CHAPTER 6

ANALYSIS OF POROUS BEARING WITH COUPLE STRESS
FLUID AS LUBRICANT
a) DOUBLE LAYERED SLIDER BEARINGS*
b) STEP BEARING**

* The first part of this chapter has been communicated to WEAR
** The second part has been communicated after revision to ACTA MECHANICA
6a: DOUBLE LAYERED SLIDER BEARINGS:

6a.1: INTRODUCTION:

A number of theories of the microcontinuum have been developed to explain the peculiar behaviour of fluids containing substructure such as polymeric fluids [19, 20]. The simplest microcontinuum theory generalizes the classical theory to allow for polar effects such as the presence of couple stress, body couples and a symmetric stress tensors, self lubricating porous bearings have advantages in overcoming the need for oil pipes, pumps etc, and simplifies the problems concerned with machine design. Motivated with this, numerous attempt have been made to understand the lubrication of various types of porous bearings such as squeeze films [93, 94], externally pressurized bearings [95], Journal bearings [96] and Slider bearings [46] with non-Newtonian fluid as lubricant. The load capacity of the bearing can be increased by reducing the seepage in to the wall of the bearing, this can be achieved by reducing the permeability. However, this is impractical since a reduction in the permeability results in the reduction of the porosity and hence reduction of the oil content within the bearing material. A double layered porous facing would be useful as it would not only increase the load capacity of the bearing because of reduced oil seepage into its wall but would also help to bring oil between the surfaces, thereby improving the performance of the bearing when it is not completely saturated with oil. Cusano [97, 98] investigated lubrication characteristics of a two layer porous journal bearing and the analysis of a double-layered porous slider bearing has been considered by Uma Srinivasan [99] with Newtonian fluid as lubricant. Slider bearings have practical applications in machine
design and in many kinds of machine elements in which rectilinear sliding motions occur, e.g. in steam and water turbines etc.

Rayleigh determined the optimum geometry for Newtonian incompressible fluid/film. Maday [100], analysed the same Rayleigh step bearing in order to verify that the optimum slider contains only one step. The recent review [61] gives details regarding work on the Rayleigh step bearing and other aspects of lubrication. Ramanaiah and Priti [101] extended the theory of lubrication to couple stress fluids and have studied the problems of slider bearing in general and have considered in particular cases of Rayleigh step bearing and inclined slider bearing without porosity effect. There is a need for the analysis of these problems with non-Newtonian fluids as lubricants having double porous layers.

In this analysis, we study the lubrication characteristics of a bearing with an arbitrary shaped upper surface moving over a fixed double layered porous plane surface. The material model taken is that of Stokes' couple stress fluid. It is characterized by two material constants \( \mu \) and \( \eta \) whereas only one parameter \( \mu \) appears in viscous fluid. The Rayleigh step bearing and inclined slider bearing are derived from general analysis as particular cases to illustrate the results qualitatively. The length of the bearing is considered to be infinite, so that edge effects can be neglected in the model. It is shown that the presence of additives in the lubricant can create a significant change in the pressure distribution resulting in a gain in the load capacity of the bearings.
6a.2: FORMULATION AND SOLUTION OF THE PROBLEM:

The geometry and co-ordinates of the two-dimensional double-layered porous slider bearing, is shown in Fig. 6a.1. It consists of two closely spaced rigid surfaces in relative motion. The lubricant between the surfaces is an incompressible couple stress fluid. The body forces and body couples are assumed to be absent. The upper surface is of arbitrary shape and it moves with constant velocity on a lower surface. The lower surface consists of double-layered porous backed by a solid wall. The surfaces are separated by a fluid filled gap of instantaneous height \( h(x) \). The basic equations of motion of fluids with Stokes' couple stress \[28\] given in cartesian tensor notation are (1.4.1) to (1.4.3). If \( q(u,v,0) \) is the velocity vector, and assuming fluid film lubrication approximation, the governing fluid flow equations in region I are

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{6a.2.1}
\]

\[
\mu \frac{\partial^2 u}{\partial y^2} - \eta \frac{\partial^4 u}{\partial y^4} = \frac{\partial p}{\partial x} \tag{6a.2.2}
\]

In the present case, the components of the stress tensor and the couple stress tensor of interest are

\[
T_{21} = \mu \frac{\partial u}{\partial y} - \eta \frac{\partial^3 u}{\partial y^3}, \tag{6a.2.3}
\]

\[
M_{23} = -2\eta \frac{\partial^2 u}{\partial y^2}. \tag{6a.2.4}
\]

The boundary conditions for region I (Fluid film) are

\[ u = u_1, \quad v = v_1 \quad \text{at} \quad y = 0, \]

\[
\frac{\partial^2 u}{\partial y^2} = 0 \quad \text{at} \quad y = 0 \quad \text{and} \quad y = h, \tag{6a.2.5}
\]
\( u = - u_0 \), \( v = 0 \) at \( y = h \),
\[ p = 0 \text{ at } x = 0, L, \]  \hspace{1cm} (6a.2.6)

where \( u_1 \) and \( v_1 \) are the velocity components in porous region.

The governing equation in region II (porous) is the flow of viscous fluid in porous matrix which satisfies
\[ \frac{\partial u_i}{\partial x} + \frac{\partial v_i}{\partial y} = 0. \]  \hspace{1cm} (6a.2.7)

The velocity components \( u_i \) and \( v_i \) in the porous region satisfy Darcy's law
\[ u_i = -\frac{k_i}{\mu} \frac{\partial p_i}{\partial x}, \]  \hspace{1cm} (6a.2.8)
\[ v_i = -\frac{k_i}{\mu} \frac{\partial p_i}{\partial y}. \]  \hspace{1cm} (6a.2.9)

where \( p_i \) is the pressure in the porous region and \( k_i \) is the permeability, in different layers. Boundary conditions for region II are
\[ p_i = 0 \text{ at } x = 0, L, \]  \hspace{1cm} (6a.2.10)
\[ p_i(x, 0) = p_i(x, 0), \]  \hspace{1cm} (6a.2.11)
\[ p_i(x, -H_i) = p_i(x, -H_i), \]  \hspace{1cm} (6a.2.12)
\[ \frac{\partial p_2}{\partial y} = 0 \text{ at } y = -(H_1 + H_2). \]  \hspace{1cm} (6a.2.13)

where \( H_1, H_2 \) are porous layer thicknesses
\[ k_1 \left( \frac{\partial p_1}{\partial y} \right) = k_2 \left( \frac{\partial p_2}{\partial y} \right), \text{ at } y = -H_1 \]  \hspace{1cm} (6a.2.14)
The conditions expressed by equations (6a.2.11) and (6a.2.12) are the continuity of pressure at the interface between the inner layer of the porous bearing and the oil film and between the two porous layers respectively. Equation (6a.2.13) states that the porous slider bearing is press fitted into an impermeable housing. Equation (6a.2.14) results from the fact that the flow normal to the boundary between the two porous layers must be equal on the two sides.

The pressure in the porous regions satisfies the Laplace equation

\[ \nabla^2 p_i = 0 \quad (i = 1, 2) \]  

(6a.2.15)

where \( p_1 \) and \( p_2 \) are the pressures in the inner and outer layers of the porous region respectively.

Integrating equation (6a.2.15) (for \( i = 1 \)) with respect to \( y \) over the wall thickness gives

\[ \left. \frac{\partial p_1}{\partial y} \right|_{y=0}^{y=-H_1} = - \int_{-H_1}^{0} \frac{\partial^2 p_1}{\partial x^2} \, dy + \left. \frac{\partial p_1}{\partial y} \right|_{y=-H_1} \]  

(6a.2.16)

From condition (6a.2.14), equation (6a.2.16) reduces to

\[ \left. \frac{\partial p_1}{\partial y} \right|_{y=0}^{y=-H_1} = - \int_{-H_1}^{0} \frac{\partial^2 p_1}{\partial x^2} \, dy + \frac{k_2}{k_1} \left. \frac{\partial p_2}{\partial y} \right|_{y=-H_1} \]  

(6a.2.17)

Again integrating equation (6a.2.15) with respect to \( y \) over the wall thickness \( H_2 \) gives

\[ \left. \frac{\partial p_2}{\partial y} \right|_{y=-H_1}^{y=-H_1 - (H_1 + H_2)} = - \int_{-H_1}^{-H_1 - (H_1 + H_2)} \frac{\partial^2 p_2}{\partial x^2} \, dy \]  

(6a.2.18)
since \( \frac{\partial P_2}{\partial y} \) at \( y = -(H_1 + H_2) \) is zero as the porous facing is press fitted into an impermeable housing.

From equations (6a.2.17) and (6a.2.18)

\[
\left( \frac{\partial P_1}{\partial y} \right)_{y=0} = - \int_{-H_1}^{0} \frac{\partial^2 P_1}{\partial x^2} \, dy - \frac{k_2}{k_1} \int_{-(H_1+H_2)}^{0} \frac{\partial^2 P_2}{\partial x^2} \, dy \quad (6a.2.19)
\]

Since the porous layer thicknesses are very small [84], using Morgan and Cameron [82] approximation and the conditions (6a.2.11), (6a.2.12) equation (6a.2.19) reduces to

\[
\left( \frac{\partial P_1}{\partial y} \right)_{y=0} = - R \frac{\partial^2 P}{\partial x^2} \quad (6a.2.20)
\]

where \( R = H_1 + \frac{k_2}{k_1} H_2 \)

Also,

\[
\left. u_1 \right|_{y=0} = - \frac{k_1}{\mu} \left( \frac{\partial P_1}{\partial x} \right)_{y=0} = - \frac{k_1}{\mu} \frac{\partial P}{\partial x} \quad (6a.2.21)
\]

\[
\left. v_1 \right|_{y=0} = - \frac{k_1}{\mu} \left( \frac{\partial P_1}{\partial y} \right)_{y=0} = - \frac{k_1}{\mu} (-R \frac{\partial^2 P}{\partial x^2})
\]

\[
= \frac{k_1 R}{\mu} \frac{\partial^2 P}{\partial x^2} \quad (6a.2.22)
\]

The solution of equation (6a.2.2) satisfying condition (6a.2.5) is

\[
u = u_1 (1 - \frac{y}{h}) - \frac{u_0}{h} y + \frac{1}{2\mu} \frac{dp}{dx} [y^2 - yh + \cosh (2y-h) + 2l^2(1 - \frac{2l}{\cosh (\frac{h}{2l})})] \quad (6a.2.23)
\]
where \( l = (\eta / \mu)^{\frac{1}{2}} \)

The volume flow rate is given by

\[
Q = \int_0^{H_1} u_1 \, dy + \int_0^h u \, dy = \int_{-H_1}^{0} u_2 \, dy + \int_{-(H_1+H_2)}^{-H_1} u_1 \, dy + \int_{-(H_1+H_2)}^{-H_1} u \, dy
\]

\[
= -\frac{u_0 h}{2} - \frac{1}{12\mu} \frac{dp}{dx} \frac{1}{s_1(l, k_1, h)}
\]

where

\[
s_1(l, k_1, h) = [h^3 - 12l^2h + 24l^3 \tanh(h) + 6k_1h + 12k_1R]^{-1}
\]

Integration of equation (6a.2.1) across the fluid film thickness gives

\[
\frac{d}{dx} \left( \frac{1}{s_1(l, k_1, h)} \frac{dx}{dh} \right) = -6\mu u_0 \frac{dh}{dx}
\]

Equation (6a.2.25) is the modified Reynolds equation. Substituting the expression (6a.2.23) into (6a.2.3), we get

\[
T_{21} = (y - \frac{h}{2}) \frac{dp}{dx} - \frac{\mu}{h} (u_1 + u_0)
\]

The frictional force on the bearing surface \( y = h \) is given by

\[
f = -\int_0^L T_{21} \, dx = \int_0^L \left[ -\frac{h}{2} \frac{dp}{dx} + \frac{\mu}{h} (u_1 + u_0) \right] dx
\]

Using the dimensionless scheme

\[
X = \frac{x}{L}, \quad H = \frac{h}{h_0}, \quad \tau = h_0 \left( \frac{\mu}{\eta} \right)^{\frac{1}{2}}
\]

\[
\bar{Q} = \frac{2Q}{u_0 h_0}, \quad \bar{p} = \frac{p h_0^2}{6\mu u_0 L}
\]
Equations (6a.2.24) and (6a.2.25) take the forms

\[ Q = \left[ H - \frac{1}{s(\tau, \bar{k}_1, H)} \frac{d\bar{P}}{dx} \right], \quad (6a.2.29) \]

and

\[ \frac{d}{dx} \left[ \frac{1}{s(\tau, \bar{k}_1, H)} \frac{d\bar{P}}{dx} \right] = \frac{dH}{dx}, \quad (6a.2.30) \]

respectively, where

\[ s(\tau, \bar{k}_1, H) = \left[ H^3 - \frac{12}{3} H + \frac{24}{3} \tanh\left(\frac{\tau H}{2}\right) \right. \]

\[ + 6\bar{k}_1 H + 12 \bar{k}_1 \bar{R}]^{-1}. \]

Also, \( \bar{P} = 0 \) at \( X = 0 \) and \( X = 1 \) \( (6a.2.31) \).

From equations (6a.2.29) and (6a.2.30), using condition (6a.2.31), we get

\[ \bar{p} = \int_0^X (H - \bar{Q}) s(\tau, \bar{k}_1, H) \, dx, \quad (6a.2.32) \]

\[ \bar{Q} = \frac{1}{\int_0^1 s(\tau, \bar{k}_1, H) \, dx} \int_0^1 H s(\tau, \bar{k}_1, H) \, dx \quad (6a.2.33) \]

The dimensionless load capacity, \( W \), and the centre of pressure, \( \bar{X} \), are

\[ W = \frac{1}{W_0} \int_0^1 \bar{P} \, dx = \int_0^1 X(\bar{Q} - H) s(\tau, \bar{k}_1, H) \, dx \quad (6a.2.34) \]

\[ \bar{X} = \frac{1}{W} \int_0^1 X \bar{p} \, dx \]

\[ = \frac{1}{2W} \int_0^1 X^2(\bar{Q} - H) s(\tau, \bar{k}_1, H) \, dx \quad (6a.2.35) \]
The dimensionless frictional force, $F$, obtained from equation (6a.2.27) is

$$F = \frac{f h_0}{\mu u_0 L} = \frac{1}{6} \int \left( \frac{6\tilde{k}_1}{H} \right) \frac{1}{(H - \bar{Q})} \frac{1}{s(\tau, \tilde{k}_1, H)} \left( H - \bar{Q} \right) \frac{1}{H} dx$$

(6a.2.36)

The basic characteristics of the slider bearing, viz., $\bar{Q}, W, X$ and $F$ are obtained by evaluating the integrals (6a.2.33) to (6a.2.36) for a given $H(x)$. 

Co-efficient of friction, $C = \frac{F}{W}$

(6a.2.37)

6a.3: POROUS STEP-BEARING:

In a porous step bearing, Fig. 6a.2, the film thickness $H(x)$ takes the form

$$H(x) = \begin{cases} A & \text{when } 0 \leq x \leq B \\ 1 & \text{when } B \leq x \leq 1 \end{cases}$$

where $A (A = \frac{h_1}{h_0})$ is a step height ratio and $B$ is the step riser location.

The volume flow rate is

$$\bar{Q} = \left[ AB \left\{ 1 - \frac{24}{\tau^3} \left( \frac{\tau}{2} - \tanh\left( \frac{\tau}{2} \right) \right) + 6\tilde{k}_1 + 12\tilde{k}_1 \bar{R} \right\} \right. $$

$$+ \left(1 - B\right) \left\{ A^3 - \frac{24}{\tau^3} \left( \frac{A\tau}{2} - \tanh\left( \frac{A\tau}{2} \right) \right) + 6\tilde{k}_1 A + 12\tilde{k}_1 \bar{R} \right\} \right]$$

$$+ \left(1 - B\right) \left\{ A^3 - \frac{24}{\tau^3} \left( \frac{A\tau}{2} - \tanh\left( \frac{A\tau}{2} \right) \right) + 6\tilde{k}_1 A + 12\tilde{k}_1 \bar{R} \right\}^{-1}$$

(6a.3.1)
The dimensionless load is
\[ W = \frac{1}{2} B(1 - B)(A - 1) \left[ B \left( 1 - \frac{24}{3} \left( \frac{1}{2} - \tanh(\frac{V}{2}) \right) \right) \right. \]
\[ + 6\bar{k}_1 + 12\bar{k}_1\bar{R} \left. \right] + (1 - B) \left\{ A^3 - \frac{24}{3} \left( \frac{A^4}{2} \right) \right. \]
\[ \left. - \tanh(\frac{A^4}{2}) + 6\bar{k}_1A + 12\bar{k}_1\bar{R} \right\}^{-1} \]  
(6a.3.2)

When \( \bar{R} \to 0 \), \( \bar{k}_1 \to 0 \), the dimensionless load, \( W \), given in equation (6a.3.2) reduces to the solid case [101].

The centre of pressure is
\[ \bar{x} = \left( \frac{1 + B}{3} \right) \]  
(6a.3.3)

The dimensionless frictional force becomes
\[ F = (A - 1)(1 - \frac{2\bar{k}_1}{A})W + \frac{1}{6} \left( \frac{B}{A} + 1 - B \right) \]  
(6a.3.4)

For \( \bar{R} \to 0 \), \( \bar{k}_1 \to 0 \) and \( \tau \to \infty \), the expressions corresponding to the classical viscous case (Gross [10]) are recovered.

6a.4: POROUS INCLINED SLIDER BEARING:

In a porous inclined slider bearing, Fig.6a.3, the film thickness \( H(x) \) is given by
\[ H(x) = A - (A - 1)x, \quad 0 \leq x \leq 1 \]
where \( A \) is the ratio of the film thickness at the ends of the bearing.

With this choice of \( H(x) \), the integrals (6a.2.33) to (6a.2.36) reduce to
\[ \bar{Q} = \frac{A}{1} \int Hs(\tau, \bar{k}_1, H) dH \]
\[ = \frac{A}{1} \int s(\tau, \bar{k}_1, H) dH \]  
(6a.4.1)
The integrals (6a.4.1) to (6a.4.4) can not be evaluated in closed form. They are calculated numerically.

When \( R \to 0, k_1 \to 0, \) \( W, X \) and \( F, \) given in expressions (6a.4.2) to (6a.4.4) reduce to solid case [101].

For \( R \to 0, k_1 \to 0 \) and \( \tau \to \infty, \) we obtain the corresponding expressions for viscous case (Gross [10])
6a.5: DISCUSSION AND CONCLUSION:

We have obtained the numerical values of the physical quantities \( W, F, C \) and \( X \) for different combinations of \( \tau, k_1, k_2 \) and \( B \) which are presented in Figs.6a.4 to 6a.10. In all these figures, the continuous curves refer to non-Newtonian fluid and dashed ones to Newtonian fluid. These results are compared with classical case as well as conventional bearing i.e. a porous bearing having a permeability \( k_1 \) and of wall thickness \( (H_1 + H_2) \).

We have plotted graphs in Figs.6a.4 to 6a.7, for the load supporting capacity, \( W \), and the co-efficient of friction, \( C \), versus \( A \) by confining to Rayleigh step slider bearings. From Fig.6a.4, we observe that the load capacity, \( W \), gradually increases with the decreasing values of \( \tau \). In the present case, an increase of 18.4\% in \( W_{\text{max}} \) is found as compared with Newtonian fluid of the same viscosity when \( \tau = 5 \) and \( B = 0.69 \). Fig.6a.5 illustrates the effect of permeability and step riser location on the load carrying capacity. As the permeability value decreases, the load capacity increases. An increase of 4.52\% for \( W_{\text{max}} \) is found for double layered couple stress fluid compared with conventional bearing when \( \tau = 10 \) and \( B = 0.7 \). The effect of double layer is to shift the point of \( W_{\text{max}} \) towards the entry region. Further, it is of interest to note that, for optimal load, \( (W_{\text{max}}) \), the bearing lengths and step height ratio are slightly smaller in the present analysis compared with conventional porous Rayleigh step bearings. The optimum value of load, \( W \), is attained for

(I) double layered: at \( A = 1.79 \), for \( B = 0.69 \), \( k_1 = 4.3 \times 10^{-5} \),

\[ k_2 = 1.2 \times 10^{-5} \] and \( \tau = 5 \)
(II) Conventional bearings: at $A = 1.83$, for $B = 0.70$, $k_1 = k_2 = 4.3 \times 10^{-5}$ and $\tau = 5$

(III) Newtonian double layer: at $A = 1.85$, for $B = 0.70$, $k_1 = 4.3 \times 10^{-5}$, $k_2 = 1.2 \times 10^{-5}$ ($\tau' = \infty$)

(IV) Newtonian conventional bearing: at $A = 1.86$, for $B = 0.70$, $k_1 = k_2 = 4.3 \times 10^{-5}$, ($\tau = \infty$)

Newtonian solid case cameron [9]:

$$\text{at } A = 1.87, \text{ for } B = 0.70 \quad (\tau = \infty).$$

In Fig.6a.6, the graphs show variation of co-efficient of friction with $A$, for different values of couple stress parameter $\tau$. As $\tau$ increases, the frictional co-efficient also increases significantly which indicates the suitability of couple stress fluid as efficient lubricant. Also, we notice that, frictional co-efficient is lesser in the case of couple stress fluid compared with Newtonian fluid. Fig.6a.7 shows the graphs of the variation of co-efficient of friction with $A$ for different permeabilities and step riser location $B$. As the step riser location is shifted towards the leading edge, the frictional co-efficient is found reduced. A decrease in permeability results in the decrease of the co-efficient of friction. Further, we notice from the profiles in Fig.6a.7 that, in the case of double layer the co-efficient of friction is smaller than that of the conventional porous Rayleigh step bearing. From equation (6a.3.3), it is clear that the location of the centre of pressure is at the same position in the case of Rayleigh step solid as well as porous bearings.

Graphs, in Figs. 6a.8 to 6a.10 are concerned with inclined slider bearings. The load capacity for a double layered porous slider bearing
is shown in Fig.6a.8. The load support falls gradually as the couple stress parameter $\tau$ increases. For decreasing values of permeability, load capacity enhances significantly. Also, it compares the load support for a double layered porous inclined slider bearing with that of the conventional porous slider bearing. The load support increases with the double layered porous slider bearing. The optimum value of load, $W$, is attained for

(I) double layer: at $A = 2.25$ for $\bar{k}_1 = 4.3 \times 10^{-5}$, $\bar{k}_2 = 1.2 \times 10^{-5}$ and $\tau = 10$

(II) conventional bearing: at $A = 2.29$, for $\bar{k}_1 = \bar{k}_2 = 4.3 \times 10^{-5}$ and $\tau = 10$

(III) Newtonian double layer: at $A = 2.3$ for $\bar{k}_1 = 4.3 \times 10^{-5}$, $\bar{k}_2 = 1.2 \times 10^{-5}$, ($\tau = \infty$)

(IV) Newtonian conventional bearing: at $A = 2.31$, for $\bar{k}_1 = \bar{k}_2 = 4.3 \times 10^{-5}$

(V) Newtonian solid case, Gross [10]: at $A = 2.32$ $\tau = \infty$

An increase of 4.7% in $W_{\text{max}}$ is found in the present case (for $\tau = 10$) as compared with Newtonian fluid and this agrees with the results of Uma Srinivasan [99]. Besides this, an increase of 2.6% in $W_{\text{max}}$ is found as compared with the conventional slider bearings ($\tau = 40$).

Fig.6a.9 shows the variation in the co-efficient of friction, $C$. As the couple stress parameter decreases, the co-efficient of friction reduces significantly and for the increasing values of permeability the co-efficient of friction increases gradually, whereas frictional force is found decreasing. In addition, the graphs in Fig.6a.9 indicates that the co-efficient of friction is significantly reduced in double layered compared to the conventional porous slider bearing. The effect of double layer is to decrease the frictional force. The graphs in Fig.6a.10, show that the centre of
pressure is shifted towards the trailing edge of the bearing as we decrease $\tau$ for given values of permeabilities which are in the neighbourhood of $A = 1.8$. Also, the centre of pressure, $\bar{X}$, is found shifting towards the leading edge of the bearing for increasing values of permeability. The increase in $\bar{X}$ indicates that the pressure profiles are distorted towards the leading edge showing that the effect of couple stress is more pronounced in the thinner region of the lubricant film. In view of advantages such as increased load capacity and reduced frictional co-efficient, fluids with couple stress offer possibilities as improved lubricants.

In the case of step bearings, the step height ratio ($A$) for optimum load capacity is for $A = 1.79$, whereas in inclined slider bearings, for optimum load, the film thickness ratio is for $A = 2.2$, so with this, it is possible to select suitable design parameters which lead to efficient lubrication of bearings. As the Rayleigh step bearing has 26.79% more load carrying capacity than that of the inclined slider bearing, the Rayleigh step bearing is preferable to slider bearings.
6b: STEP BEARING:

6b.1: INTRODUCTION:

A squeeze film is a fluid layer situated between surfaces that are approaching each other. The squeezing action of the approaching surfaces causes fluid to flow towards less constrained surroundings. For very thin films, viscous forces offer a high resistance to such fluid motion, which in turn, tends to inhibit the approach of the bounding surfaces. In this way, a squeeze film serves to offset potential contact between the surfaces. Squeeze films are encountered in numerous applications of lubrication technology. In accordance with the lubrication theory, the flow in the squeeze film is assumed to be dominated by viscous and pressure forces.

As the self lubricating porous bearings have advantages, with this motivation several investigators have done the analysis of porous bearings which include the work by Cameron et al [102], Rouleau [103], Cusano [104], Murti [105], Prakash and Vij [106] and Malik et al [107]. They have confined their studies to Newtonian fluids as lubricants. Step slider squeeze bearing has been discussed earlier by Diprima [108]. Ramanaiah and Dubey [109], Maiti [110] and Shukla and Isa [111] have extended the analysis to micropolar fluids for step bearings and have predicted some advantages over Newtonian lubricants. Squeezed film step bearings using non-Newtonian power law lubricant have been studied by Shukla and Isa [112] and it has been shown that the load capacity increases as the flow behaviour index of the power law fluid increases. The effects of couple stresses on the velocity field are found to be quite larger.
for smaller values of the non-dimensional parameter $\tau = h_0 (\mu / \eta)^{\frac{1}{3}}$. The thrust area of the step bearings is in clutch plates, gear box, journal and thrust bearings.

This section gives an analysis of the characteristics of the porous squeezed film step bearings using couple stress fluid as a lubricant.

6b.2: MATHEMATICAL FORMULATION AND SOLUTION OF THE PROBLEM:

The geometry of the two dimensional step bearing is shown in Fig.6b.1. It consists of two closely spaced surfaces in which the lower one is fixed porous matrix whereas the upper rigid stepped surface is approaching normally towards the lower plate with velocity $V_0$. Without loss of generality, the co-ordinates may be fixed to the lower bounding surface of the squeeze film, as is shown in the figure. The plates are separated by a couple stress fluid-filled gap. The fluid in the gap comprises the squeeze film. We use Darcy's law in the porous matrix and an appropriate modified Reynolds equation in the fluid film region. Body forces and body couples are assumed to be negligible.

If we assume fluid film lubrication applicable to thin films, then the governing equations are (6a.2.1) and (6a.2.2) and the component of couple stress tensor is (6a.2.4)

In the porous material, the velocity components $u_1$ and $v_1$ are related to the pressure $p_1$ and expressed by equations (1.6.3) to (1.6.5).

The relevant boundary conditions in region I are,

$$u = u_1, \quad v = v_1 \quad \text{at} \quad y = 0$$

$$u = 0, \quad v = \dot{h} = \frac{dh}{dt} = - V_0 \quad \text{at} \quad y = h$$
and the no-couple stress conditions

\[ \frac{\partial^2 u}{\partial y^2} = 0 \quad \text{at} \quad y = 0 \text{ and } y = h. \]

Also,

\[ p = 0 \text{ at } x = 0, L \quad \text{(6b.2.2)} \]

where \( p \) is the pressure in the fluid film region.

In region II:

The boundaries at \( x = 0 \) and \( x = L \) are open to environments having the same uniform pressure, i.e.

\[ p_1 = 0 \text{ at } x = 0, L \quad \text{(6b.2.3)} \]

where the environment pressure may be taken to be zero without loss of generality.

Furthermore, if the porous material is backed by a solid wall, then

\[ \frac{\partial p_1}{\partial y} = 0 \text{ at } y = -\delta \quad \text{(6b.2.4)} \]

and the continuity of pressure at the interface is

\[ p(x,0) = p_1(x,0). \quad \text{(6b.2.5)} \]

It follows from the equations (1.6.4), (1.6.5) and (1.6.3) that the pressure field \( p_1(x,y) \) is governed by Laplace equation,

\[ \frac{\partial^2 p_1}{\partial x^2} + \frac{\partial^2 p_1}{\partial y^2} = 0. \quad \text{(6b.2.6)} \]

Integrating equation (6b.2.6) with respect to \( y \) over the porous layer thickness and using equation (6b.2.4), we have
As the porous layer thickness $\delta$ is small, using Morgan and Cameron [82] approximation, equation (6b.2.7) reduces to

$$\frac{\partial P_1}{\partial y} \bigg|_{y=0} = -\delta \frac{\partial^2 P_1}{\partial x^2} = -\delta \frac{\partial^2 P}{\partial x^2}.$$  

(6b.2.8)

As the thickness of the porous layer is very small (Torzilli and Mow [84]), the error in the analysis due to this approximation is negligible.

From equations (6b.2.5) and (6b.2.8), we have

$$u_1 \bigg|_{y=0} = -\frac{k_1}{\mu} \left( \frac{\partial P_1}{\partial x} \right) = -\frac{k_1}{\mu} \frac{\partial P}{\partial x}$$  

(6b.2.9)

$$v_1 \bigg|_{y=0} = -\frac{k_1}{\mu} \left( \frac{\partial P_1}{\partial y} \right) = -\frac{k_1}{\mu} \delta \frac{\partial^2 P}{\partial x^2}.$$  

(6b.2.10)

The general solution of equation (6a.2.2) subject to the boundary conditions (6b.2.1)

$$u = u_1 - \frac{u_1}{h} y + \frac{1}{2 \mu} \frac{dp}{dx} [y^2 - y h + 2 h^2 \cosh(\frac{y}{h} - h) \cosh(\frac{h}{2})]$$

$$\left(1 - \frac{h}{2 \mu} \frac{dp}{dx} \frac{1}{S(l,h)} \right)$$  

(6b.2.11)

where $l = (\eta / \mu)^{1/2}$. The volume flow rate is given by

$$Q = \int_0^h u dy = \frac{u_1 h}{2} - \frac{1}{12 \mu} \frac{dp}{dx} \frac{1}{S(l,h)}$$  

(6b.2.12)
where \( S(l, h) = [h^3 - 12l^2(h - 2l \tanh(\frac{h}{2l}))]^{-1} \).

Integration of equation (6a.2.1) across the fluid film using equation (6b.2.1) and the boundary conditions (6b.2.1) yields

\[
Q = V_0 x + \frac{k_1 \delta}{\mu} \frac{dp}{dx} \quad \text{(6b.2.13)}
\]

In the case of porous step bearing the upper surface is approaching the lower porous bed normally with constant velocity \( V_0 \), the modified Reynolds equation determining pressure can be written from equations (6b.2.12) and (6b.2.13) in the form

\[
\frac{dp_i}{dx} = -12 \mu V_0 x \left[ \frac{1}{S(l, h_i)} + 6k_1 h_i + 12k_1 \delta \right]^{-1} \quad \text{(6b.2.14)}
\]

where \( i = 2, \ h = h_2, \ p = p_2 \) in the region \( 0 \leq x \leq L_1 \)

and \( i = 3, \ h = h_3, \ p = p_3 \) in the region \( L_1 \leq x \leq L_2 \).

Integrating equation (6b.2.14) and using the boundary conditions

\[
p_2 = p_3 \quad \text{at} \quad x = L_1 \quad \text{(6b.2.15)}
\]

and

\[
p_3 = 0 \quad \text{at} \quad x = L
\]

we get,

\[
p_2 = 6 \mu V_0 \left[ \frac{1}{S(l, h_2)} + 6k_1 h_2 + 12k_1 \delta \right]^{-1} (L_1^2 - x^2)
\]

\[
+ \left[ \frac{1}{S(l, h_3)} + 6k_1 h_3 + 12k_1 \delta \right]^{-1} (L_2^2 - L_1^2) \quad \text{(6b.2.16)}
\]

and

\[
p_3 = 6 \mu V_0 \left[ \frac{1}{S(l, h_3)} + 6k_1 h_3 + 12k_1 \delta \right]^{-1} (L^2 - x^2) \quad \text{(6b.2.17)}
\]
The load capacity \( W \) which the film is capable of supporting is equal to the integrated pressure force exerted by the fluid on the bounding surface at \( y = h \), that is,

\[
W = b \int_0^{L_1} p_2 \, dx + b \int_0^{L_2} p_3 \, dx,
\]

which on using equations (6b.2.16) and (6b.2.17) becomes

\[
W = 4 \mu V_0 b \left[ \left\{ \frac{1}{S(1,h_2)} + 6k_1 h_2 + 12k_1 \delta \right\}^{-1} L_1^3 \right.
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where
\[
\frac{1}{S(t, H)} = [H^3 - \frac{24}{\tau^3} \left( \frac{\tau H}{2} - \tanh\left(\frac{\tau H}{2}\right)\right)] \quad (6b.2.20)
\]
and
\[
\frac{1}{S(t, 1)} = [1 - \frac{24}{\tau^3} \left( \frac{\tau}{2} - \tanh\left(\frac{x}{2}\right)\right)].
\]

Taking \( h_2 = h_3 + h_s \), where \( h_s \) is the step height, equation (6b.2.18) can be written as
\[
\frac{W}{4 \mu V_0 b} = \left( h_3 + h_s \right)^3 - 12 L_2^2 \left( h_3 + h_s \right) - 21 \tanh\left(\frac{h_3 + h_s}{2}\right)
\]
\[
+ 6 k_i (h_3 + h_s) + 12 k_i \delta \right]^{-1} L_1^3
\]
\[
+ \left[ h_3^3 - 12 L_2^2 (h_3 - 21 \tanh\left(\frac{h_3}{2}\right)) \right]
\]
\[
+ 6 k_i h_3 + 12 k_i \delta \right]^{-1} \left[ \frac{L_2^2}{2} (3 L_2^2 - L_2^1) - L_1^3 \right]. \quad (6b.2.21)
\]

Writing \( V_0 = -\frac{dh_3}{dt} \) in equation (6b.2.21), the squeezing time for reducing the film thickness from an initial value \( h_0 \) of \( h_3 \) to a final value \( h_f \) is given by
\[
T = \frac{\mu h_0^2 t}{4 \mu b L_3^3} = \int_{\bar{h}_f} \frac{1}{S(t, \bar{h}_3, \bar{h}_s, k_i)} dh_3
\]
\[
\left[ \frac{L_2^2}{2} (3 - \bar{L}_2^2) - \bar{L}_1^3 \right]
\]
\[
+ \int_{\bar{h}_f} \frac{1}{S(t, \bar{h}_3, k_i)} dh_3 \quad (6b.2.22)
\]

where
\[
\bar{h}_f = \frac{h_f}{h_0}, \quad \bar{h}_3 = \frac{h_3}{h_0}, \quad \bar{h}_s = \frac{h_s}{h_0}.
\]
For $\tau \to \infty$ (which corresponds to Newtonian lubricant) the load capacity and time of approach are as follows

$$W = \left[ \frac{1}{H^3} + 6\kappa_1 + 12\kappa_1 \delta \right]^{-1} \tilde{L}_1^3$$

$$+ \left[ 1 + 6\kappa_1 + 12\kappa_1 \delta \right]^{-1} \left[ \frac{L_2}{2} (3 - \tilde{L}_2) - \tilde{L}_1^3 \right]$$  \hspace{1cm} (6b.2.23)

and

$$T = \frac{1}{\tilde{h}_f} \int \frac{(1 - 3 \cdot \tilde{h}_s \tilde{h}_3)}{\tilde{h}_3} \tilde{L}_1^3 \\tilde{h}_3 \, d\tilde{h}_3$$

$$= \frac{1}{\tilde{h}_f} \left[ \frac{L_2}{2} (3 - \tilde{L}_2) - \tilde{L}_1^3 \right]$$

$$+ \frac{1}{\tilde{h}_f} \left[ \frac{L_2}{2} (3 - \tilde{L}_2) - \tilde{L}_1^3 \right]$$ \hspace{1cm} (6b.2.24)

Further $\kappa_1 \to 0$ (Solid surfaces) the load capacity and time of approach reduce to classical case obtained by Shukla and Isa [112] for $n = 1$. 
6b.3: DISCUSSION AND CONCLUSION:

Our interest is to study the characteristics of couple stress fluid as a lubricant, and hence the attention is focussed on the load capacity and time of approach. The required physical quantities $\tilde{W}$ and $\tilde{T}$ are obtained for different combinations of $\tau, k_1, \overline{L}_1$ and $\overline{L}_2$ and their variations are exhibited in Figs. 6b.2 to 6b.6. In these figures, the dotted lines represent the variations in $\tilde{W}$ and $\tilde{T}$ in the case of Newtonian fluid and solid ones to couple stress fluid. The variations of load capacity with $H$ and $\tau$ are shown in Fig.6b.2. It is observed that the effect of couple stress is significant for small values of $\tau$ and also as the couple stress parameter $\tau$ tends to infinity we arrive at the classical viscous case. It shows that the load capacity is higher for couple stress fluid compared with Newtonian fluid. Fig.6b.3 shows the effect of permeability on the load support. As the permeability parameter increases, the load carrying capacity monotonically decreases. From the results shown in Fig.6b.4, it can be concluded that, the optimum load is attained for the combination of step location lengths $\overline{L}_1 = 0.3$ and $\overline{L}_2 = 0.7$. For the couple stress fluid it is 43.3% higher compared with viscous fluid. Also, for a specified step height ratio the load capacity in the present analysis is much higher than viscous case.

In Fig.6b.5 squeeze film time is plotted for different values of couple stress fluid parameters. The squeezing time of the step bearing with couple stress fluid is longer than that with a Newtonian fluid. This is a very desirable finding since longer squeeze film time results in a smaller co-efficient of friction and the almost negligible rate of wear of the bearing. From Fig.6b.6, it is observed that approaching
time of surfaces is greater when the porosity of the surface decreases. Also, it is noticed that, the time of approach is significantly delayed for the smaller values of step height ratio.