Denote by $A$ the class of all analytic functions $f(z)$ in the unit disk $E = \{ z \mid |z| < 1 \}$. A function $f(z)$ in $A$ is said to be univalent in $E$, if it never takes the same value more than once, that is $f(z_1) \neq f(z_2)$ if $z_1 \neq z_2$ for $z_1, z_2 \in E$.

A function which is analytic and univalent in the unit disk $E$ may further be normalized by the conditions $f(0) = 0$ and $f'(0) = 1$. The class of analytic functions which are univalent in the unit disk $E$ and which are normalized by the conditions $f(0) = f'(0) - 1 = 0$ will be denoted by $S$. The class $S$ has been studied widely for over seventy years. Much work has been done in determining the estimates for $|a_n|, \ |f(z)|, \ |f'(z)|$ and other extremal properties. The Bieberbach's conjecture namely $|a_n| \leq n, \ n = 2, 3, \ldots$, $f(z) \in S$ is settled now by L.de Branges.

A function $f(z) \in S$ is said to be starlike of order $\alpha$, $(0 \leq \alpha < 1)$, denoted by $f(z) \in S^*(\alpha)$, if

$$\text{Re}(zf'(z)/f(z)) > \alpha, \ z \in E.$$  

A function $f(z) \in S$ is said to be convex of order $\alpha$, denoted by $f(z) \in K(\alpha)$ if

$$\text{Re}(zf''(z)/f'(z) + 1) > \alpha, \ z \in E.$$
Note that $S^* = S \cdot (0)$ and $K = K(0)$, the classes of starlike and convex functions respectively. The classes $S(a)$ and $K(a)$ are introduced by Robertson [29].

Denote by $E$ the class of functions of the form

$$f(z) = z^{-1} + a_0 + a_1 z + \ldots$$

that are analytic in the punctured disk $U = \{ z \mid 0 < |z| < 1 \}$ with a simple pole at $z = 0$. A function $f(z)$ in $E$ is said to be meromorphically starlike of order $\alpha$ if

$$\text{Re} \left( - \frac{zf'(z)}{f(z)} \right) > \alpha, \ z \in E,$$

is said to be meromorphically convex of order $\alpha$ if

$$\text{Re} \left( - \frac{zf''(z)}{f'(z) + 1} \right) > \alpha, \ z \in E.$$

These classes are denoted by $E^*(\alpha)$ and $E^c(\alpha)$ respectively. Note that $E^* = E \cdot (0)$ and $E^c = E^c(0)$, the classes of meromorphically starlike and convex functions respectively. The concept of order for meromorphic starlike functions was introduced by Pommernke [28]. A function $f(z)$ in $E$ is said to be close-to-convex with respect to $g(z)$ if there exists a meromorphically starlike and univalent function $g(z) = d/z + \sum_{n=0}^{\infty} b_n z^n (d \neq 0)$ such that

$$\text{Re} \left( zf'(z)/g(z) \right) > 0, \ z \in E.$$

This class is denoted by $E_c$. (A function $g(z) = d/z + \sum_{n=0}^{\infty} b_n z^n (d \neq 0)$...
which is analytic in $U$ with a simple pole at $z = 0$ is said to be starlike if $\text{Re}(zg'(z)/g(z)) < 0$, $z \in E$.

Ruscheweyh [30] introduced the symbol

$$D^n f(z) = f(z)^* \frac{z}{(1 - z)^{n+1}}, \quad n = 0, 1, 2$$

for function $f(z)$ in $A$ with normalizations $f(0) = 0$ and $f'(0) = 1$, where (*) denotes the Hadamard product of two power series and obtained the new criterion for univalence. Later the symbol $D^n f(z)$ was termed as the $n$th order Ruscheweyh derivative of $f$ [4]. Further certain properties of various subclasses of $S$ have been studied by different authors like Al-Amiri [4], R. Singh and S. Singh [35] and Goel and Sohi [11]. Ganigi and Uralegaddi [8] introduced the symbol

$$D^n f(z) = \frac{1}{z(1 - z)^{n+1}} * f(z), \quad n = 0, 1, 2$$

for the functions $f(z)$ in $E$ and obtained the new criteria for meromorphic univalence.

In the first chapter functions of the form $f(z) = z - \sum_{n=2}^\infty |a_n| z^n$ with fixed second coefficient which satisfy $\text{Re}(f(z)/z) > \alpha$ or $\text{Re}(f'(z)) > \alpha$ for $z \in E$ are considered. Coefficient inequalities, extreme points, distortion theorems, closure and covering theorems and comparable results etc., are obtained for these functions. Certain results of Uralegaddi [39] are improved by involving the second coefficients in the power expansion.
of $f$. Next some distortion theorems for the fractional calculus of $f(z)$ belonging to $P_p[a]$ and $Q_p[a]$ are obtained. Also the functions of the form $f(z) = a_1 z - \sum_{n=2}^{\infty} a_n z^n$ ($a_1 > 0, a_n > 0$) with fixed second coefficients are considered. Al-Amiri [1], Finkelstein [7], Tepper [38] and Silverman and Silvia [34] studied the subclasses of univalent functions with fixed second coefficient.

Further meromorphic univalent functions with negative coefficients are introduced and coefficient inequalities, distortion, covering and closure theorems, convolution properties, and class preserving integral operators are determined for these functions that are meromorphically starlike of order $\alpha$ and convex of order $\alpha$. Silverman [33] studied the univalent functions with negative coefficients. Gupta and Jain [14] studied certain classes of univalent functions with negative coefficients.

In the second chapter some sufficient conditions for close-to-convexity and starlikeness are obtained for certain subclasses of $\Sigma$. Next differential operators for certain meromorphic functions are considered. In particular the following result of Bajpai [5] is extended: If $f(z)$ is a member of $\Sigma^\gamma$, then so is $F(z) = z^{-2} \int_0^z f(t) dt$. Further mapping properties of $f$. $f(z) = (1 - \alpha) F(z) + \alpha z F'(z), \alpha > 0$ when $F$ belongs to different subclasses of $S$ with fixed second coefficients are studied. The earlier results of Al-Amiri [2], Livingston [19] and Noor, Aloboudi and Aldihan [25] are obtained as particular cases.
In the third chapter Ruscheweyh derivatives and certain analogy of Ruscheweyh derivatives are studied. Results obtained include the earlier results of several authors as particular cases. Further among other things new criteria for p-valent meromorphic convex functions are obtained.