CHAPTER - II

Review of Past Work
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REVIEW OF PAST WORK

2.1 INTRODUCTION

Since the pioneering experiments of Darcy (1856) on the flow of water through filter sands, a succession of important analytical treatments coupled with experimental support have appeared on the subject of porous media flow, thereby subjecting it to continuous exploitation in various aspects by different investigators. A brief and relevant review of the past work related to seepage flow is presented in this chapter.

2.2 PAST WORK ON GROUNDWATER FLOW IN NUTSHELL

It was in 19th century, groundwater hydrology began as a quantitative science. Flow of water through porous structure was demonstrated first by French Civil Engineer, Henry Darcy. He postulated a mathematical law that governs the flow of ground water, which later becomes most widely referred and used law, as Darcy's law.

Henry Darcy (1803-1856)
Dupuit (1863), based on Darcy’s Law, derived an equation for the rate of flow of water to a well. Thiem (1906) modified Dupuit’s equation to calculate hydraulic properties of an aquifer by means of a pump test.

Later part of the 19th Century saw the development of a more comprehensive understanding of the relationship of ground water to the geological formations in which it occurs. Chamberlin (1885) published the first hydrogeologic report, “The Requisite and Qualifying Conditions of Artesian Wells” in 1885. It provided a theoretical basis for the scientific study of the occurrence of ground water prompting an explosion of activity in the evaluation of ground water resources in the United States.

King (1892) wrote “Principles and Conditions of the Movement of Groundwater” which introduced a number of concepts related to the movement of ground water due to gravity and presented water level contour maps. He was the first to observe that in humid areas water table was a subdued reflection of surface topography.

As early as 1901, Forchheimer was the first to realize that Darcy’s law was not universally valid for flow through porous media. It was stated that at higher Reynolds number, linear relationship between hydraulic gradient and velocity of flow breaks down.

Theis (1935) proposed an equation describing the decline of the piezometric (potentiometric) surface in a fully confined aquifer due to the withdrawal of water from a well. This work formed the basis to quantify the flow of water to wells in confined or semi confined aquifers. Formation of a
regional cone of depression and its impact on the dynamic equilibrium of the aquifer was also described.

Hubbert (1940) in his book, "The Theory of Ground Water Motion", proposed a theoretical foundation for Darcy equation. It was demonstrated that Darcy’s law for groundwater flow is analogous to Ohm’s law for the flow of electricity.

Jacob (1940) devised a graphical method of interpreting aquifer test data for a pumping well in a fully confined aquifer.

Ergun (1952) proposed a modified form of Forchheimer's expression to describe flow through packed columns made of spheres, cylinders, round sand and coarse materials.

Hantush and Jacob (1955) solved the problem of quantifying non-steady flow to a well in a leaky or semi-confined aquifer.

Ahmed and Sunada (1969) derived expressions for Forchheimer coefficients from Navier Stokes equation and established that these coefficients are not strictly constants as was hitherto assumed and found to depend on Reynolds Number of flow.

Nasser (1970) made use of this form of expression to study radial non-Darcy flow through crushed rock.

Arbhabirama and Dinoy (1973) suggested the use of square root of intrinsic permeability as characteristic length to define friction factor and
Reynolds Number. A graph similar to Moody diagram of pipe flow was presented for porous media flow.

Volker (1975) presented the results of an experimental study using screened gravel as media packed in converging boundaries, simulating flow to a well. The partial differential equation with suitable boundary values was solved using finite difference method.

Subramanya and Madav (1978) have reported extensive studies on linear and non-linear flow, through porous media and have brought out an elaborate report on the various problems encountered in seepage flow.

Somerton and Wood (1988) investigated effect of wall of the permeameter on the resistance and the viscous effects were accounted for by incorporating the Brinkman’s extension term (1947) in the momentum equation. The wall zone is assumed to extend from the inner side of the wall to one-radius of the particle used as media.

Nield (1991) analyzed the ability of the Brinkman-Forchheimer equation to adequately model flow in a porous medium and at a porous-medium and clear-fluid interface. It was demonstrated that certain terms in the equation require modification, and that there was a difficulty when using this equation to deal with a stress boundary condition.

Sohvas (1992) reported the results on the relation between shape of particles and soil permeability.
Burcharth and Andersen (1995) presented a theoretical development for the seepage flow, with a special reference to various coefficients appearing in different equations.

Van Gent (1995) investigated permeability characteristics to study flow through coarse granular material. The results are of importance for studying the flow in permeable structures such as rubble mound structures and gravel beaches. The contributions of laminar and turbulence friction terms have been determined as well as the importance of inertial resistance.

Pradeep Kumar and Venkatraman (1995) carried out an experimental program to study the behaviour of large size crushed rock and glass spheres on a converging permeameter. The applicability of the well known quadratic and power laws to analyse the flow behaviour in a converging permeameter was examined.

Masuoka and Takatsu (1996) examined the macroscopic governing equations for the turbulent flow through porous media consisting of packed spheres.

Antohe and Lage (1997) presented two equation turbulence model for incompressible flow within a fluid saturated and rigid porous medium.

Lee and Rang (1997) examined an incompressible fluid flow across a bank of circular cylinders and modeled as a non-Darcy flow through a porous medium. Continuity equation and momentum equation in pore scale were solved on a Cartesian grid system. Darcy-Forchheimer drag was determined from the resulting volumetric flow rate under a known pressure drop.
Thiruvengadam and Pradeep Kumar (1997) verified the validity of application of the Forchheimer equation for steady non-Darcy radial flow through coarse granular media and glass spheres.

Bingjun Li et al., (1998) investigated non-Darcy flow in rock fill material. Based on pipe flow theory, definition for mean hydraulics radius in rock fill material, theoretical relationships between friction coefficient and Reynolds number and relationships between hydraulic gradient and bulk seepage velocity were presented.

Hwa-Chong Tien and Kwang – Sheng Chalang (1999) analyzed non-Darcy flow in a vertical slot filled with porous matrix. The non-Darcy model includes the Forchheimer extensions along with the convection terms to describe the flow in porous media.

Shijie Lice and Jacob (1999) applied a volume averaging approach to derive governing equations for purely viscous homogeneous flows in porous media.

Rhodes Trussell and Melissa Chang (1999) reviewed the development of understanding of flow through porous media with a focus on understanding the clean-bed head loss in water filters. A model of the Forchheimer form is developed by typical existing empirical models to current hydrodynamic theory, and constants are developed for three filter media: glass beads, crushed silica sand, and crushed anthracite coal.

Skjetne and Aurianit (1999) derived a cubic weak inertia correction term for Darcy’s law which is valid for any matrix of anisotropy.
In turbulent models for flow in a porous medium, the next one starts with the

Warne et al. (2000) proposed two distinct approaches for developing

\[ \text{proportionality appearing therein} \]

the motion equation proposed by Ward (1994 and 1996) and the constant of

behave by coarse porous media. Their study mainly pertains to

Venkataraman and Rama Mohan Rao (2000) described how

\[ \text{mathematical relationships were highlighted} \]

methodology was suggested and the differences between those

were investigated. A new double-decomposition (time and volume)

energy in flow through porous media. Accordingly, two methods of applying

Peters and Lemos (2000) tried to analyze the role of turbulent kinetic

\[ \text{draining systems} \]

results, a method was proposed to determine hydraulic properties and
describe the flow situations in different coarse materials. Based on the

quadratic and power, and had shown that these two are equally suitable to

Bouder and Zimmer (2000) studied the applicability of two laws.

\[ \text{surface velocity at the macroscopic scale} \]

term has been found to be tensorial and proportional to the square of the

networks as well as the tortuosity and porosity of the media. The non-Darcy

special correlation-numeric. Non-Darcy flow had been investigated in those

anisotropy were then proposed. \( \text{viz.} \) size-induced, concentration-induced and

in porous media at the macroscopic scale. Three structural models of

Wang et al. (1999) tried to understand the non-Darcy flow behavior
macroscopic equations using the extended Darcy-Forchheimer model. The second method considers the microscopic balance equations. In both cases, time and volume averaging operators are applied in a different order.

Karakas and Kavvs (2000) derived a conservation equation for the random ground water flow seepage velocity under general conditions of hydraulic stochastic functions.

Fourar and Henormand (2001) presented a new model for describing two-phase flow at high velocities in porous media and fractures. It was based on the generalization of the single-phase Forchheimer's equation by introducing a multiplying factor for the superficial velocity during two-phase flow. This factor was assumed to depend on the saturation and field properties but not on the flow regime.

Macedo et al., (2001) studied the influence of turbulent effects on a fluid flow through a porous medium by numerically solving the set of Reynolds - averaged Navier-Stokes equations. A good agreement with the Forchheimer's equation is observed.

Buddhima and Sujeewa (2002) proposed an explicit solution for the critical hydraulic gradient required to move a base particle with in a pore channel. The theoretical model was tested in the laboratory using five gravel filters and a cohesionless base soil consisting of very fine river sand.

Imam Wahyudi et al., (2002) reported results of a study conducted to examine several sands with a large spread of particles size in order to validate the modeling of both Darcy's and Forchheimer's law parameters.
Tortuosity values calculated from pressure drop experiments were found to be within range of values found in the literature.

Payne and Song (2002) established exponential decay bounds for solutions of the Forchheimer equations in semi-infinite pipe flow through a porous medium when homogeneous initial and lateral surface boundary conditions are applied.

Legrand (2002) analyzed and compared experimentally the capillary model based on the square root of permeability as characteristic length, considering the effect of the pore diameter and the tortuosity.

Brown et al., (2003) presented a review and progress of post Darcy work in the conference held as a remembrance of 200th birth anniversary of Darcy.

Murali et al., (2004) brought out the limitations of not specifying the range of applicability of equations relating linear and nonlinear parameters with the size of the medium.

Mehmet Emin Birpinar and Zekai Sen (2004) developed an analytical solution for radial groundwater flow towards a fully penetrating well in a leaky aquifer. The results are presented as a set of type curves for a leaky aquifer with non-Darcian groundwater flow.


Fourar and Henormand (2004) examined 3D effect on flows at high velocities through homogeneous porous media. Results of numerical
simulations of a steady incompressible Newtonian fluid flowing through a 2D and a 3D periodic porous media performed at various Reynolds numbers are presented. The flow patterns were described and pressure and shear stress at the fluid / solid interface were analyzed.

Xiao-Hong Wang and Zhi-Feng Liu (2004) studied the scaling relations for the fluid permeability and the inertial parameter in the Forchheimer equation based on solving the Navier-Stokes equations in the two-dimensional percolation porous media for 500 different configurations. 

Note: In addition to the above references, relevant past work is cited while discussing various concepts such as methods of approach, governing equations, porosity, wall and tortuosity effects.

2.3 THE CONCEPTS

2.3.1 Methods of Approach

Aim of all the investigators on seepage flow had been to relate flow resistance to its behaviour in terms of measurable properties of the fluid and the medium. Three methods of approach to achieve this objective have been described. They are

i) Correlation based on pipe flow analogy,

ii) Correlation based on flow characteristics of the medium,

iii) Correlation based on drag on the individual particle.

Pipe flow analogy consists in relating micro flow properties of granular medium to those of flow through a pipe by replacing linear dimensions of the latter with the corresponding hydraulic equivalents of medium.
In the second approach, flow characteristics, rather than structural parameters of granular medium are considered. One or two overall flow parameters are determined from experiments and are then used to predict flow pattern in all regimes of flow. For example, Darcy’s law, Forchheimer equation etc. A detailed discussion is presented in Sec. 2.3.6.

In the third approach, two non-dimensional numbers to quantify resistance to flow and flow behaviour when a fluid flows through the coarse granular media are derived by the use of dimensional analysis. These numbers are Resistance Coefficient $\lambda$ and Reynolds Number $Re$ respectively and are given by

$$\lambda = \frac{Idg}{V^2} \frac{1}{G_1(Z), G_2(f), G_3(D/d), G_4(r_0), G_5(s)}$$

(2.1)

and

$$Re = \frac{\rho vd}{\mu} G_2(f)$$

(2.2)

Where

- $G_1(Z)$ = Function expressing effects of shape of the particle on resistance,
- $G_2(f)$ = Function expressing effect of porosity on resistance,
- $G_3(D/d)$ = Function expressing effect of wall on resistance,
- $G_4(r_0)$ = Function expressing effect of surface roughness on resistance,
- $G_5(s)$ = Function expressing particle size distribution,

Relationship between these two variables is obtained from experiments for different fluids and media. These two parameters, in general, are also used to characterize the flow pattern in a granular medium.
which is generally classified based on the relative influence of viscous and inertial forces and the term regime is often used to identify the flow behaviour. When the former predominates, regime is referred to as laminar regime, and the turbulent regime indicates the predominance of inertial forces. Further, intermediate range is divided into steady inertial and turbulent transitional regimes.

Analysis of the experimental data of the various investigators (Wright (1969), Subramanya and Madhav (1978), Kovacs (1981)) discloses that the behaviour of flow through porous media may be analyzed by classifying the flow regimes into five categories as illustrated in Fig. 2.1. They are (i) Micro seepage, (ii) Darcy regime, (iii) Non linear laminar regime, (iv) Turbulent transitional regime and (v) Fully turbulent regime.

As the thesis is not concerned with micro seepage flow in which velocities are extremely small — of the order of $10^{-5}$ cm/sec — and flow may become non Newtonian, it is not discussed here any more.

![Fig. 2.1 Flow regimes in porous media](image-url)
In the Darcian zone, flow is represented by Darcy's law. In this zone, micro velocity is stationary and head loss is proportional to velocity of flow. Maximum velocity occurs nearly in the centre of each flow passage. Viscous forces are very large as compared with inertial forces in this regime. Effect of surface forces is not felt.

In non linear laminar regime, although both viscous and inertial forces influence the motion, effect of inertial forces gradually increases and causes the flow to deviate from linear relationship as expressed by Darcy's law. Micro velocity is stationary and flow is still laminar in this regime. Stationary vortices are formed at the upper end of the regime.

In turbulent transitional regime micro velocity fluctuates with a regular frequency and head loss becomes more dependent on square of the velocity. Inertial actions predominate and vortices are shed at regular intervals from individual grains. Turbulence may begin in stages as the velocity of flow is increased.

In fully turbulent regime, flow is turbulent and micro velocity fluctuates randomly about an average value. Head loss is more or less dependent on square of the velocity.

Flow through porous medium is stochastic in nature. It is not possible to predict with any accuracy the limit of a particular regime although the change from one regime to another regime is gradual and not abrupt as observed in the case of flow through straight open channels or flow in pipes. However, Reynolds number has been generally accepted as a parameter to
classify the flow regimes, and Kovacs (1969) and Wright (1968 and 1969) have suggested numerical limits of Reynolds number for various zones, which are given in Table 2.1.

Table 2.1: Limits of Reynolds number for various regimes

<table>
<thead>
<tr>
<th>Sl. No.</th>
<th>Regime</th>
<th>Range of Reynolds Number</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Wright</td>
</tr>
<tr>
<td>1</td>
<td>Darcy regime</td>
<td>$0.6 \frac{V d_p}{\nu (1-f)}$</td>
</tr>
<tr>
<td>2</td>
<td>Non linear laminar</td>
<td>$1 - 5$</td>
</tr>
<tr>
<td>3</td>
<td>Turbulent transition</td>
<td>$90 - 120$</td>
</tr>
<tr>
<td>4</td>
<td>Fully turbulent</td>
<td>$&gt; 800$</td>
</tr>
</tbody>
</table>

where

$\nu = \text{Velocity of flow}$

d = \text{Volume diameter of the particle}$

$\frac{d (1-f) \alpha_s}{4f} = \text{Size of the particle as defined by Kovacs and}$

f = \text{Porosity}$

$\nu = \text{Kinematic viscosity}$

$\alpha_s = \text{Shape factor of the particle}$

Proper definitions of the characteristic length and characteristic pore velocity are the primary requirements to analyze seepage flow problems. Further, it requires the consideration of porosity, wall and tortuosity effects on the analysis of the data and hence in developing relationships among different variables.
2.3.2 Characteristic Length Dimension

Size of the pore is an important parameter in the study of seepage flow. However, it cannot be determined directly, since the pore system forms a very complicated surface and geometrically difficult to describe. Therefore, size of the particle has been chosen as the characteristic length parameter in defining seepage flow characteristics. Volume diameter, which is the diameter of a sphere having the same volume, is used to denote the size of the particle. (Scheidegger (1960) and Kovacs (1981)).

Hydraulic radius, defined as the ratio of void ratio (e) to specific surface (S) of pore space is used as characteristic length in the study of seepage flow problems. (Scheidegger (1960) and Kovacs (1981))

In addition to volume diameter and hydraulic radius, square root of intrinsic permeability is also found in the literature. But, as it cannot be determined directly, it is of limited use. (Arhabirama and Dinoy (1973), Venkataraman and Rama Mohan Rao (2000)).

2.3.3 Porosity Effect

Another important parameter influencing flow phenomenon is the porosity, which depends upon the degree of packing. A certain percentage variation in porosity of the medium brings about a much larger change in resistance to flow than what would be caused by an equal change of percentage in any other parameter. It exercises greatest influence of all the functions on the flow resistance. Porosity is the main parameter linking micro
and macro velocities, pore size and particle size of any particular assemblage of similarly shaped particles.

The basic assumptions in defining a porosity function are:

i) no pores are sealed off,
ii) pores are distributed at random,
iii) pores are reasonably uniform in size,
iv) medium is of moderate porosity,
v) slip phenomenon is absent.

In order to consider the effect of porosity and transform macro values to micro values correction for porosity is required. The correction has two components; namely, (i) the correction to be applied for velocity and (ii) the correction to be applied to the size of the particle. Therefore, porosity enters the governing equation through the characteristic pore velocity and characteristic length. Hence, complete porosity function will be the product of that part entering via micro velocity and that part entering via pore dimension.

It is seen from past work as quoted by Wright (1968) that the formulae purporting to represent the influence of porosity in the resistance equation are numerous. However, it is observed that, majority of the formulae were proved to be applicable over a limited range of porosity only. There is a large variation of this function with the regime of flow, and some of them lack analytical foundation, and majority of the functions do not reflect the effect of shape of the particles, which also affects the resistance.
Macro velocity is corrected for porosity to obtain micro velocity as

\[ V_v = \frac{V_b}{f} \]  

(2.3)

and the size of the pore is obtained from size of the particle as

\[ d_{por} = \frac{d_p f}{(1-f)} \]  

(2.4)

where

- \( V_v \) = Micro velocity of flow
- \( V_b \) = Macro velocity of flow
- \( d_{por} \) = Size of the pore
- \( d_p \) = Size of the particle (volume diameter)
- \( f \) = Porosity of the medium.

This function gives reliable results for values of porosity between 30% and 70%. Further, Carman (1937) also adopts, a porosity function of the same form as suggested by Rose, which gave consistent results in the range of 0.3 < f < 0.5, which is the common range occurring in the field.

### 2.3.4 Wall Effect

It is not possible to reproduce identical conditions in the laboratory as those that exist in the field. Permeameters used in laboratory are of different shapes and finite sizes. Results of any experimental study cannot be applied directly to field conditions. They require correction, called wall effect, to take care of influence of the permeameter boundary.

Total surface area of contact near the wall of a permeameter is larger and hence the resistance offered by the container wall to flow is significant. Further, in laminar regime, where energy loss is primarily due to shearing
forces, overall resistance may considerably be affected because of presence of a finite boundary.

Flow paths in the immediate vicinity of the wall, where particles are loosely packed, are less tortuous and hence velocities are high. Studies on velocity distribution across a packed bed through which a fluid is flowing disclose that effect of wall on resistance-flow pattern relationship needs to be considered.

Rose and Rizk (1949) plotted the ratio, (resistance of finite bed / resistance of infinite bed) against the parameter (D/dp), where D is the size of the permeameter and dp is the size of the particle, along with the data of many other investigators and observed for Reynolds numbers less than 400 the magnitude of wall correction is greater than unity; and for Reynolds numbers greater than 400 the factor is less than unity. Finally, they opined that the effect of wall can be neglected when the ratio of size of the permeameter to the size of the media is greater than 50.

According to Dudgeon (1966 and 1967), the marked dependence of velocity on porosity causes a much higher than average flow rate to occur near the wall. Error in taking mean velocity to represent one dimensional velocity can be estimated from porosity-permeability data and measurements of local porosity in the wall zone, compared with the overall average porosity. Dudgeon estimated the effect experimentally, by measuring flow through inner zone and wall zone separately in a specially designed permeameter and proposed wall correction factors as listed in Table 2.2.
Table 2.2: Wall correction factors (Proposed by Dudgeon)

<table>
<thead>
<tr>
<th>Regime</th>
<th>Wall correction factors</th>
<th>Estimated</th>
<th>Measured</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear regime</td>
<td></td>
<td>1.06</td>
<td>1.10</td>
</tr>
<tr>
<td>Non-linear regime</td>
<td></td>
<td>1.06</td>
<td>1.07</td>
</tr>
</tbody>
</table>

Nasser (1970) adopted a wall correction factor, proposed by Mc Corquodale and NG (1969 and 1970), which was obtained on the basis of studies conducted by Dudgeon (1967). It is a function of the area of cross section of the permeameter and size of the particle. However, Nasser’s work does not indicate as to how the wall effect has to be considered when the flow is radial.

2.3.5 Tortuosity Effect

Tortuosity ($\tau$) is one of the fundamental geometrical properties of the porous medium. It was introduced to account for the fact that the pores are not straight in a real porous medium. Length of the most probable path is longer than the overall length of the porous medium. Tortuosity is defined as the length of the actual flow path of particle divided by the overall (average) path of fluid particle through the porous medium i.e. sinuosity of actual flow path in the porous medium. Hence, tortuosity ($\tau$) takes the value greater than one. Values of $\tau < 1$ are not physically correct.

It is thus, a dimensionless textural constant related to the shape and orientation of the pores. Therefore, tortuosity is a macroscopic measure of
both the sinuousness of the flow path and variation of pore size along the flow path. Tortuosity is expressed mathematically as

$$\tau = \frac{l_a}{L}$$  \hspace{1cm} (2.5)

where \( l_a \) = Actual fluid flow path through porous medium,

\( L \) = Average path of fluid particle through porous medium.

According to Carman (1937) hydraulic gradient which is a representative of driving force in the porous medium may be corrected for tortuosity effect as:

$$\left( \frac{Vh_f}{L} \right) = \frac{Vh_f}{l_{\text{eff}}} \times \frac{l_{\text{act}}}{L}$$  \hspace{1cm} (2.6)

$$i_h = i_{\text{act}} \tau$$  \hspace{1cm} (2.7)

$$i_{\text{act}} = \frac{i_h}{\tau}$$  \hspace{1cm} (2.8)

Comiti and Renaud (1989) proposed an equation for pore velocity based on the presentation of the porous medium by a bundle of identical tortuous pores, as

$$V_c = \frac{V_f}{f} \tau$$  \hspace{1cm} (2.9)

Kopenen et al., (1997) discussed the concept of tortuosity of fluid flow in porous media. A clear correlation between the average tortuosity of the flow paths and the porosity of the substance was established, and the findings were presented in the form of a graph between porosity (\( \phi \)) and tortuosity (\( \tau \)).
2.3.6 Governing Equations – General Remarks

Some of the relevant works concerning various forms of equations employed in seepage flow are reviewed in this section. Each approach is better understood when its performance is compared with those of others.

Darcy (1856) was the first to relate superficial velocity and hydraulic gradient by means of an experimentally determined coefficient, called the Darcy Coefficient or Coefficient of permeability, (k) and is given by

\[ V_0 = k_i \]  \hspace{1cm} (2.10)

Darcy's law is applicable when flow is laminar. Till the beginning of this century, this equation was used irrespective of the flow regime. However, the validity of this equation became questionable for flows with high Reynolds numbers.

It was recognized that there was an upper limit to the validity of this law in terms of velocity. Maximum diameter of the uniform sand in which the laminar flow occurs is found to be around half a millimeter.

Experiments for the determination of Reynolds number at which the Darcy's law is expected to break down have been reviewed by many investigators. It is evident from the review of these investigations, that there is no agreement on the value of Reynolds number, at which Darcy's law would no longer be valid. Todd (1959) and Scheidegger (1960) quote the range as from 1.0 to 10 and 0.1 to 75 respectively. This wide variation in the
range of Reynolds number is due in part to the actual indeterminacy of the effects of size, shape and distribution of particles and porosity effect.

Forchheimer (1901), based on the results of experiments on a sand model for well flow and studying the transient pressure change in porous media by the diffusion equation, based on adding terms of a high order in the equation, was the first to propose an equation covering linear and post-linear ranges in a quadratic form as,

\[ i = a_1 V + b_1 V^2 \]  \hspace{1cm} (2.11)

in which \( a_1 \) and \( b_1 \) are constants determined by properties of the fluid and porous medium and are known as Darcy and non-Darcy parameters. It is obvious from the above equation that \( a_1 V \) represents the rate of energy loss in the linear regime and \( b_1 V^2 \) for fully developed turbulent regime. As mentioned in Scheidegger (1960), equation (2.11) was later refined by Forchheimer himself (1901), by adding a third term to make equation fit experimental data better as

\[ i = a_1 V + b_1 V^2 + c_1 V^3 \]  \hspace{1cm} (2.12)

In order to take into account the effect of transitional conditions of seepage flow, the two term Forchheimer equation was modified as (Rose (1951))

\[ i = a_1 V + b_1 V^2 + c_1 V^{1.5} \]  \hspace{1cm} (2.13)

Forchheimer equation has been further generalized to contain a time dependent term after Polubarinova-Kochina (1952), as

\[ i = a_1 V + b_1 V^2 + c_1 \frac{dv}{dt} \]  \hspace{1cm} (2.14)
For steady flow conditions, Eq. (2.14) reduces to Eq. (2.11). Further, Eqs. (2.12) and (2.13) seem to be the more representative ones containing linear, turbulent and transitional regimes. However, according to McCorquodale (1969) these equations were 'only slightly' better than the two-term Forchheimer equation. In general, Eq. (2.11) is used in computations because of its simplicity.

Widely used forms of hydraulic gradient-velocity of flow relationships are summarized in Table 2.3.

Nature and factors influencing \( a_f \) and \( b_f \) of Forchheimer equation have become the subject of investigation. Ahmed and Sunada (1969) derived expressions for \( a_f \) and \( b_f \) from Navier-Stokes equations as

\[
a_f = \frac{\mu}{\rho g k_o}
\]

and

\[
b_f = \frac{1}{g \sqrt{c k_o}}
\]

where

- \( \mu \) = Dynamic viscosity of the fluid
- \( \rho \) = Density of the fluid
- \( k_o \) = Intrinsic permeability = \( cd_p^2 \)
- \( d_p \) = Size of the particle
- \( c \) = Media constant
- \( g \) = Acceleration due to gravity

### Table 2.3: Different Forms of Non-Linear Equations

<table>
<thead>
<tr>
<th>Sl. No.</th>
<th>Equation</th>
<th>Proposed by</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>( i = a_f V + b_f V^2 )</td>
<td>Forchheimer (1901)</td>
<td>Empirical</td>
</tr>
<tr>
<td>2.</td>
<td>( i = a_f V + b_f V^2 + c_f V^3 )</td>
<td>Forchheimer (1901)</td>
<td>Empirical</td>
</tr>
<tr>
<td>3</td>
<td>( i = p V^l )</td>
<td>Missbach (1931)</td>
<td>Empirical</td>
</tr>
<tr>
<td>4</td>
<td>( V_f = c r^b f^n )</td>
<td>Willkins (1955)</td>
<td>Semi-empirical</td>
</tr>
</tbody>
</table>
It was established that $a_r$ and $b_r$ are not constants and are dependent on Reynolds number of flow. Curtis and Lawson (1970) expressed doubts on the possibility of obtaining a single-generalized curve relating friction factor and Reynolds number presented by Ahmed and Sunada as they neglected turbulent fluctuations in the derivation. It was pointed out that the expressions for $a_r$ and $b_r$ are similar to those proposed by Irmay (1959). They suggested the use of an exponential form of equation.

Ranganadha Rao and Suresh (1970) observed that the practical usefulness of the equations proposed by Ahmed and Sunada based on Forchheimer equation is limited, because the intrinsic permeability in the equations cannot be determined directly. They noted that there is a large difference in the values of $a_r$ and $b_r$ obtained for media of approximately the same size used by various investigators. They attributed this large difference to neglecting the effects of porosity, shape, orientation of the particle and tortuosity in the equations for $a_r$ and $b_r$.

Commenting on Ahmed and Sunada’s study, Todd (1970) opined that $a_r$ term involving turbulent characteristics in porous media should have been considered in the expressions for $a_r$ and $b_r$. Mavis (1971) suggested that Ahmed and Sunada could have stated the limits of applicability of their work and the antecedent conditions on the values of intrinsic permeability.

Another heuristic correlation between $i$ and $V$ is (Scheidegger (1960) and Subramanya and Madav (1978))

$$i = pV^{1.8} \quad (2.17)$$

Exponent 1.8 is not well established.
Wide use of an equation of the form,

\[ i = p V^j \]  \hspace{1cm} (2.18)

is also found in literature. (Scheidegger (1960) and Subramanya and Madhav (1978)). In this equation, \( p \) is a coefficient determined by the properties of the medium and the fluid and \( j \) is an exponent lying between 1 and 2. \( j \) equals 1 for laminar flow and tends to 2 as the flow regime approaches turbulent flow conditions. A glance at the nature of \( p \) indicates that this single parameter accounts for the effects of various properties of the medium, namely, size, shape, surface area, tortuosity and porosity and properties of the fluid, that is, density and viscosity. Conclusions from such equations may be misleading or confusing.

Willkins (1955) while investigating the permeability properties of a wide range of gravel and glass spheres, derived the equation of the form:

\[ V_p = c \mu^\alpha r^\beta \rho^n \]  \hspace{1cm} (2.19)

where

- \( V_p \) = Seepage velocity,
- \( r \) = Hydraulic mean radius
- \( \frac{\text{Void ratio}}{\text{Surface area per unit volume}} \)
- \( c, \alpha \) and \( \beta \) = constants
- \( n = \frac{i}{j} = \text{Constant} \)
- \( \mu = \text{Dynamic viscosity of the fluid} \)

It is obvious that Eq. (2.19) is an improved form of Eq. (2.18) as majority of the media properties are given separate identify in Eq. (2.19).
Parkin (1963) used this form of equation to study the hydraulic and stability problems of inbuilt spillway dams.

2.4 CONCLUDING REMARKS

Various regimes of flow that occur in a seepage flow have been reviewed. The concepts such as characteristic length dimension, characteristic pore velocity the wall effect and tortuosity effect have been discussed along with the past work carried on them. A detailed study on various forms of governing equations including the drawbacks is also presented in this chapter.