterizing the temperature level of the medium. is a measure of the absorption coefficient and lies between zero and one. is the Alven wave velocity and is the adiabatic sound velocity. ( ) has been taken to be unity in these calculations.

<table>
<thead>
<tr>
<th>Magneto-acoustic wave</th>
<th>Radiation Induced Wave</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>339.4 : 24.49 : 147.002</td>
</tr>
<tr>
<td></td>
<td>0.06 : 0.53 : 3.85</td>
</tr>
<tr>
<td>0.5</td>
<td>424.27 : 21.08 : 123.79</td>
</tr>
<tr>
<td></td>
<td>0.01 : 0.51 : 2.54</td>
</tr>
<tr>
<td>0.8</td>
<td>509.12 : 21.64 : 106.91</td>
</tr>
<tr>
<td></td>
<td>0.8 : 0.48 : 2.09</td>
</tr>
</tbody>
</table>