CHAPTER 3

A STUDY OF ANISOTROPIC POROUS FINITE SQUEEZE FLUID FILM BEARING LUBRICATED WITH COUPLE STRESS FLUIDS

3.1 Introduction:

In order to explain the peculiar behaviour of complex fluids, a number of micro-continuum theories have been developed and these theories have been used to discuss different physical phenomena. In particular, couple stress fluid theory as proposed by Stokes [107] has also been applied to explain some practical problems, even lubrication problems.

On the other hand, with the development of bearing design, porous bearings are becoming important. Porous bearings are self lubricating bearings, which have advantages in overcoming the need of continuous oil supply and simplify the problems with machine design. It has also been observed that a coating of polymer materials on bearing surfaces reduces corrosion effect of the bearing surfaces and makes the bearings less noisy. In many practical cases, the coating surfaces behave as porous layers. Thus the knowledge of porous bearings lubricated with different types of fluids, even with
couple stress fluids are essential to design many real bearings. Ramanaiah and Sarkar [91] used the couple stress theory to explain the effect of stresses in squeeze films and thrust bearings. Ramanaiah [93] also considered a study of squeeze films between two finite nonporous plates lubricated by fluids with couple stresses. A generalized equation for squeezing flow of a power law fluid between two plane annuli was derived by Elkouh et al. [38]. Wu [122] studied a squeeze film lubrication of rectangular porous plates by assuming that the porosity is isotropic in nature and the lubricant is Newtonian in character. Following a similar technique, used by Wu, Wheelar and Balasubramanyam [121] included the effects of anisotropic permeability of the porous surface and slip velocity at the interfaces in a squeeze film between two porous rectangular plates, considering the lubricant as Newtonian one. Prakash and Vij [89] developed an interesting relation between time and height for a squeeze film between porous parallel plates. In order to present couple stress effects, Bujurke et al. [12] studied the squeeze film lubrication between pore-elastic rectangular plates with special reference to synovial joints. Das and Bhattacharjee [23] presented combine effects of porosity and couple stresses in fluids, even in squeezed slider bearings, considering isotropic nature of porosity. Majumdar et al. [80] considered a study on design of externally pressurized rectangular porous thrust bearings. Recently,
Naduvinamani et al. [82] analyzed the problem of squeeze film lubrication of a short porous journal bearing lubricated with couple stress fluids.

3.2 Basic Equations:

The basic equations of motion of fluids with couple stress given in Cartesian tensor notation are (3.3, 3.10)

\[ \dot{\rho} + \rho \nabla \cdot \mathbf{v} = 0 , \quad (3.1) \]

\[ \rho \ddot{\mathbf{v}} = \rho \mathbf{b} + \mathbf{T} \quad (3.2) \]

\[ \rho \mathbf{g} + \mathbf{e}_{ijk} \mathbf{T}_{jk} + \mathbf{M}_{ij} = 0 , \quad (3.3) \]

Where \( \mathbf{v} \) is the velocity vector, \( \mathbf{b} \) the body force per unit mass, \( \mathbf{g} \) the body couple per unit mass, \( \rho \) the density, \( \mathbf{T} \) the stress tensor and \( \mathbf{M} \) the couple stress tensor. The superscript dot denotes material time derivative and a subscript followed by comma denotes partial differentiation. The constitutive equations for the stress tensor and the couple stress tensor are

\[ \mathbf{T}_{ij} = (- \rho + \lambda \nabla \cdot \mathbf{v} + \mu (\mathbf{v} \cdot \nabla) + \frac{1}{2} (\rho \mathbf{e}_{ijk} \mathbf{g}_k) + \eta \nabla^2 (\mathbf{v} \cdot \nabla) \mathbf{v} , \quad (3.4) \]

\[ \mathbf{M}_{ij} = 2 \eta e_{i\alpha \beta} v\beta, \alpha i + 2 \eta e_{i\alpha \beta} v\beta, \alpha j , \quad (3.5) \]
where \( \delta_{ij} \) is the Kronecker delta, \( e_{ijk} \) the permutation tensor, \( p \) the pressure, \( \lambda \) and \( \mu \) the classical viscosity coefficients related by the relation \( \lambda = -\frac{2}{3} \mu, \left( \frac{\lambda}{\mu} \right)^{1/2} = l \) characterizes the material length of couple stress fluids and, \( \eta \) and \( \hat{\eta} \) the new material constants of the couple stress fluid. It is to note that the substitution of \( T_{ij} \) and \( M_{ij} \) from eqns. (3.4) and (3.5) satisfies equation (3.3) identically.

We assume that the lubricant between the squeezed surfaces is incompressible with couple stresses, body forces and body couples are absent. Under these assumptions, the equation of continuity (3.1) and the simplified form of governing equations after substitution of the value of \( \lambda \) in terms of \( \mu \) and \( T_{ij} \) from eqn. (3.4) in eqn. (3.2) are:

\[
\mathbf{v}_{ij} = 0 \quad , \quad \rho \mathbf{v}_i = -p_i + (\frac{\mu}{3} + \eta \nabla^2) \mathbf{v}_{k,ki} + (\mu - \eta \nabla^2) \mathbf{v}_{ij} \quad (3.6)
\]

In this problem, a squeezing flow of the couple stress fluid between two rectangular plates is considered, in which the upper non-porous plate is squeezing normally towards the lower porous plate (i.e. along z-direction). The geometry of the problem in consideration is presented in Fig. 3.1. The porous region is extended from \( z = -h_1 \) to \( z = 0 \) and lubricant region is extended from \( z = 0 \) to \( z = h \). Under the usual assumption of hydrodynamic lubrication
applicable to a thin film in three dimensional Cartesian coordinate system, equations (3.6) and (3.7) reduce to the form:

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 ,
\]  

(3.8)

\[
\frac{\partial p}{\partial x} = \mu \left( \frac{\partial^2 u}{\partial x^2} \right) - \eta \left( \frac{\partial^4 u}{\partial x^4} \right)
\]  

(3.9)

\[
\frac{\partial p}{\partial y} = \mu \left( \frac{\partial^2 v}{\partial y^2} \right) - \eta \left( \frac{\partial^4 v}{\partial y^4} \right)
\]  

(3.10)

\[
\frac{\partial p}{\partial z} = 0 .
\]  

(3.11)

Assuming no slip velocity and no couple stress at the upper solid surface \( z = h \), the boundary conditions for the velocity components in fluid region are:

\[ u = v = 0 \]  

(3.12a)

\[ w = - \frac{\partial h}{\partial t} \]  

(3.12b)

\[
\frac{\partial^2 u}{\partial z^2} = \frac{\partial^2 v}{\partial z^2} = 0
\]  

(3.12c)
We have assumed that the flow of lubricant in the anisotropic porous matrix of thickness $h_i$ in the lower plate is governed by a modified Darcy's law:

$$v'_i = - \frac{1}{\mu} \{k_i f_i(h_i) p'_{,i}\}, \quad i = \text{x, y, z} \quad (3.13)$$

where $v'_i$ are velocity components, $p'$ is the pressure in the porous region and, $k_x$, $k_y$ and $k_z$ are constant permeability coefficients in $x$, $y$ and $z$-directions respectively, $f_i(h_i)$ are assumed functions stated in the next section.

Assuming slip velocity (3.13), and no couple stress at the porous interface $z = 0$, the boundary conditions for the velocity components in porous interface are:

$$\frac{\partial u}{\partial z} = \frac{\alpha_y}{\sqrt{k_y}} (v - v'), \quad (3.14a)$$

$$\frac{\partial v}{\partial z} = \frac{\alpha_y}{\sqrt{k_y}} (v - v'), \quad (3.14b)$$

$$w = w' = - \frac{1}{\mu} k_z f_z(h_1) \left( \frac{\partial p'}{\partial z} \right)_{z=0}, \quad (3.14c)$$

$$\frac{\partial^2 u}{\partial z^2} = \frac{\partial^2 v'}{\partial z^2} = 0, \quad (3.14d)$$
where $\alpha_x$ and $\alpha_y$ are the parameters characterizing the velocity slip at the porous wall-fluid film interface and the relation (3.14c) presents continuity of velocity at the interface along $z$-direction. $u'$, $v'$ and $w'$ (i.e. $v'_1$) are respectively velocity components in the porous medium along $x$, $y$ and $z$-directions and are given by (3.13), satisfying the equation of continuity:

$$\frac{\partial u'}{\partial x} + \frac{\partial v'}{\partial y} + \frac{\partial w'}{\partial z} = 0.$$  \hspace{1cm} (3.15)

### 3.3 Solution of Equations:

The solutions of equations (3.9) and (3.10) subject to the appropriate boundary conditions in (3.12) and (3.14) can be written as

$$u = \frac{1}{2\mu} \frac{\partial p}{\partial x} \left[(z - h)\{z - h\beta_x - 2\gamma_x/\tanh\frac{h}{2t}\} + 2l^2\varphi\right],$$  \hspace{1cm} (3.16)

$$v = \frac{1}{2\mu} \frac{\partial p}{\partial y} \left[(z - h)\{z - h\beta_y - 2\gamma_y/\tanh\frac{h}{2t}\} + 2l^2\varphi\right],$$  \hspace{1cm} (3.17)

where $\beta_i = 1 + (2\alpha_i/\sqrt{k_i}/h)/(1 + h\alpha_i/\sqrt{k_i})$, $\gamma_i = 1/(1 + h\alpha_i/\sqrt{k_i})$,

$$\varphi = 1 - \{\cosh\frac{2z - h}{2t}\}/\{\cosh\frac{h}{2t}\}.$$
On substituting the values of $u$ and $v$ from the relations (3.16) and (3.17) in the equation (3.8) and then integrating over the film thickness from $z = 0$ to $z = h$, with the boundary conditions (3.12b) and (3.14c), we obtain

$$
\frac{\partial}{\partial x} \{ S_x(l,h) \frac{\partial p}{\partial x} \} + \frac{\partial}{\partial y} \{ S_y(l,h) \frac{\partial p}{\partial y} \} = - 12 \mu \{ \frac{\partial h}{\partial t} + \}
$$

$$
\frac{1}{\mu} k_x f_x h_1 \left| \frac{\partial p}{\partial z} \right|_{z = 0}, \quad (3.18)
$$

where $S_i(l,h) = h^3 (1 + \beta_i) - 6h^2 \gamma_i \tanh \frac{h}{2l} - 12l^2 \{ h - 2lh \tanh \frac{h}{2l} \},$

$$
i = x, \ y
$$

Substitution of velocity components in porous region from (3.13) in the equation (3.15) gives the elliptic equation on pressure $p'$:

$$
k_x f_x(h_1) \frac{\partial^2 p'}{\partial x^2} + k_y f_y(h_1) \frac{\partial^2 p'}{\partial y^2} + k_z f_z(h_1) \frac{\partial^2 p'}{\partial z^2} = 0, \quad (3.19)
$$

From the above analysis, especially from the relations (3.16) & (3.17), it is clear that lubrication characteristics are governed by velocity components along $x$- & $y$-directions. Further, Darcy's law for Newtonian fluids in porous region will fail to introduce the effect of couple stress parameter in porous region. So, in order to introduce the effect of couple stress parameter on the expressions for velocity components in porous region, the velocity components in porous region have been assumed in the form analogous to the relation
(3.13), a modified form of Darcy's law; and thereby \( f_i(h) \) are defined as

\[
f_i(h) = S_i(l, h), \quad i = x, y,
\]

\[
= 1 \quad i = z,
\]

where \( S_i(l, h) \) is a modified form of \( S_i(l, h) \), defined after the relation (3.18), just replacing \( h \) by \( h_1 \), even in the expression for \( \gamma_i \).

The modified Reynolds equation for pressure in the lubricant region i.e., the equation (3.18) reduces to the corresponding Newtonian one (3.7) and the relation (3.13) reduces to well known Darcy's law when \( l \rightarrow 0 \). Equations (3.18) and (3.19) have been solved with the help of following boundary conditions for pressures \( p \) and \( p' \) and their continuity at the common interface:

\[
p(x, 0) = p(0, y) = p(x, b) = p(a, y) = 0, \quad (3.20a)
\]

\[
p'(x, 0, z) = p'(0, y, z) = p'(x, b, z) = p'(a, y, z) = 0, \quad (3.20b)
\]

\[
\frac{\partial p'}{\partial z} |_{z=0} = 0 \quad (i.e. \ w' = 0), \quad (3.20c)
\]

\[
p(x, y) = p'(x, y, 0). \quad (3.20d)
\]
Solutions for \( p \) and \( p' \) from (3.18) and (3.19) have been obtained in two different methods for a comparative study and for analyzing lubricant characteristics such as load bearing capacity and time height relation.

### 3.4 Method 1: Finite difference method

Equations (3.18) and (3.19) are written in the following finite difference form and solved simultaneously by an iterative method, using Mathcad 7 and satisfying boundary conditions (3.20):

\[
\begin{align*}
\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} + \frac{\partial^2 p}{\partial z^2} &= 0, \\
\frac{\partial p}{\partial x} &= 0, \\
\frac{\partial p}{\partial y} &= 0, \\
\frac{\partial p}{\partial z} &= 0.
\end{align*}
\]

\[
p'_{ij,k} = \left( k_x(p'_{i+1,j,k} + p'_{i-1,j,k}) + k_y(p'_{i,j+1,k} + p'_{i,j-1,k}) + k_z(p'_{i,j,k+1} + p'_{i,j,k-1}) \right) / 2(k_x + k_y + k_z),
\]

\[
(3.21)
\]

where \( i = 1, 2, \ldots, r \) : nodes along the length of the plate, \( a \)

\( j = 1, 2, \ldots, m \) : nodes along the breadth of the plate, \( b \)

\( k = 1, 2, \ldots, n-1 \) : nodes along the width of the porous plate, \( h \)

\[
p_{ij,n} = \left( S_x(l,h)(p_{i+1,j,n} + p_{i-1,j,n}) + S_y(l,h)(p_{i,j+1,n} + p_{i,j-1,n}) + 12 \mu \Delta^2 \frac{dh}{dt} + 12 \Delta k_x(p'_{ij,n-1}) \right) / \psi(l,h),
\]

\[
(3.22)
\]

where \( \psi(l,h) = 2 \left( S_x(l,h) + S_y(l,h) + 6 \Delta k_z \right) \)
\( p'_{i,j,k} \) = pressure at the point \((i,j,k)\) in the porous region,

\( p'_{i,j,n} = p_{i,j,n} \) = pressure at \( z = 1 \),

\( \Delta \) = nodal length.

The simultaneous solutions of equations (3.21) and (3.22) give the pressure distribution. The pressure \( p_{i,j,n} \) gives the film pressure at the point \((i, j)\) at \( z = 1 \).

**Method 2: Separation of variables (an approximate method)**

Solving equation (3.19) by variable separable method with the help of assumed boundary conditions, we get

\[
p'(x,y,z) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} a_{mn} \cosh\{\gamma_{mn} (h_1 + z)\} \sin(\alpha_m x) \sin(\beta_n y),
\]

(3.23)

where \( \alpha_m = \frac{m\pi}{a}, \hspace{1cm} \beta_n = \frac{n\pi}{b}, \hspace{1cm} \gamma_{mn} = \left\{ (\alpha_m^2 k_x + \beta_n^2 k_y) / k_z \right\}^{1/2} \)

and the coefficients \( a_{mn} \) are to be determined (\( a \) is the length and \( b \) is the breadth of the plates).

Equation (3.18) can also be solved in terms of an infinite series consisting of a set of orthogonal functions satisfying appropriate boundary conditions in the form

\[
p(x,y) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} b_{mn} \sin(\alpha_m x) \sin(\beta_n y).
\]

(3.24)
The pressure continuity equation (20d) suggests that

\[ b_{mn} = a_{mn} \cosh(\gamma_{mn} h_1). \]  

(3.25)

Substituting the relations from (3.23) - (3.25) in (3.18) and then using the orthogonality of functions in the series, we get

\[ a_{mn} = -192 \mu \frac{dh}{dt} \zeta_{mn}, \]  

(3.26)

where \( \zeta_{mn} = a_m b_n \left[ \left( a_m^2 S_x(l, h) + b_n^2 S_y(l, h) \right) \cosh(\gamma_{mn} h_1) + 12k_z \gamma_{mn} \sinh(\gamma_{mn} h_1) \right], \) \( m \) and \( n \) are odd.

The load carrying capacity \( w \), which is balanced by the fluid film thrust on the plate i.e. equal to the integration of film pressure over the surface area of the squeezing plate, is given by

\[ w = \int_0^a \int_0^b p(x, y) \, dx \, dy. \]  

(3.27)

For the method I, the relation (3.27) can be discretized in the following form:

\[ w = \sum_{i=1}^{r} \sum_{j=1}^{m} p_{i,j,n} \Delta^2, \]

(3.28)
Again for the method II, (3.27) gives the following result in closed form:

\[ w = \left\{ \frac{768 \mu ab \frac{dh}{dt}}{(\pi^4)} \right\} \sum_{m=1,3}^{\infty} \sum_{n=1,3}^{\infty} \xi_{mn}, \]  

(3.29)

where \( \xi_{mn} = \left[ m^2 n^2 \left\{ m^2 \pi^2 b^2 S_x(l, h) + n^2 \pi^2 a^2 S_y(l, h) + 12k_z \gamma_{mn} a^2 b^2 \tanh(\gamma_{mn} h_t) \right\} \right]^{-1}. \)

The time-height relation can be obtained in both methods by integrating the values of \( \frac{dh}{dt} \) over an instantaneous squeezed interval \([h_0, h]\), where \( h_0 \) is the initial film height (i.e. \( h \) at \( t = 0 \)). For method II, it can be expressed in the integral form:

\[ t = \left\{ \frac{768 \mu ab \frac{dh}{dt}}{(\pi^4)} \right\} \sum_{m=1,3}^{\infty} \sum_{n=1,3}^{\infty} h_0 h_0^{\xi_{mn}} dh, \]  

(3.30)

where \( m = 1, 3, \ldots \infty, \quad n = 1, 3, \ldots \infty \)

For a comparative study of results in two methods and for variation of results with variation of parametric values, we use the following non-dimensional forms of pressure \( p \), load bearing capacity \( w \), instantaneous squeeze time \( t \) and their constituents:

\[ X = \frac{x}{a}, \quad Y = \frac{y}{b}, \quad H = \frac{h}{h_0}, \quad H_1 = \frac{h_1}{h_0}, \quad K_x = \frac{k_x}{h_0}, \quad K_y = \frac{k_y}{h_0}, \]
\[ K_z = \frac{k_z}{h_0}, \quad \lambda = \frac{a}{b}, \quad L = l/h_0, \quad P = - h_0^3 p/\{ab \mu \frac{dh}{dt}\}, \]
\[ W = - h_0^3 w/\{a^2 b^2 \mu \frac{dh}{dt}\}, \quad T = - h_0^2 \nu/\{a^2 b^2 \mu\}. \]
3.5 Numerical consideration:

For finite difference method (method I), number of nodes in x-, y- & z-directions have been taken conveniently, depending on the choice of parameters $h_i$ and $\lambda$. Further for boundary conditions (3.20 a) and (3.20 b), we consider

$$p_{i,0,k} = p_{0,j,k} = p_{i,-1,k} = p_{i,1,k} = 0,$$

[p_{i,0,k} means $p_{i,j,k}$ at the node $(i,0,k)$]

$$p'_{i,0,k} = p'_{0,j,k} = p'_{i,-1,k} = p'_{i,1,k} = 0,$$

$$p_{i,m,k} = p_{r,j,k} = p_{i,m+1,k} = p_{r+1,j,k} = 0,$$

$$p'_{i,m,k} = p'_{r,j,k} = p'_{i,m+1,k} = p'_{r+1,j,k} = 0,$$

$$p_{i,j} = p'_{i,j,n}.$$

This numerical technique is used only for calculation of fluid pressure, in order to present pressure curves and make a comparative study of results obtained in two methods in consideration.

For the method of variable separable (method II), it is seen that the value of $P$ is altered in the fourth place of decimal or in its higher place, even at the peak of the pressure curve, when the summation in the relation (3.24) is taken up to $n = 100$, $m = 100$ from $n = 50$, $m = 50$. Thus we have considered highest values of $m$ & $n$ as 200 each instead of $\infty$. 
3.6 Results and Discussion:

In order to present some results evaluated based on the numerical technique stated above, we have kept some parametric values fixed unless stated otherwise, e.g., we have taken $\alpha_x = 0.01$ and $\alpha_y = 0.01$. Values of pressure $P$, obtained by two separate methods, and values of load bearing capacity $W$ and time height relation obtained by method II have been presented numerically and graphically.

Fig. 3.2 presents values of $P$ along the direction of $X$ at the middle of the breadth (i.e. $Y = 0.5$). Though values of pressure can be presented in three-dimensional form, but it cannot be presented distinctly for a comparative study. Thus this sectional view has been considered only for a comparative study of $P$ obtained in two methods of solution in consideration. The comparative study in this figure infers that the error in solution of equation (3.18) introduced by variable separable method (an approximate method) is less than 5% in comparison with that in numerical method (method I) and thereby the erroneous in the second method can be neglected in many comparative studies. Only for this reason, we have not calculated load bearing capacity $W$ by method I, which is time consuming one.
Fig. 3.3 presents values of load bearing capacity $W$ with the increasing value of couple stress parameter $L$ for three values of porosity parameter $K_z$ at two positions of film thickness $H$ (assuming $K_x = 0.01$ & $K_y = 0.1$). It is observed that $W$ increases with $L$ and the increasing trend is in good agreement with the existing results obtained for non-porous squeezing film of couple stress fluid (Ramanaiah G. [89]). But the interesting inference in the figure is that the increasing rate diminishes for higher values of $L$. This effect on increasing rate is caused by porosity parameter $K_z$ and this effect is significant for higher values of $K_z$. The figure further reveals that $W$ increases significantly with the decrease of $H$ and this increasing rate is enhanced by couple stress parameter $L$.

In fig. 3.4, values of load bearing capacity $W$ have been presented in a range of length-breadth ratio $\lambda$ ($= \frac{a}{b}$) for two values of couple stress parameter $L$. For each value of $L$ three curves have been presented considering three values of $K_y$ keeping $K_x$ fixed. It is observed that in each curve, $W$ first increases, attains a maximum value $W_{\text{max}}$ and then decreases with the increase of $\lambda$. It is further observed that $W_{\text{max}}$ depends on both couple stress parameter $L$ and porosity parameter $K_y$. For the curves (2) & (5), $\alpha_x = \alpha_y$, $K_x = K_y$ (i.e. isotropic consideration) and $W_{\text{max}}$ is attained at $\lambda = 1$, which infer that for an isotropic porous plate, the load bearing capacity is
maximum when the squeezing plate is a square in size and that it is independent of the couple stress parameter $L$. The other curves show that for $W_{\text{max}}$, the position of $\lambda$ depends on both porosity and couple stress parameters. It is inferred from the figure that for maximum load bearing capacity of a squeezing plate on a lubricated anisotropic porous plate, the squeezing plate need not be a square but it should be longer along the side in which permeability is larger.

In fig. 3.5, values of $W$ have been presented for a range of length-breadth ratio $\lambda$ for three fixed values of $L$. For the values of $W$ in this figure, values of $\alpha_x$, $\alpha_y$, $K_x$, $K_y$ are so chosen that the porous layer is isotropic in nature. It is observed that $W$ attains its maximum value at $\lambda = 1$ for each value of $L$. This study infers that for isotropic porous layer, the value of $W$ is the maximum when the squeezing plate is a square.

Similar phenomenon with values of $W$ have been observed in figure 3.6. Figure 3.6 presents values of $W$ in a range of $\lambda$ for three values of $K_z$. It is observed that the variation of $W$ with $\lambda$ is more significant for lower values of $K_z$.

Figure 3.7, present values of $W$ in a range of film thickness $H$ for three values of $L$. It is observed that $W$ increases rapidly with the
decrease of H and increases significantly for higher values of L. These results are in good agreement with the existing result in earlier theoretical investigation. However, no comparative study of our results has been made with earlier results for mode of non-dimensionalization is not identical in many cases.

In figure 3.8, values of load bearing capacity W against length-breath ratio λ have been presented with variation of K_y, keeping K_x fixed. Thus this figure clearly presents the effect of anisotropic nature of porosity. The figure reveals that for the maximum load capacity of a squeezing plate over an anisotropic porous plate, the squeezing plate needs not be a square. For maximum load bearing capacity, it is lengthen along the direction in which permeability co-efficient is larger.

Figure 3.9, presents logarithmic values of squeeze time against squeezed film thickness for two values of couple stress parameter L. The figure indicates that squeeze time of a squeeze bearing depends on couple stress parameter L i.e. the nature of fluid in general. Though not presented in the figure, it is observed from further study that squeeze time also depends on anisotropic nature of porous plate. But the variation of squeeze time produced by anisotropic parameter is negligible in comparison with the variation
of squeeze time produced by the variation of \( L \). The figure infers that larger the value of \( L \) longer the squeeze time, even in anisotropic porous bearing and it is very much in conformity with the existing literature of porous bearing.

Table 3.1 presents a study of percentage increase in load bearing capacity \( W \) in the present non-Newtonian consideration in comparison with Newtonian consideration. It is observed that the percentage increase is more significant for higher values of couple stress parameter \( L \). But the increasing rate is considerably diminished by the increase of permeability parameters. Similar trend has also been observed in study of percentage increase in pressure \( P \), but no results have been presented in order to minimize the length of the chapter.

3.7 Conclusion:

From the above analysis, following conclusions have been drawn:

1. For comparative studies of a lubricating problem, variable separable method for solution of elliptic equation for pressure in porous region gives good accuracy.

2. Pressure in lubricant region depends on anisotropic nature of porous plates of a porous squeeze bearing.
3. For anisotropic porous layer of a porous squeeze bearing, the maximum load capacity is obtained for a rectangular plate and for the maximum load capacity; the length-breadth ratio of the rectangular plate $\lambda$ depends on the nature of porosity especially in $x$-, $y$-directions.

4. Time height relation of an anisotropic porous squeeze bearing significantly varies with the variation of nature of porosity.
Table 3.1 Relative increase in load bearing capacity in percentage

$W_R$ for $K_X = 0.01$, $\lambda = 1$ and $H_1 = 0.5$ at $H = 1.5$

$W_R = \left( \frac{W_{non-Newtonian} - W_{Newtonian}}{W_{Newtonian}} \right) \times 100$

<table>
<thead>
<tr>
<th>$K_y$</th>
<th>$K_z$</th>
<th>$L = 0.2$</th>
<th>$L = 0.4$</th>
<th>$L = 0.6$</th>
<th>$L = 0.6$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.001</td>
<td>0.0</td>
<td>194.5</td>
<td>726.5</td>
<td>1578.1</td>
<td>2703.2</td>
</tr>
<tr>
<td></td>
<td>0.01</td>
<td>191.4</td>
<td>713.3</td>
<td>1547.4</td>
<td>2647.6</td>
</tr>
<tr>
<td></td>
<td>0.10</td>
<td>187.9</td>
<td>692.8</td>
<td>1501.2</td>
<td>2589.3</td>
</tr>
<tr>
<td>0.01</td>
<td>0.0</td>
<td>189.4</td>
<td>688.4</td>
<td>1456.9</td>
<td>2435.7</td>
</tr>
<tr>
<td></td>
<td>0.01</td>
<td>187.5</td>
<td>680.1</td>
<td>1437.4</td>
<td>2408.9</td>
</tr>
<tr>
<td></td>
<td>0.10</td>
<td>184.4</td>
<td>671.1</td>
<td>1408.4</td>
<td>2373.5</td>
</tr>
<tr>
<td>0.1</td>
<td>0.0</td>
<td>181.6</td>
<td>673.5</td>
<td>1411.7</td>
<td>2316.8</td>
</tr>
<tr>
<td></td>
<td>0.01</td>
<td>175.8</td>
<td>653.9</td>
<td>1389.1</td>
<td>2251.9</td>
</tr>
<tr>
<td></td>
<td>0.10</td>
<td>167.8</td>
<td>616.9</td>
<td>1353.7</td>
<td>2185.3</td>
</tr>
</tbody>
</table>

$W_{Newtonian}$ means load capacity in Newtonian consideration ($L = 0$)

$W_{non-Newtonian}$ means load capacity in non-Newtonian consideration ($L \neq 0$)
Fig. 3.1 *Squeeze film geometry of finite porous bearing with sides a & b*
Fig. 3.2 Comparative values of $P$ with increasing values of $X$ for $L=1.5$, $\alpha_x = 0.01$, $\alpha_y = 0.01$, $K_x = 0.01$, $K_y = 0.01$, $K_z = 0.01$, $H_x = 0.5$, $\lambda = 1$ and $H = 0.5$ at $Y=0.5$ (a sectional view)

- Method 1
- Method 2
Fig. 3.3 \( W \) against \( L \) for \( \alpha_x = 0.01, \alpha_y = 0.01, \lambda = 1 \) and \( H_1 = 0.5 \) \( \text{---} \) for \( H = 0.4 \), \( \text{---} \) for \( H = 0.6 \) (1), (4) for \( K_z = 0.1 \); (2), (5) for \( K_z = 0.01 \); (3), (6) for \( K_z = 0.001 \)
Fig. 3.4. $W$ against $\lambda$ for $\alpha_x = 0.01$, $\alpha_y = 0.01$, $K_x = 0.01$, $K_z = 0.01$, $H_i = 0.5$; for $L = 0.5$, for $L = 1$
(1), (4) for $K_y = 0.01$; (2), (5) for $K_y = 0.01$;
(3), (6) $K_y = 0.1$
Fig. 3.5 Values of $W$ with increasing values of $\lambda$ for $K_x = 0.01$, $\alpha_x = 0.01$, $\alpha_y = 0.01$, $H_1 = 0.5$ and $H = 0.5$ for $L = 1.5$, $L = 1$, and $L = 0.5$.
Fig. 3.6  Values of $W$ with increasing values of $\lambda$ for $L = 1.5, \alpha_x = 0.01, \alpha_y = 0.01, H = 0.5$ and $H = 0.5$

- for $K_z = 0.1$,
- for $K_z = 0.01$,
- for $K_z = 0.001$
Fig. 3.7 Values of $W$ with increasing values of $H$ for $K_z = 0.1$, $\alpha_x = 0.01$, $\alpha_y = 0.01$, $H_1 = 0.5$ and $\lambda = 1$

- for $L = 1.5$,
- for $L = 1$,
- for $L = 0.5$
Fig. 3.8  Values of $W$ with increasing values of $\lambda$, for $L = 1.5$, $\alpha_x = 0.01$, $\alpha_y = 0.01$, $K_x = 0.01$, $H_1 = 0.5$ and $H = 0.5$ for $K_y = 0.001$, for $K_y = 0.01$, for $K_y = 0.1$
Fig. 3.9  Variation of non-dimensional time with squeezed film thickness $H$ for $H_f = 0.5$, $\lambda = 1$ and $K_z = 0.01$