CHAPTER III

UNSTABLE FLOW OF VISCOUS INCOMPRESSIBLE FLUID PAST A SEMI-INFINITE POROUS PLANE WALL WITH STEP FUNCTION

CHANGE IN SUCTION AT SLIP FLOW REGIME
3.1 Introduction

In Chapter II we gave an account of the study of the unsteady flow of viscous fluid past a porous semi-infinite plate where the suction velocity changes by step function. The same problem is now considered here for the flow in regime of slight rarefaction. This corresponds to the slip flow regime where the Knudsen number is in the range $0.01 < K_n < 0.1$:

$$K_n = \frac{M_1}{Re}$$

where $M_1$ is Mach number and $Re$ is the Reynolds number. In the slip flow regime, so defined, the mean free path is of the order of 1 to 10 percent of the boundary layer thickness or other characteristic dimension of the flow field. Slip flow effects may thus be expected to be approximately of this order.

Schaaf has studied the hydrodynamic Rayleigh problem in rarefied gas dynamics using first order velocity slip boundary condition. Soundalgebra et al. have extended this problem to MHD. Similar problem has also been studied by Datta. Pop has studied the unsteady flow of an electrically conducting fluid past an infinite flat plate with applied transverse magnetic field under slip flow boundary condition. He assumed the Knudsen number and the magnetic Reynolds number to be small. In his analysis the skin friction was found to increase with slip boundary condition.

In this Chapter the combined effects of velocity slip
and magnetic field on unsteady MHD flow of an electrically conducting viscous incompressible rarefied gas past semi-infinite flat porous plate with step function change in suction velocity, are studied. The assumption of incompressible rarefied electrically conducting gas and of uniform electrical conductivity is physically realisable in the entire flow field, for the subsonic flows of relatively hot gas. Such flow problems could arise, for example, in magneto-gas-dynamics (MGD) generators, that use relatively low pressure and temperature gases, in aerodynamic and MGD space flight propulsion systems.

3.2 Formulation of the problem

We are concerned with viscous, incompressible electrically conducting rarefied gas of finite electrical conductivity past semi-infinite flat porous plate. All the assumptions made here are the same as that with the problem of MHD case considered in Chapter II, except that here, we apply first order slip boundary condition in the analysis.

The equation of motion for the problem considered is given by eq. (2.4.2).

The boundary conditions on \( u^* \) are (1) slip flow boundary condition that permits a slip velocity \( u_\parallel \) at the plate, \( y = 0 \), i.e.,
\( u' = u'_{1} = L_{1} \left( \frac{\partial u_{1}}{\partial y} \right)_{y=0} \) (first order velocity slip) \( \ldots \) \( (3.2.1) \)

where \( L_{1} \) is the slip coefficient given by the expression

\[
L_{1} = \int^{2-\rho_{1}}_{\rho_{1}} I \]

and \( I = \mu \int^{\frac{\pi}{2}}_{0} \frac{1}{r} \int^{\frac{1}{2}}_{0} \int^{\frac{1}{2}}_{0} \frac{1 + \rho_{1}}{\frac{1}{2}} \mu \frac{RT}{p} \) is the mean free path of the gas molecules and \( \rho_{1} \) is Maxwell's reflection coefficient and \( R \) is the gas constant and

ii) free stream boundary condition is, \( u'(y,t) = U_{\infty} \) as \( y \to \infty \)

The boundary conditions on \( V' \) are

\[
V'(o,t) = V_{1} \quad \text{(constant)} \quad t \leq 0
\]
\[
= V_{2} \quad \text{(constant)} \quad t > 0
\]

The initial condition is given by

\[
\sqrt{\frac{\sigma \rho_{0}}{\gamma}} \left( \frac{V_{1}^{2}}{4y^{2}} + \frac{V_{1}}{\gamma} \right) \frac{V_{1}}{\gamma} \frac{y}{y} \] \( u' = U_{\infty} \left( \frac{\frac{2}{\gamma} + \frac{V_{1}^{2}}{4y^{2}} + \frac{V_{1}}{\gamma}}{\left( \frac{2}{\gamma} + \frac{V_{1}^{2}}{4y^{2}} + \frac{V_{1}}{\gamma} \right) L_{1} + 1} \right) \)

(3.2.2)

with the same non-dimensional parameters introduced in chapter
II for MHD case and with \( h = \frac{L_1}{V_1} \) = rarefaction parameter, the equation of motion, boundary and initial conditions are given respectively as,

\[
\frac{\partial u}{\partial \tau} + \frac{V'}{V_1} \frac{\partial u}{\partial \eta} = \frac{\partial^2 u}{\partial \eta^2} + Ku \tag{3.2.3}
\]

\[
u(0, \tau) = h \frac{\partial u}{\partial \eta} , \quad u (\infty, \tau) = 1
\]

\[
\frac{V'}{V_1} = -1 \text{ for } \tau \leq 0 \tag{3.2.4}
\]

\[
= -\tau \text{ for } \tau > 0
\]

and

\[
u(\gamma, 0) = 1 - \frac{\eta}{1 + \eta h} \tag{3.3.5}
\]

### 3.3 Solution and Discussion

The solution of eq. (3.2.3) is obtained with Laplace transform technique. The velocity field subject to boundary conditions (3.2.4) and initial condition (3.2.5) is given by,

\[
u = 1 - \frac{\eta}{\eta h + 1} + \]

\[
- \frac{y}{2h} \eta - \frac{c \eta}{2} + (c^2 - a) \tau
\]

\[
+ \frac{\text{erfc} \left( \frac{\eta}{2 \sqrt{\gamma}} - c \sqrt{\tau} \right)}{2h} \left\{ \frac{1}{c^2} - \frac{b}{b^2 - c^2} \frac{b}{c(b^2 - c^2)} \right\} -
\]
\[ \begin{align*}
&- \frac{\gamma n}{2h} + c_1 (c_2^2 - d_2)^\frac{1}{2} \\
&\text{erfc} \left( \frac{\eta}{2\sqrt{\gamma}} + c \sqrt{\gamma} \right) \left\{ \frac{1}{c} + \frac{b}{b^2 - c^2} + \frac{b^2}{c (b^2 - c^2)} \right\} - \\
&- \frac{\gamma n}{2} + \frac{e}{2h} \frac{b}{b^2 - c^2} e^{\eta n} (b^2 - d)^\frac{1}{2} \\
&\text{erfc} \left( \frac{\eta}{2\sqrt{\gamma}} + b \sqrt{\gamma} \right) - \\
&- \frac{e}{2h} \frac{b}{b^2 - c^2} e^{\eta n} (b^2 - d)^\frac{1}{2} \\
&\text{erfc} \left( \frac{\eta}{2\sqrt{\gamma}} + \sqrt{\gamma} c \right) \left\{ \frac{1}{b^2 - d} + \frac{b^2}{\sqrt{(b^2 - d)^3}} \right\} + \\
&+ \frac{e}{2h} \frac{b}{b^2 - c^2} e^{\eta n} (b^2 - d)^\frac{1}{2} \\
&\text{erfc} \left( \frac{\eta}{2\sqrt{\gamma}} + b \sqrt{\gamma} \right) (3.3.1)
\end{align*} \]

where
\[ b = \frac{1 + h}{2} \quad , \quad c = (\frac{\gamma}{2} - H) \quad , \quad d = \frac{\gamma^2}{4} + H \]

For hydrodynamic case i.e., for H = 0, the velocity field \( v_c \) given by
\[ u = 1 = \frac{e^{n} (1 - \gamma) r - \gamma}{h + 1} + \\
+ \frac{1}{2h} e^{n} (c_1 \gamma + (c_2^2 - d_1)^\frac{1}{2} \text{erfc} \left( \frac{\eta}{2\sqrt{\gamma}} - c_1 \sqrt{\gamma} \right) \left\{ \frac{1}{c_1} - \frac{b_1}{b_1^2 - c_1^2} \right\} - \]
\[ + \frac{b_1^2}{(b_1^2 - c_1^2)c_1} \]
\[
\frac{-\frac{m}{2h}}{e^{\text{erfc}(\frac{\eta}{2b} + c_1\eta) + (c_1^2 - d_1)\eta}} \left\{ \frac{1}{b_1 - c_1} + \frac{b_1}{b_2 - d_1} + \frac{b_1^2}{(b_1 - c_1)c_1} \right\} - \\
- \frac{\gamma \eta}{2} + \eta b_1 + (b_1^2 - d_1)\gamma \\
- \frac{b_1 e}{h(b_1 - c_1)} \text{erfc}(\frac{\eta}{2b} + b_1\sqrt{\gamma}) - \\
- \frac{\eta}{2} - \sqrt{d_1} \gamma \\
- \frac{e}{2h} \text{erfc}(\frac{\eta}{2b} + \sqrt{d_1} \gamma) \left\{ \frac{1}{\sqrt{d_1}} - \frac{b_1}{b_2 - d_1} + \frac{b_1^2}{\sqrt{d_1}(b_1 - d_1)} \right\} = \\
+ \frac{e}{2h} \text{erfc}(\frac{\eta}{2b} + \sqrt{d_1} \gamma) \left\{ \frac{1}{\sqrt{d_1}} - \frac{b_1}{b_2 - d_1} + \frac{b_1^2}{\sqrt{d_1}(b_1 - d_1)} \right\} = \\
- \frac{\gamma \eta}{2} + \eta b_1 + (b_1^2 - d_1)\gamma \\
+ \frac{b_1 e}{h(b_1 - d_1)} \text{erfc}(\frac{\eta}{2b} + b_1\sqrt{\gamma}) \right) (3.3.2)
\]

where
\[
b_1 = 1 + h \frac{\gamma}{2}, \quad c_1 = (\frac{\gamma}{2} - 1), \quad d_1 = \frac{\gamma^2}{4}
\]

For suction parameter \( \gamma = 2 \) i.e., when the suction velocity is doubled the solution for velocity field in hydrodynamic case is given by

\[
u = 1 - \frac{e}{h+1} \left\{ 1 - \text{erfc}(\frac{\eta}{2b}) \right\} - \frac{(b_2-1)\eta + (b_2^2-1)\gamma}{(b_2-1)h} - \frac{1}{h+1} - \frac{b_2}{(b_2-1)h} \gamma
\]
To study the effect of suction, magnetic field and the rarefaction on the velocity field, the equation (3.3.1) is used to plot the velocity profiles. The velocity profiles plotted in Fig. (3.1) represent the effect of suction. Near the surface of the wall the velocity of the fluid is greatly influenced by the suction. The thickness of the boundary layer decreases as the suction parameter $\gamma$ increases. Fig. (3.2) shows that the effect of rarefaction is to enhance the velocity field, and the boundary layer growth is arrested by the influence of rarefaction parameter $h$. In the layers adjacent to the wall the velocity of the fluid is smaller with higher value of $h$.

The effect of the magnetic field is represented by Fig. (3.3). With increase in magnetic field there is increase in velocity of the fluid at every point of the fluid. Fig. (3.4) shows the effect of time on the boundary layer growth. With higher value of time $\tau$ the thickness of the boundary layer away from the leading edge decreases. The velocity with the layers just adjacent to the wall decreases with increasing time $\tau$. 

\[-2\gamma
- \frac{e}{2n} \text{erfc} \left( \frac{\eta}{2\sqrt{\nu}} - \sqrt{\nu} \right) \sqrt{1 - \frac{b_2^2}{b_2^2 - 1}} + \frac{b_2^2}{b_2^2 - 1} \right] +
\]

\[+ \frac{1}{2n} \text{erfc} \left( \frac{\eta}{2\sqrt{\nu}} + \sqrt{\nu} \right) \sqrt{1 + \frac{b_2^2}{b_2^2 - 1}} + \frac{b_2^2}{b_2^2 - 1} \right] \]

\[(3.3.3)\]

where

\[b_2 = \frac{1+h}{n}\]
The shearing stress at the wall is given by

\[ \tau_c = \int |V_1| u_{\infty} \left\{ \frac{1}{h} - \frac{e}{h(h+1)} + \frac{b_1}{h^2(b_1^2 - d)} \int_1^{(b_1^2 - d)} \gamma \right\} \] 

\[ \left[ \frac{\gamma b_1^2}{b_1^2 - d} - \frac{1}{b_1^2 - d} \right] \] 

\[ \left[ \frac{\gamma b_1^2}{b_1^2 - d} - \frac{1}{b_1^2 - d} \right] \] 

\[ + \frac{\text{erf}(c\sqrt{\gamma})}{2h,c} \left[ \frac{2b_1^2}{b_1^2 - d} - \frac{\gamma b_1^2}{b_1^2 - d} \right] \] 

\[ + \frac{\text{erf}(c\sqrt{\gamma})}{2h,c} \left[ \frac{\gamma b_1^2}{b_1^2 - d} - \frac{1}{b_1^2 - d} \right] \] 

\[ - \frac{\text{erf}(c\sqrt{\gamma})}{2h,c} \left[ \frac{2b_1^2}{b_1^2 - d} - \frac{\gamma b_1^2}{b_1^2 - d} \right] \] 

For hydrodynamic case i.e. \( \gamma = 0 \), the shear stress at the wall is given by

\[ \tau_c = \int |V_1| u_{\infty} \left\{ \frac{1}{h} - \frac{e}{h(h+1)} + \frac{b_1}{h^2(b_1^2 - d)} \int_1^{(b_1^2 - d)} \gamma \right\} \] 

\[ \left[ \frac{\gamma b_1^2}{b_1^2 - d} - \frac{1}{b_1^2 - d} \right] \] 

\[ - \frac{b_1}{b_1^2 - d} \frac{e}{h^2} \left[ \frac{c_1^2}{b_1^2} + \frac{b_1^2}{b_1^2} \right] + \frac{b_1}{h\sqrt{\gamma}} \int \frac{1}{b_1^2 - c_1} - \frac{1}{b_1^2 - d} \]
For suction parameter $\gamma = 2$ i.e. when the suction velocity doubles in step change the shear stress reduces to a relatively simple expression

$$\tau_0 = \int |v_1| u_h \left\{ \frac{\text{erf}(b_2 \sqrt{\gamma}) - 1}{h + 1} \cdot \frac{e^{(b_2^2-1)\gamma}}{b_2} + \frac{2}{(b_2^2-1)h} \int_{1-\text{erf}(b_2 \sqrt{\gamma})} \right\}$$

$$= \frac{b_2^2}{h} \cdot \frac{b_2}{b_2^2-1} + \frac{1}{4h} \int_{1-\text{erf}(b_2 \sqrt{\gamma})} \frac{2b_2+1}{b_2+1} \right\}$$

The numerical results of shear stress at the wall are represented in Figures (3.5), (3.6) and (3.7). The effect of suction on the shear stress is represented in Fig. (3.5). Negative values of shear stress are observed for small values of time $\gamma$, which is owing to the velocity slip at the boundary. The shear stress increases with time $\gamma$ and for certain range
of there is increase in shear stress with increase in suction velocity. Fig. (3.6) shows the effect of rarefaction parameter on the shear stress. With increase in h there is decrease in shear stress. The impact of the magnetic field on the shear stress at the wall is to enhance it; higher magnetic field gives higher value of shear stress.

3.4 Conclusions

1. Increase in suction increases the velocity of the fluid for all time \( V \) and magnetic field in slip flow regime.

2. The rarefaction parameter enhances the velocity field.

3. With higher magnetic field velocity is more.

4. The shearing stress increases for certain time \( V \) with increase in suction velocity. The same is true with magnetic field effect on shear stress.

5. Shear stress on the wall reduces as a result of rarefaction.