CHAPTER VII

EXTERNAL BREMSSTRAHLUNG SPECTRUM DIFFERENTIAL
IN PHOTON ENERGY

Experimentally Determined Pulse-height Distribution:

Pulse-height distribution corresponding to external bremsstrahlung spectrum differential in photon energy, in the energy range 100 KeV to 1800 KeV has been determined using the linear extrapolation method. This pulse-height distribution has been converted into the true photon spectrum by the method of Liden and Starfelt\(^1\) and compared with the Bethe-Heitler theory\(^2,3\).

External bremsstrahlung intensity has been measured using a scintillation spectrometer having a single channel pulse-height analyser. Bremsstrahlung intensity was measured with a channel width of 40 KeV about the mean energies of 100, 200, 300, ..., 1800 KeV. The intensity in each channel was recorded for different durations of time in order to obtain the required statistical accuracy (\(\approx 5-6\%\)). In each case the recorded intensity was normalised to correspond to the same number of counts above 660 KeV due to the source bremsstrahlung alone.
Again in each channel the true bremsstrahlung intensity was obtained by the difference method, and the geometrical arrangement used is shown in Fig. 1.

As in the previous case, it has been observed that the bremsstrahlung intensity first builds up to a maximum value for $t \approx 0.4R$ and then decreases for higher values of target thicknesses, even when the target thickness is a fraction of the range of beta particles of maximum energy. In this investigation also, only that part of the external bremsstrahlung intensity which increases with the target thickness is considered.

We have seen that the slopes of the curves log $(I_{BB} A/t)$ versus $t$ for photon energies above 400 keV are the same as those of above 50 keV, showing that even the number of beta particles which retain their energy above 400 keV vary as a function of $t$ in the same way as the number of beta particles which retain their energy above 50 keV. vary as a function of $t$ even in foils of different $Z$. Since the bremsstrahlung intensity of any particular photon energy is produced by all the beta particles of energies equal to or greater than that photon energy, we may expect the production of external bremsstrahlung at each photon energy to follow
the same dependence on the thickness of the target as the number of beta particles that retain their energy above that photon energy. This means that we may expect the external bremsstrahlung intensity at each photon energy to vary as $t e^{-\frac{E}{t}}$ with $t$, even at higher photon energies. So, the data recorded for each photon energy band has been analysed in the same way as in the case of integrated external bremsstrahlung measurements above 50 keV and above 400 keV, for $t < 0.4R$, for the target elements Cu, Cd, and Pb.

In figures 15, 16 and 17 we have plotted the experimentally determined values of $\log \left\{ I_{EB}(E)A/t \right\}$ versus $t$ (mg/cm²) for each band of photon energy covering the range from 100 keV to 1800 keV, for targets of Cu, Cd and Pb respectively, where $I_{EB}(E)$ is the true external bremsstrahlung intensity corresponding to the mean energy $E$ of the band. The solid lines are the least square fit straight lines and we see that the experimentally observed points lie on the respective straight lines for all photon energies of interest and for all target materials used. That $\log \left\{ I_{EB}(E)A/t \right\}$ versus $t$ is a straight line shows that the external bremsstrahlung intensity at each photon energy band also follows a relation of the type expressed in equation 1, $Ch\sqrt{V}$, as expected.
FIG 15
TARGET THICKNESS $t$ (mg/cm$^2$)

LOG ($l_{EB}/t$)

$Cu$
It is interesting to note that the least square fit lines shown in Figures 15, 16 and 17, are parallel to each other for each target element over the entire photon energy range of interest ranging from 100 KeV to 1800 KeV. In Fig. 18, we have plotted the values of \( \Sigma g \) as a function of external bremsstrahlung photon energy. We see that the values of \( \Sigma g \) for Cu, Cd and Pb agree with each other within 15 to 20 percent.

That the slopes are independent of the atomic number of the target means that the deviation from linear increase of bremsstrahlung intensity with \( t \) is due to slowing-down of beta particles as said earlier. The interesting point is that the slopes remain almost the same for each element over the photon energy range of 100 KeV to 1800 KeV. This means that the spectrum of beta particles remains almost the same for \( t < 0.4R \) and the number of beta particles that retain their energy above any fixed photon energy (in the range of 100 KeV to 1800 KeV) vary with \( t \) as \( t^{\Sigma g} \), where \( \Sigma g \) is almost independent of the fixed energy of beta particles.

However, there is a slight tendency of \( \Sigma g \) to increase at photon energies above about 1.2 MeV. Since the fraction of the beta particles that have the energy above 1.2 MeV is comparatively small, we cannot expect, on the continuous slowing-down model, the spectrum of
beta particles at very high energy to remain constant as the thickness of the foil increases. The number of beta particles that retain their energy above this fixed high energy decreases rapidly with increase of target thickness and so it is natural that the value of $\sum_\beta$ at these higher energies should increase with energy.

However, the intercept of each straight line of Figures 15, 16 and 17, gives in arbitrary units, the logarithm of the bremsstrahlung intensity per atom in the limit of target thickness $t$ (mg/cm$^2$) tending to zero for the relevant band of the photon spectrum. For reasons mentioned before, this intensity is independent of the effects of the slowing-down of the beta particles and the attenuation of the bremsstrahlung photons within the target. The antilogarithms of these intercepts yield the experimentally observed external bremsstrahlung pulse-height distribution per atom of the target in the limit of the target thickness $t$ tending to zero, in arbitrary units. From this pulse-height distribution one can obtain the true bremsstrahlung spectrum differential in photon energy after applying relevant corrections.

Evaluation of the True External Bremsstrahlung Spectrum from Observed Pulse-height Distribution:
The observed pulse height distribution which results from photons that fall on a scintillation detector is related to the incident photon spectrum in a complicated way because of the various processes that take place in the detector. We mention here these processes and discuss the procedures one may adopt to obtain the true photon spectrum from the observed pulse height distribution.

If monoenergetic photons are incident on a scintillation detector, a continuous spectrum having characteristic peaks of various energies is recorded. The photoelectric absorption of the incident photon in a crystal leads to the so called photopeak having a pulse height proportional to the incident photon energy. Because of the statistical nature of the light output in the crystal and the electron multiplication in the photomultiplier, the pulse-height due to photoelectric absorption is gaussian in shape. If the incident photon undergoes in the crystal compton scattering, the compton scattered electron carries only a fraction of the incident photon energy. Since such electrons can have continuous distribution of energy, one observes a continuous pulse height distribution called the compton distribution. Due to the back scattering of the incident photons from the photomultiplier window a peak of definite energy, the back scatter peak is observed. Thus we see that
even monochromatic photons lead a continuous distribution of pulse height.

When the photons having continuous spectrum of energy, like the bremsstrahlung, falls on a crystal the photons of each energy lead to a continuous pulse height distribution and the observed pulse height distribution is a sum of such distributions. Further the efficiency of the crystal is also a function of energy of the incident photons and the geometry in the sense that it depends on the size of the crystal and on the position and extension of the gamma source.

So one has to correct the observed pulse-height distribution for various effects such as the background, internal bremsstrahlung, dead time of the analyser, energy resolution, compton electron distribution and the gamma detection efficiency of the crystal for the geometry used, before comparing the observed results with theory. This problem has been investigated by many authors. It is interesting to observe that Bustard and Silverman have remarked that "the continuous gamma-ray pulse-height spectra, such as occur from bremsstrahlung are for practical purposes identical to the incident photon spectra.... Bremsstrahlung spectra have been found to require even less of a correction. For this reason,
when investigating bremsstrahlung generation, it is likely that the measured pulse height spectra can be used without correction. Nevertheless we have followed the method of Liden and Starfelt in converting the observed pulse-height distribution to the true photon spectrum, a brief review of which is given below:

1. **Dead Time of the Analyser:**

   The observed pulse-height distribution should be corrected for the finite dead time of the analyser. In the present investigation as the counting rates are small, this correction is negligible.

2. **Background and Internal Bremsstrahlung:**

   In general background counts due to extraneous sources and due to internal bremsstrahlung should be determined and subtracted from the observed counts in each channel. But as we adopt a difference method our experimental results are free from the background and the internal bremsstrahlung.

3. **Energy Resolution:**

   Since the photopeak has a distribution, the number of pulses at any energy is due to photons of neighbouring energy also. So one has to correct the observed pulse-height distribution obtained by plotting the antilogarithma
of the intercepts $\log\left\{ \frac{I_{EB}(E_\gamma)}{A/t} \right\}$ as a function of photon energy for the finite resolution. If $N_1 (E_\gamma)$ is the number of pulses observed at $E_\gamma$ from this distribution, the number of pulses $N_2 (E_\gamma)$ corrected for the finite resolution of the detector is given according to Morton by

$$N_2 (E_\gamma) = N_1 (E) - \frac{K E}{4} \frac{d^2 N_1 (E)}{dE^2}$$

where $E$ is the pulse height corresponding to the photon energy $E_\gamma$ and $K$ is a constant which is related to the full width at the half maximum for the monochromatic photons by the relation

$$K = \frac{\sigma^2 (E_\gamma)}{2 E_{\text{FWHM}} \ln 2}$$

We have determined $K$ by measuring the full widths at half maximum for monochromatic gamma-ray lines - 662 KeV (Cs$^{137}$), 840 KeV (Mn$^{54}$), 1114 KeV (Zn-65) and 1280 KeV (Na-22). This correction for finite energy resolution has been found to be small.

4 Compton Electron Distribution:

The correction due to the Compton electron distribution is the most significant one and should be evaluated more accurately. If $N_0 (E_\gamma) dE_\gamma$ photons are incident on the crystal with photon energies between $E_\gamma$
and $E_{\gamma} + dE_{\gamma}$, $N_{p}(E_{\gamma})dE_{\gamma}$ photons are recorded due to photoelectric absorption and $N_{e}(E_{\gamma})dE_{\gamma}$ photons are found in the Compton electron distribution. The maximum energy of the Compton scattered electron, $E_{x}\gamma$, is given by the relation

$$E_{x}\gamma = \frac{2E_{\gamma}^2}{m_{0}c^{2}} \left( 1 + \frac{2E_{\gamma}}{m_{0}c^{2}} \right)^{-3}.$$

The probability that such a Compton electron will have an energy between $E$ and $E + dE$, produced by a photon of energy $E_{\gamma}$, is given by $C(E, E_{\gamma})$. The total number of Compton electrons at any energy $E$ due to all incident photons from zero to $E_{\text{max}}$ is given by

$$N_{e}(E) = \int_{0}^{E_{\text{max}}} C(E, E_{\gamma}) N_{e}(E_{\gamma})dE_{\gamma} \quad - 4.$$

In general, a flat Compton electron distribution is assumed for any given photon energy over the entire range from zero to $E_{\gamma}$; the following approximation is made.

$$C(E, E_{\gamma}) = C(E_{\gamma}) = \frac{1}{E_{\gamma}} \quad \text{for} \quad 0 < E_{\gamma} < E_{\gamma}^{\text{max}}$$

and

$$C(E, E_{\gamma}) = C(E_{\gamma}) = 0 \quad \text{for} \quad E > E_{\gamma}^{\text{max}}.$$
The observed number of pulses within any band of photon energy is due to the photoelectric absorption of photons of that energy and due to the Compton electrons produced from photons of higher energy.

In order to obtain the pulse-height distribution in each energy band due to photoelectric absorption only, one has to subtract the pulses due to Compton electrons. This is achieved by adopting the following procedure. The observed pulse-height distribution $N_2(E_f)$ is first extrapolated to the end point. The whole observed spectrum is divided into equal parts by taking $dE_f$ equal to 40 KeV. Then by starting at the highest energy (that is in the last band), for which all pulses are due to the photoelectric absorption of photons of that energy only, the total number of Compton electrons due to photons of this energy $E_f$ is evaluated using the relation

$$N_e(E_f) = N_2(E_f) \frac{1 - K(E_f)}{K(E_f)} .$$

Now the number of Compton electrons that are present in any band is given by $N_e(E_f)dE_f$. This is subtracted from the relevant energy bands. This procedure is repeated going from highest energy band to successive lower energy bands and the pulse height distribution due
photoelectric absorption alone is determined. For the evaluation of Compton distribution the experimentally determined values of $K(E')$ have been used.

**X-ray Escape and Backscatter:**

The correction for the iodine K x-rays and the backscattering of the incident photons have been neglected, as our primary interest is in the region of 400 KeV to 1800 KeV of the spectrum. These two effects contribute only at very low photon energies.

**Geometrical and Gamma-detection Efficiency:**

The correction for the geometrical and gamma-detection efficiency of the detector have been arrived at as follows. The extended source geometrical factor for the present experimental arrangement has been calculated according to Jaffey and it turns out to be 0.28 percent. The photopeak efficiency or gamma detection efficiency $\varepsilon_p(E')$ at any energy $E'$ is given as the product of the peak-to-total ratio $K(E')$ and the intrinsic efficiency $\varepsilon(t)(E')$, that is,

$$\varepsilon_p(E') = K(E') \varepsilon(t)(E').$$

The actual values of peak-to-total ratios have been determined by obtaining the complete pulse height spectra for the following monoenergetic gamma sources kept in the place of the target: 148 KeV (Ce$^{141}$), 280 KeV (Hg$^{203}$), 325 KeV (Cr$^{51}$),
662 KeV (Cs\textsuperscript{137}), 840 KeV (Mn\textsuperscript{54}), 1114 KeV (Zn\textsuperscript{65}) and 1280 KeV (Na\textsuperscript{22}). The area under the photopeak divided by the sum of the areas under photopeak and Compton distribution gives the peak-to-total ratio. In Fig. 1, we have presented the values of \( K(E_p) \) thus determined along with the peak-to-total ratios calculated by Berger and Dogget\textsuperscript{6} using the Monte Carlo method for collimated radiation incident centrally on the end face of a cylindrical NaI(Tl) crystal of the same size as used in our experiment; their values agree with our results within a few percent. Also presented in the same figure is the ratio \( \tau/\mu \), where \( \tau \) is the linear photoelectric absorption coefficient and \( \mu \) is the total linear absorption coefficient of NaI crystal. Since the highest monochromatic source available to us was 1.28 MeV, we have extrapolated \( K(E_p) \) beyond 1.28 MeV keeping in view with the results of Berger and Dogget on one side and the fraction \( \tau/\mu \) on the other; since \( K(E_p) \) does not change appreciably over this high energy range this extrapolation may be considered to be fairly accurate. 

The values of the intrinsic efficiency for the present geometry of 7 cm have been obtained by interpolating the theoretically calculated values of Welicki et al.\textsuperscript{7} for different distances and for the right cylindrical crystal of 1 inch in diameter and 1 inch thick, and are...
PEAK-TO-TOTAL RATIOS FOR THE GAMMA-RAY DETECTOR EMPLOYING A NaI(Tl) CRYSTAL OF 1 INCH IN DIA AND 1 INCH THICK, ○ EXPERIMENTALLY DETERMINED VALUES, — $\frac{T}{\mu}$ VALUES, × MONTE CARLO CALCULATIONS OF BEPPER AND DOGGET
CALCULATED INTRINSIC EFFICIENCY OF ULTRASONIC N' IN 0.1 INCH IN THE CRYSTAL 1 INCH IN THICK CURVE OBTAINED BY INTERPOLATING THE CALCULATED VALUE OF WOE'JICKI ET AL.
FIG 21

GEOMETRICAL AND γ-DETECTION EFFICIENCY CURVE
presented in Fig. 20. The geometrical and gamma detection efficiency as a function of photon energy is presented in Fig. 21.

Correction is also made for the attenuation of the external bremsstrahlung intensity in the perspex beta stopper (3 of Fig. 1).

Thus the true external bremsstrahlung spectra have been evaluated for targets of Cu, Cd and Pb.

Comparison with Theory:

Before comparing the experimentally determined external bremsstrahlung spectrum with theory we should decide about the external bremsstrahlung production cross section formula to be used. For this purpose we consider the experimental arrangement and other relevant factors. In the experimental arrangement we have adopted, the detector subtends a semivertical angle of 10° at the centre of the target foil. Though the external bremsstrahlung intensity is measured in the forward direction, for the reasons given below, it should be considered as being due to isotropic distribution of the beta particles in the foil. From a good geometry transmission experiment it is clear (please see Fig. 11) that for targets in the range from 100 mg/cm² to 400 mg/cm², the transmitted beta intensity varies from
50 to 10 percent of the incident intensity. This along with the fact that the elastic scattering cross section is proportional to the atomic number of the target, shows that the incident electrons undergo many collisions and lose their initial sense of direction. Therefore, so far as the bremsstrahlung emission from the targets of different thicknesses in the range of 100 to 400 $\text{mg/cm}^2$ is concerned, there is no definite relation between the initial direction of incidence of the beta particles on the target and the direction of emission of external bremsstrahlung photons from the target. So the bremsstrahlung intensity detected by the crystal should be considered as due to electrons that are isotropically distributed within the target foil, though in the experimental arrangement, the source, the target and the crystal have well defined geometrical relations. Since we have obtained the bremsstrahlung intensity per atom in the limit of target thickness $t$ tending to zero by extrapolating the results obtained from targets in the range of 100 to 400 $\text{mg/cm}^2$, the experimentally determined external bremsstrahlung spectrum should be compared with the theoretical spectrum that is differential in photon energy only.

The bremsstrahlung production cross section that is differential in photon energy should be integrated over the incident beta spectrum. In the Fr-Y-90 beta source
used, Sr-90 and Y-90 are in secular equilibrium with end point energies 0.54 MeV and 2.26 MeV respectively. As remarked earlier the Sr-90 beta spectrum does not contribute to external bremsstrahlung spectrum at all above 540 KeV and is found to be quite negligible even at 400 KeV. As our main interest is in the high energy part of the external bremsstrahlung spectrum, we have considered only the Y-90 beta spectrum for the calculation of the theoretical external bremsstrahlung spectrum. If \( \frac{d\sigma}{dE} (E_y) \) is the external bremsstrahlung production cross section that is differential in photon energy and \( n(E_\beta) \) is the incident spectrum of beta particles on a target atom, the external bremsstrahlung spectrum is given by

\[
N(E_y) = \int_{E_y}^{E_{\text{max}}} \frac{d\sigma(E_y)}{dE_y} n(E_\beta) dE_\beta,
\]

where \( E_y \) is the energy of the emitted photon, \( E_\beta \) is the energy of the beta particle and \( E_{\text{max}} \) is the end point energy of the incident beta spectrum. It is customary to express the energy in the units of \( m_e C^2 \) and the momentum in the units of \( m_e C \), where \( m_e \) is the rest mass of the electron and \( C \) is the velocity of light. In terms of these units the external bremsstrahlung differential cross section \( \frac{d\sigma}{d\hat{k}} \), where \( \hat{k} = \frac{E}{m_e C^2} \) is given by the equation (2) (of Chapter II) according to the Bethe-Heitler theory.
The spectrum of beta particles is given according to the Fermi theory, by
\[ n(E_0)dE_0 = A G(Z, p_0) E_0^2 (E_{\text{max}} - E_0)^2 dE_0 \]  
(8)

where \( E_{\text{max}} \) is the end point energy of the \( \text{Y}-90 \) spectrum, \( A \) is a constant, \( G(Z, p_0) \) is the modified Fermi function. In these calculations the tabulated values of Fermi function by Rose et al.\(^9\) (Oak Ridge National Laboratory Report No. 1222) have been used. From experimental results of Braden et al.\(^9\) it is clear that the experimentally determined \( \text{Y}-90 \) beta spectrum is well represented by equation (8), in which a shape correction has been omitted. We integrated the beta spectrum numerically and selected the value of \( A \) such that \( n(E_0)dE_0 \) is unity. Using this spectrum normalised to unity the values of \( N(k) \) have been obtained by numerical integration of Equation (7). From this the number of bremsstrahlung photons per MeV energy interval at any energy \( E \) per atom of the target per beta particle of \( \text{Y}-90 \), has been obtained. The theoretical spectra for Cu, Cd and Pb targets are given in Fig. 22 by solid lines.

As we do not have the necessary facilities for evaluating the absolute strength of the \( \text{Sr}-\text{Y}-90 \) source, we have not evaluated the absolute bremsstrahlung production cross section from our experimental results.
In order to compare the experimentally determined results with theory, we have normalised the experimentally determined external bremsstrahlung spectrum of Pb target at 600 KeV. The experimentally determined spectra of Cu and Cd targets have not been independently normalised at any point, but corresponds to the normalisation of spectrum relevant to Pb target at 600 KeV. The dashed curve of Fig. 22 represent the experimentally determined external bremsstrahlung spectra for targets of Cu, Cd and Pb.

From Fig. 22, we see that there is good agreement between the experimentally determined spectrum and the theoretical spectrum in the energy range of 400 to 1000 KeV in the case of Pb for which the spectrum was normalised at 600 KeV. The external bremsstrahlung spectra of both Cu and Cd are in agreement with theory about the photon energy of 600 KeV, even though the experimentally determined spectra have not been normalised at any point. However, in the case of all the three targets, the disagreement with theory increases appreciably at higher photon energies. The disagreement with theory at photon energies less than 400 KeV is evidently due to the contribution of the Sr-90 beta spectrum which has been neglected in the theoretical calculations.
External bremsstrahlung investigations using targets thick enough to stop beta particles in the target have shown that the disagreement with theory increases with photon energy and also with the atomic number of the target. In order to see the Z-dependence of the experimentally determined external bremsstrahlung spectrum, in Fig. 23 we have plotted $1/Z^2$ times the experimentally determined external bremsstrahlung spectrum; $X$ corresponds to spectrum relevant to Cu target, $O$ corresponds to Cd target and $*$ to Pb target. The dashed curve is the theoretically evaluated bremsstrahlung spectrum for $Z = 1$.

We see that the experimental points relevant to Cd and Pb agree well with each other in the entire energy range of 100 KeV to 1800 KeV. This shows that the experimentally determined spectrum is proportional to $Z^2$ in the case of Cd and Pb. For these targets the disagreement with theory does not depend on the atomic number of the target. However, we see that the disagreement between the experimentally determined spectrum increases with photon energy as has been observed by others. In fact, the experimental values are 10 percent higher than theoretical values at 1.0 MeV and 250 percent higher at 1.8 MeV.

The results of Cu disagree with those of Cd and Pb at higher photon energies and are consistently above the points of Cd and Pb by about 20 to 30 percent. This
$\frac{1}{Z^2} \times \text{NO OF PHOTONS PER MeV} \times 90^\circ \beta$ \text{PER ATOM}$
disagreement between the results of Cd and Pb, and Cu is more than the estimated error of 5 percent. This may be either due to some unsuspected systematic error or may be genuine. This is not definitely due to the contribution of electron-electron bremsstrahlung whose contribution is not expected to be more than 3 percent; nor can this be attributed to the difference method in which the external bremsstrahlung produced by the perspex stopper is assumed to be negligible. However, the interesting point to notice is that there is no tendency for the disagreement between the experimentally determined spectrum of copper and the spectra of Cadmium and Lead to increase at higher photon energies.

From our results we see that the shapes of the experimentally determined external bremsstrahlung spectra for targets of atomic number in the range between Z=29 to 82, agree with each other within about 25 percent. It may be remarked that the results obtained with the thick targets show that the disagreement with theory increases at higher photon energies and with the atomic number. This Z-dependence of the disagreement with theory observed in experiments using thick targets may be attributed to the deviation from expected Z-dependence of the stopping power of the beta particles which appears in the evaluation of the theoretical spectrum relevant
to thick targets, rather than to the deviation from the $z^2$-dependence of the external bremsstrahlung production cross section.
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