Chapter 3

Interval Valued Equiprime, 3-Prime and C-Prime L-Fuzzy Ideals

In this chapter, we define interval valued equiprime, 3-prime and c-prime L-fuzzy ideals of nearrings using interval valued t-norms and interval valued t-conorms. We characterize interval valued prime L-fuzzy ideals in terms of their level subsets. We find interrelations among different interval valued prime L-fuzzy ideals. We relate interval valued fuzzy points with different interval valued prime fuzzy ideals.

3.1 Introduction

Prime ideal in rings is a natural extension of the concept of a prime number. The prime ideal notion is extended from commutative rings to nearrings in many ways. The frequently used generalized prime ideal notions in nearrings are equiprime, 3-prime and c-prime (refer Booth, Groenewald and Veldsman [26], Veldsman [110], Groenewald [55]). We define interval valued equiprime, 3-prime and c-prime L-fuzzy
ideals of nearrings. Then we characterize these prime ideal notions in terms of their level subsets.


We study interrelations among different interval valued L-fuzzy prime ideals. We prove that interval valued equiprime L-fuzzy ideal and interval valued 3-prime L-fuzzy ideal coincide in a distributive nearring. We characterize interval valued prime fuzzy ideals in terms of interval valued fuzzy points.

### 3.2 Interval Valued Equiprime, 3-Prime and C-Prime L-Fuzzy Ideals

**Definition 3.2.1.** An i-v L-fuzzy ideal \(\hat{\mu}\) of \(N\) is called an i-v **equiprime L-fuzzy ideal** if for all \(x, y, a \in N\),

\[
C_I(\hat{\alpha}, \hat{\mu}(a)) \geq T_I(\hat{\beta}, C_I(\hat{\alpha}, \inf_{r \in N} \hat{\mu}(arx - ary))) \quad \text{or} \quad C_I(\hat{\alpha}, \hat{\mu}(x - y)) \geq T_I(\hat{\beta}, C_I(\hat{\alpha}, \inf_{r \in N} \hat{\mu}(arx - ary))).
\]
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**Definition 3.2.2.** An i-v L-fuzzy ideal \( \hat{\mu} \) of \( N \) is called an *i-v 3-prime L-fuzzy ideal* if for all \( a, b \in N \),
\[
C_I(\hat{\alpha}, \hat{\mu}(a)) \geq T_I(\hat{\beta}, C_I(\hat{\alpha}, \inf_{r \in N} \hat{\mu}(arb))) \quad \text{or} \quad C_I(\hat{\alpha}, \hat{\mu}(b)) \geq T_I(\hat{\beta}, C_I(\hat{\alpha}, \inf_{r \in N} \hat{\mu}(arb))).
\]

**Definition 3.2.3.** An i-v L-fuzzy ideal \( \hat{\mu} \) of \( N \) is called an *i-v c-prime L-fuzzy ideal* if for all \( a, b \in N \),
\[
C_I(\hat{\alpha}, \hat{\mu}(a)) \geq T_I(\hat{\beta}, C_I(\hat{\alpha}, \hat{\mu}(ab))) \quad \text{or} \quad C_I(\hat{\alpha}, \hat{\mu}(b)) \geq T_I(\hat{\beta}, C_I(\hat{\alpha}, \hat{\mu}(ab))).
\]

**Lemma 3.2.4.** Let \( \hat{\mu} \) be an i-v equiprime L-fuzzy ideal of \( N \).

(i) For \( x, y, a \in N \) if \( C_I(\hat{\alpha}, \hat{\mu}(ax - ary)) \geq \hat{\beta} \) for all \( r \in N \) then \( C_I(\hat{\alpha}, \hat{\mu}(a)) \geq T_I(\hat{\beta}, \hat{\beta}) \) or \( C_I(\hat{\alpha}, \hat{\mu}(x - y)) \geq T_I(\hat{\beta}, \hat{\beta}) \). Further, if associated i-v t-norm \( T_I \) of \( \hat{\mu} \) is idempotent then \( a \in \hat{\mu}_\beta \) or \( x - y \in \hat{\mu}_\beta \).

(ii) For \( a, b \in N \), \( C_I(\hat{\alpha}, \hat{\mu}(a0)) \geq T_I(\hat{\beta}, \hat{\beta}) \) and \( C_I(\hat{\alpha}, \hat{\mu}(a - b0)) \geq T_I(C_I(\hat{\alpha}, \hat{\mu}(a)), T_I(\hat{\beta}, T_I(\hat{\beta}, \hat{\beta}))) \). Further, if associated i-v t-norm \( T_I \) of \( \hat{\mu} \) is idempotent then \( a0 \in \hat{\mu}_\beta \) and \( C_I(\hat{\alpha}, \hat{\mu}(a - b0)) \geq T_I(\hat{\beta}, C_I(\hat{\alpha}, \hat{\mu}(a))) \).

(iii) If there exist \( a, b \in N \) such that \( aN = b \) then \( C_I(\hat{\alpha}, \hat{\mu}(b)) \geq T_I(\hat{\beta}, \hat{\beta}) \). Further, if associated i-v t-norm \( T_I \) of \( \hat{\mu} \) is idempotent then \( b \in \hat{\mu}_\beta \).

(iv) Let \( \hat{\mu} \) be an i-v equiprime L-fuzzy ideal of \( N \) with thresholds \( \hat{\alpha} = [m, m] \) and \( \hat{\beta} = [M, M] \). For \( x, y, a \in N \), if \( \hat{\mu}(ax - ary) = \hat{\mu}(0) \) for all \( r \in N \) then \( \hat{\mu}(a) \geq \hat{\mu}(0) \) or \( \hat{\mu}(x) = \hat{\mu}(y) \).

*Proof.* To prove (i), let \( x, y, a \in N \) be such that \( C_I(\hat{\alpha}, \inf_{r \in N} \hat{\mu}(ax - ary)) \geq \hat{\beta} \) for all \( r \in N \). Then \( C_I(\hat{\alpha}, \inf_{r \in N} \hat{\mu}(ax - ary)) \geq \hat{\beta} \). As \( \hat{\mu} \) is an i-v equiprime L-fuzzy ideal of \( N \), we get
\[
C_I(\hat{\alpha}, \hat{\mu}(a)) \geq T_I(\hat{\beta}, C_I(\hat{\alpha}, \inf_{r \in N} \hat{\mu}(ax - ary))) \geq T_I(\hat{\beta}, \hat{\beta}) \quad \text{or} \quad C_I(\hat{\alpha}, \hat{\mu}(x - y)) \geq T_I(\hat{\beta}, C_I(\hat{\alpha}, \inf_{r \in N} \hat{\mu}(ax - ary))) \geq T_I(\hat{\beta}, \hat{\beta})
\]
(monotonicity of i-v t-norm).

Hence \( C_I(\hat{\alpha}, \hat{\mu}(a)) \geq T_I(\hat{\beta}, \hat{\beta}) \) or \( C_I(\hat{\alpha}, \hat{\mu}(x - y)) \geq T_I(\hat{\beta}, \hat{\beta}) \).

Now, suppose \( T_I \) is an idempotent i-v t-norm. Then \( C_I(\hat{\alpha}, \hat{\mu}(a)) \geq \hat{\beta} \)
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or \( C_I(\hat{\alpha}, \hat{\mu}(x - y)) \geq \hat{\beta} \). Hence \( a \in \mu_\beta \) or \( x - y \in \mu_\beta \).

To prove (ii) by the property of an i-v equiprime L-fuzzy ideal, for all \( x, y, p \in N \) we get

\[
C_I(\hat{\alpha}, \hat{\mu}(p)) \geq T_I(\hat{\beta}, C_I(\hat{\alpha}, \inf_{r \in N} \hat{\mu}(prx - pry)))
\]  \hspace{1cm} (3.2.1)

\[
C_I(\hat{\alpha}, \hat{\mu}(x - y)) \geq T_I(\hat{\beta}, C_I(\hat{\alpha}, \inf_{r \in N} \hat{\mu}(prx - pry)))
\]  \hspace{1cm} (3.2.2)

By taking \( p \) and \( x \) as \( a0 \) and \( y \) as 0 in Equation 3.2.1 and Equation 3.2.2, we get

\[
C_I(\hat{\alpha}, \hat{\mu}(a0)) \geq T_I(\hat{\beta}, C_I(\hat{\alpha}, \inf_{r \in N} \hat{\mu}(a0ra0 - a0r0))) = T_I(\hat{\beta}, C_I(\hat{\alpha}, \hat{\mu}(0))) \geq T_I(\hat{\beta}, \hat{\beta}).
\]

Therefore \( C_I(\hat{\alpha}, \hat{\mu}(a0)) \geq T_I(\hat{\beta}, \hat{\beta}) \). Suppose \( T_I \) is an idempotent i-v t-norm. Then

\[
C_I(\hat{\alpha}, \hat{\mu}(a0)) \geq \hat{\beta}. \text{ Hence } a0 \in \mu_\beta.
\]

Consider \( C_I(\hat{\alpha}, \hat{\mu}(a - b0)) = C_I(\hat{\alpha}, \hat{\mu}(a + (-b0))) \)

\[
\geq T_I(\hat{\beta}, T_I(C_I(\hat{\alpha}, \hat{\mu}(a)), C_I(\hat{\alpha}, \hat{\mu}(-b0)))) \geq T_I(\hat{\beta}, T_I(C_I(\hat{\alpha}, \hat{\mu}(a)), T_I(\hat{\beta}, C_I(\hat{\alpha}, \hat{\mu}(b0))))).
\]

Suppose \( T_I \) is an idempotent i-v t-norm. Then \( C_I(\hat{\alpha}, \hat{\mu}(a - b0)) \)

\[
\geq T_I(C_I(\hat{\alpha}, \hat{\mu}(a)), T_I(\hat{\beta}, T_I(\hat{\beta}, \hat{\beta})))
\]

\[
= T_I(C_I(\hat{\alpha}, \hat{\mu}(a)), T_I(\hat{\beta}, C_I(\hat{\alpha}, \hat{\mu}(a)))) = T_I(\hat{\beta}, C_I(\hat{\alpha}, \hat{\mu}(a))).
\]

To prove (iii), suppose there exists \( a, b \in N \) such that \( aN = b \). Then \( a0 = b \).

By (ii), we get \( C_I(\hat{\alpha}, \hat{\mu}(b)) \geq T_I(\hat{\beta}, \hat{\beta}) \). Suppose \( T_I \) is an idempotent i-v t-norm. Then

\[
C_I(\hat{\alpha}, \hat{\mu}(b)) \geq \hat{\beta}. \text{ Hence } b \in \mu_\beta.
\]

To prove (iv), let \( a, x, y \in N, \hat{\alpha} = [m, m] \) and \( \hat{\beta} = [M, M] \). Then

\[
\hat{\mu}(a), C_I(\hat{\alpha}, \hat{\mu}(x - y)) = \hat{\mu}(x - y) \text{ and } T_I(\hat{\beta}, C_I(\hat{\alpha}, \inf_{r \in N} \hat{\mu}(arx - ary)))
\]

\[
= \inf_{r \in N} \hat{\mu}(arx - ary). \text{ Let } \hat{\mu}(arx - ary) = \hat{\mu}(0) \text{ for all } r \in N \Rightarrow \inf_{r \in N} \hat{\mu}(arx - ary) = \hat{\mu}(0).
\]

As \( \hat{\mu} \) is an i-v equiprime L-fuzzy ideal of \( N \), we get

\[
\hat{\mu}(a) \geq \hat{\mu}(0)
\]  \hspace{1cm} (3.2.3)

or

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\[ \hat{\mu}(x - y) \geq \hat{\mu}(0). \]  \hspace{1cm} (3.2.4)

Now, suppose \( \hat{\mu}(x - y) \geq \hat{\mu}(0) \). Then \( \hat{\mu}(x) = \hat{\mu}(x - y + y) \geq T(\hat{\mu}(x - y), \hat{\mu}(y)) \geq T(\hat{\mu}(0), \hat{\mu}(y)) \) (property of i-v L-fuzzy ideal and monotonicity of i-v t-norm). Therefore

\[ \hat{\mu}(x) \geq T(\hat{\mu}(0), \hat{\mu}(y)) \] \hspace{1cm} (3.2.5)

We have \( C(\hat{\alpha}, \hat{\mu}(0)) \geq \hat{\beta} \). As \( \hat{\beta} = [M, M] \), we get \( \hat{\mu}(0) \geq [M, M] \Rightarrow \hat{\mu}(0) = [M, M] \).

From Equation (3.2.5), we get \( \hat{\mu}(x) \geq T([M, M], \hat{\mu}(y)) = \hat{\mu}(y) \). Hence \( \hat{\mu}(x) \geq \hat{\mu}(y) \).

Similarly we can prove \( \hat{\mu}(y) \geq \hat{\mu}(x) \). Therefore \( \hat{\mu}(x) = \hat{\mu}(y) \). \hfill \Box

**Proposition 3.2.5.** Let \( \hat{\mu} \) be an i-v 3-prime L-fuzzy ideal of \( N \). For \( a, b \in N \), if

- \( C_I(\hat{\alpha}, \hat{\mu}(anb)) \geq \hat{\beta} \) for all \( n \in N \) then \( C_I(\hat{\alpha}, \hat{\mu}(a)) \geq T_I(\hat{\beta}, \hat{\beta}) \) or
- \( C_I(\hat{\alpha}, \hat{\mu}(b)) \geq T_I(\hat{\beta}, \hat{\beta}) \). Further if associated i-v t-norm \( T_I \) of \( \hat{\mu} \) is idempotent then \( a \in \hat{\mu}_\beta \) or \( b \in \hat{\mu}_\beta \).

**Proof.** Let \( a, b \in N \) such that \( C_I(\hat{\alpha}, \hat{\mu}(anb)) \geq \hat{\beta} \) for all \( n \in N \).

Then \( C_I(\hat{\alpha}, \inf_{n \in N} \hat{\mu}(anb)) \geq \hat{\beta} \). As \( \hat{\mu} \) is an i-v 3-prime L-fuzzy ideal of \( N \), we get \( C_I(\hat{\alpha}, \hat{\mu}(a)) \geq T_I(\hat{\beta}, C_I(\hat{\alpha}, \inf_{n \in N} \hat{\mu}(anb))) \geq T_I(\hat{\beta}, \hat{\beta}) \) (by monotonicity of i-v t-norm) or \( C_I(\hat{\alpha}, \hat{\mu}(b)) \geq T_I(\hat{\beta}, C_I(\hat{\alpha}, \inf_{n \in N} \hat{\mu}(anb))) \geq T_I(\hat{\beta}, \hat{\beta}) \) (by monotonicity of i-v t-norm). Suppose \( T_I \) is an idempotent i-v t-norm. Then \( C_I(\hat{\alpha}, \hat{\mu}(a)) \geq \hat{\beta} \) or \( C_I(\hat{\alpha}, \hat{\mu}(b)) \geq \hat{\beta} \). Hence \( a \in \hat{\mu}_\beta \) or \( b \in \hat{\mu}_\beta \). \hfill \Box

**Proposition 3.2.6.** Let \( \hat{\mu} \) be an i-v c-prime L-fuzzy ideal of \( N \). For \( a, b \in N \) if

- \( C_I(\hat{\alpha}, \hat{\mu}(ab)) \geq \hat{\beta} \) then \( C_I(\hat{\alpha}, \hat{\mu}(a)) \geq T_I(\hat{\beta}, \hat{\beta}) \) or \( C_I(\hat{\alpha}, \hat{\mu}(b)) \geq T_I(\hat{\beta}, \hat{\beta}) \). Further if associated i-v t-norm \( T_I \) of \( \hat{\mu} \) is idempotent then \( a \in \hat{\mu}_\beta \) or \( b \in \hat{\mu}_\beta \).

**Proof.** Let \( a, b \in N \) such that \( C_I(\hat{\alpha}, \hat{\mu}(ab)) \geq \hat{\beta} \). As \( \hat{\mu} \) is an i-v c-prime L-fuzzy ideal of \( N \), we get \( C_I(\hat{\alpha}, \hat{\mu}(a)) \geq T_I(\hat{\beta}, C_I(\hat{\alpha}, \hat{\mu}(ab))) \geq T_I(\hat{\beta}, \hat{\beta}) \) or \( C_I(\hat{\alpha}, \hat{\mu}(b)) \geq T_I(\hat{\beta}, C_I(\hat{\alpha}, \hat{\mu}(ab))) \geq T_I(\hat{\beta}, \hat{\beta}) \) (by monotonicity of i-v t-norm). Suppose \( T_I \) is an idempotent i-v t-norm. Then \( C_I(\hat{\alpha}, \hat{\mu}(a)) \geq \hat{\beta} \) or \( C_I(\hat{\alpha}, \hat{\mu}(b)) \geq \hat{\beta} \). Hence \( a \in \hat{\mu}_\beta \) or \( b \in \hat{\mu}_\beta \). \hfill \Box
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Theorem 3.2.7. Let \( \hat{\mu} \) be an i-v L-fuzzy subset of \( N \). If for all \( \hat{k} \in (\hat{\alpha}, \hat{\beta}] \) the level set \( \hat{\mu}_{\hat{k}} \) is an equiprime ideal of \( N \) then \( \hat{\mu} \) is an i-v equiprime L-fuzzy ideal of \( N \). Conversely, if \( \hat{\mu} \) is an i-v equiprime L-fuzzy ideal of \( N \) with associated i-v t-norm \( T_I \) is idempotent then \( \hat{\mu}_{\hat{k}} \) is an equiprime ideal of \( N \) for all \( \hat{k} \in (\hat{\alpha}, \hat{\beta}] \).

Proof. \((\Rightarrow)\) As \( \hat{\mu}_{\hat{k}} \) is an ideal of \( N \) for all \( \hat{k} \in (\hat{\alpha}, \hat{\beta}] \) by Theorem 2.2.8, we get \( \hat{\mu} \) is an i-v L-fuzzy ideal of \( N \). We will prove \( \hat{\mu} \) is an i-v equiprime L-fuzzy ideal of \( N \). In a contrary way suppose there exist \( x, y, a \in N \) such that

\[
C_I(\hat{\alpha}, \hat{\mu}(a)) < T_I(\hat{\beta}, C_I(\hat{\alpha}, \inf_{r \in N} \hat{\mu}(arx - ary))) \quad \text{and} \quad C_I(\hat{\alpha}, \hat{\mu}(x - y)) < T_I(\hat{\beta}, C_I(\hat{\alpha}, \inf_{r \in N} \hat{\mu}(arx - ary))).
\]

Choose \( \hat{k} = T_I(\hat{\beta}, C_I(\hat{\alpha}, \inf_{r \in N} \hat{\mu}(arx - ary))). \) Then

\[
\hat{k} \leq \hat{\beta} \land C_I(\hat{\alpha}, \inf_{r \in N} \hat{\mu}(arx - ary)) \leq \hat{\beta} \land C_I(\hat{\alpha}, \hat{\mu}(arx - ary))) \quad \text{for all} \ r \in N
\]

(by monotonicity of i-v t-norm)

\[
\Rightarrow \hat{k} \leq \hat{\beta} \quad \text{and} \quad \hat{k} \leq C_I(\hat{\alpha}, \hat{\mu}(arx - ary))) \quad \text{for all} \ r \in N \quad \text{(by property of lattice)}
\]

\[
\Rightarrow \hat{k} \leq \hat{\beta} \quad \text{and} \ arx - ary \in \hat{\mu}_{\hat{k}} \quad \text{for all} \ r \in N. \quad \text{Also} \ C_I(\hat{\alpha}, \hat{\mu}(a)) < \hat{k} \quad \text{and} \ C_I(\hat{\alpha}, \hat{\mu}(x - y)) < \hat{k} \Rightarrow a \notin \hat{\mu}_{\hat{k}} \quad \text{and} \ \ x - y \notin \hat{\mu}_{\hat{k}} \quad \text{and} \ \hat{\alpha} < \hat{k}. \quad \text{Hence for} \ \hat{k} \in (\hat{\alpha}, \hat{\beta}], \ arx - ary \in \hat{\mu}_{\hat{k}} \quad \text{for all} \ r \in N \quad \text{however} \ a \notin \hat{\mu}_{\hat{k}} \quad \text{and} \ x - y \notin \hat{\mu}_{\hat{k}}. \quad \text{A contradiction to the fact that} \ \hat{\mu}_{\hat{k}} \quad \text{is an equiprime ideal of} \ N \quad \text{for all} \ \hat{k} \in (\hat{\alpha}, \hat{\beta}]\).

To prove the converse, suppose the associated i-v t-norm \( T_I \) of \( \hat{\mu} \) is idempotent and \( a, x, y \in N \) such that \( arx - ary \in \hat{\mu}_{\hat{k}} \) for all \( r \in N \). Then \( C_I(\hat{\alpha}, \hat{\mu}(arx - ary))) \geq \hat{k} \) for all \( r \in N \Rightarrow C_I(\hat{\alpha}, \inf_{r \in N} \hat{\mu}(arx - ary))) \geq \hat{k}. \) As \( \hat{\mu} \) is an i-v equiprime L-fuzzy ideal of \( N \), we get \( C_I(\hat{\alpha}, \hat{\mu}(a)) \geq T_I(\hat{\beta}, C_I(\hat{\alpha}, \inf_{r \in N} \hat{\mu}(arx - ary))) \geq T_I(\hat{\beta}, \hat{k}) \geq T_I(\hat{k}, \hat{k}) = \hat{k} \)

(by monotonicity and idempotent property of i-v t-norm)

or

\[
C_I(\hat{\alpha}, \hat{\mu}(x - y)) \geq T_I(\hat{\beta}, C_I(\hat{\alpha}, \inf_{r \in N} \hat{\mu}(arx - ary)))
\]

\[
\geq T_I(\hat{\beta}, \hat{k}) \geq T_I(\hat{k}, \hat{k}) = \hat{k} \quad \text{(by monotonicity and idempotent property of i-v t-norm)}
\]

Hence \( a \in \hat{\mu}_{\hat{k}} \) or \( x - y \in \hat{\mu}_{\hat{k}} \). Therefore \( \hat{\mu}_{\hat{k}} \) is an equiprime ideal of \( N \).

Theorem 3.2.8. Let \( \hat{\mu} \) be an i-v L-fuzzy subset of \( N \). If for all \( \hat{k} \in (\hat{\alpha}, \hat{\beta}] \) the level set
\( \hat{\mu}_k \) is a 3-prime ideal of \( N \) then \( \hat{\mu} \) is an i-v 3-prime L-fuzzy ideal of \( N \). Conversely, if \( \hat{\mu} \) is an i-v 3-prime L-fuzzy ideal of \( N \) with associated i-v t-norm \( T_I \) is idempotent then \( \hat{\mu}_k \) is a 3-prime ideal of \( N \) for all \( \hat{k} \in (\hat{\alpha}, \hat{\beta}] \).

**Proof.** (\( \Rightarrow \)) As \( \hat{\mu}_k \) is an ideal of \( N \) for all \( \hat{k} \in (\hat{\alpha}, \hat{\beta}] \) by Theorem 2.2.8, we get \( \hat{\mu} \) is an i-v L-fuzzy ideal of \( N \). We will prove \( \hat{\mu} \) is an i-v 3-prime L-fuzzy ideal of \( N \). In a contrary way suppose there exist \( a, b \in N \) such that 

\[
C_I(\hat{\alpha}, \hat{\mu}(a)) < T_I(\hat{\beta}, C_I(\hat{\alpha}, \inf_{r \in N} \hat{\mu}(arb))) \quad \text{and} \quad C_I(\hat{\alpha}, \hat{\mu}(b)) < T_I(\hat{\beta}, C_I(\hat{\alpha}, \inf_{r \in N} \hat{\mu}(arb))).
\]

Choose \( \hat{k} = T_I(\hat{\beta}, C_I(\hat{\alpha}, \inf_{r \in N} \hat{\mu}(arb))) \) then \( \hat{k} \leq \hat{\beta} \land C_I(\hat{\alpha}, \hat{\mu}(arb)) \leq \hat{\beta} \land C_I(\hat{\alpha}, \hat{\mu}(arb)) \) for all \( r \in N \Rightarrow \hat{k} \leq \hat{\beta} \) and \( \hat{k} \leq C_I(\hat{\alpha}, \hat{\mu}(arb))) \) for all \( r \in N \Rightarrow \hat{k} \leq \hat{\beta} \) and \( arb \in \hat{\mu}_k \) for all \( r \in N \). Also \( C_I(\hat{\alpha}, \hat{\mu}(a)) < \hat{k} \) and \( C_I(\hat{\alpha}, \hat{\mu}(b)) < \hat{k} \Rightarrow a \notin \hat{\mu}_k \) and \( b \notin \hat{\mu}_k \) and \( \hat{\alpha} < \hat{k} \). Hence for \( \hat{k} \in (\hat{\alpha}, \hat{\beta}], arb \in \hat{\mu}_k \) for all \( r \in N \) however \( a \notin \hat{\mu}_k \) and \( b \notin \hat{\mu}_k \).

A contradiction to the fact the \( \hat{\mu}_k \) is a 3-prime ideal of \( N \) for all \( \hat{k} \in (\hat{\alpha}, \hat{\beta}] \).

To prove the converse, suppose the associated i-v t-norm \( T_I \) of \( \hat{\mu} \) is idempotent and \( a, b \in N \) such that \( arb \in \hat{\mu}_k \) for all \( r \in N \). Then \( C_I(\hat{\alpha}, \hat{\mu}(arb))) \geq \hat{k} \) for all \( r \in N \)

\[
\Rightarrow C_I(\hat{\alpha}, \inf_{r \in N} \hat{\mu}(arb))) \geq \hat{k}.
\]

As \( \hat{\mu} \) is an i-v 3-prime L-fuzzy ideal of \( N \), we get 

\[
C_I(\hat{\alpha}, \hat{\mu}(a)) \geq T_I(\hat{\beta}, C_I(\hat{\alpha}, \inf_{r \in N} \hat{\mu}(arb))) \quad \text{or} \quad C_I(\hat{\alpha}, \hat{\mu}(b)) \geq T_I(\hat{\beta}, C_I(\hat{\alpha}, \inf_{r \in N} \hat{\mu}(arb)))
\]

\[
\Rightarrow C_I(\hat{\alpha}, \hat{\mu}(a)) \geq T_I(\hat{\beta}, \hat{k}) \geq T_I(\hat{\beta}, \hat{k}) = \hat{k} \quad \text{or} \quad C_I(\hat{\alpha}, \hat{\mu}(b)) \geq T_I(\hat{\beta}, \hat{k}) \geq T_I(\hat{\beta}, \hat{k}) = \hat{k}.
\]

Hence \( a \in \hat{\mu}_k \) or \( b \in \hat{\mu}_k \). Therefore \( \hat{\mu}_k \) is a 3-prime ideal of \( N \).

**Theorem 3.2.9.** Let \( \hat{\mu} \) be an i-v L-fuzzy subset of \( N \). If for all \( \hat{k} \in (\hat{\alpha}, \hat{\beta}] \) the level set \( \hat{\mu}_k \) is a c-prime ideal of \( N \) then \( \hat{\mu} \) is an i-v c-prime L-fuzzy ideal of \( N \). Conversely, if \( \hat{\mu} \) is an i-v c-prime L-fuzzy ideal of \( N \) with associated i-v t-norm \( T_I \) is idempotent then \( \hat{\mu}_k \) is a c-prime ideal of \( N \) for all \( \hat{k} \in (\hat{\alpha}, \hat{\beta}] \).

**Proof.** (\( \Rightarrow \)) As \( \hat{\mu}_k \) is an ideal of \( N \) for all \( \hat{k} \in (\hat{\alpha}, \hat{\beta}] \) by Theorem 2.2.8, we get \( \hat{\mu} \) is an i-v L-fuzzy ideal of \( N \). We will prove \( \hat{\mu} \) is an i-v c-prime L-fuzzy ideal of \( N \). In a contrary way suppose there exist \( a, b \in N \) such that 

\[
C_I(\hat{\alpha}, \hat{\mu}(a)) < T_I(\hat{\beta}, C_I(\hat{\alpha}, \hat{\mu}(arb))) \quad \text{and} \quad C_I(\hat{\alpha}, \hat{\mu}(b)) < T_I(\hat{\beta}, C_I(\hat{\alpha}, \hat{\mu}(arb))).
\]

Choose \( \hat{k} = T_I(\hat{\beta}, C_I(\hat{\alpha}, \hat{\mu}(arb))) \) then
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\[ \hat{k} \leq \hat{\beta} \land C_I(\hat{\alpha}, \hat{\mu}(ab)) \Rightarrow \hat{k} \leq \hat{\beta} \text{ and } \hat{k} \leq C_I(\hat{\alpha}, \hat{\mu}(ab)) \Rightarrow \hat{k} \leq \hat{\beta} \text{ and } ab \in \hat{\mu}_k. \]

Also \( C_I(\hat{\alpha}, \hat{\mu}(a)) < \hat{k} \) and \( C_I(\hat{\alpha}, \hat{\mu}(b)) < \hat{k} \Rightarrow a \notin \hat{\mu}_k \text{ and } b \notin \hat{\mu}_k \text{ and } \hat{\alpha} < \hat{k}. \)

Hence for \( \hat{k} \in (\hat{\alpha}, \hat{\beta}], ab \in \hat{\mu}_k \) however \( a \notin \hat{\mu}_k \text{ and } b \notin \hat{\mu}_k \).

A contradiction to the fact that \( \hat{\mu}_k \) is a c-prime ideal of \( N \) for all \( \hat{k} \in (\hat{\alpha}, \hat{\beta}] \).

To prove the converse, suppose the associated i-v t-norm \( T_I \) of \( \hat{\mu} \) is idempotent and \( a, b \in N \) such that \( ab \in \hat{\mu}_k \). Then \( C_I(\hat{\alpha}, \hat{\mu}(ab)) \geq \hat{k} \).

As \( \hat{\mu} \) is an i-v c-prime L-fuzzy ideal of \( N \), we get \( C_I(\hat{\alpha}, \hat{\mu}(a)) \geq T_I(\hat{\beta}, C_I(\hat{\alpha}, \hat{\mu}(ab))) \) or

\[ C_I(\hat{\alpha}, \hat{\mu}(b)) \geq T_I(\hat{\beta}, C_I(\hat{\alpha}, \hat{\mu}(ab))) \Rightarrow C_I(\hat{\alpha}, \hat{\mu}(a)) \geq T_I(\hat{\beta}, \hat{k}) \geq T_I(\hat{k}, \hat{k}) = \hat{k} \text{ or } \]

\[ C_I(\hat{\alpha}, \hat{\mu}(b)) \geq T_I(\hat{\beta}, \hat{k}) \geq T_I(\hat{k}, \hat{k}) = \hat{k}. \]

Hence \( a \in \hat{\mu}_k \text{ or } b \in \hat{\mu}_k \).

Therefore \( \hat{\mu}_k \) is a c-prime ideal of \( N \).

Now we provide an example to study properties of i-v L-fuzzy ideal by changing t-norms and t-conorms.

**Example 3.2.10.** Let \( N = \{0, a, b, c\} \) be a set with binary operations \(+\) and \(\cdot\) defined as in Table 3.1.

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<tr>
<th>+</th>
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Table 3.1: Nearring for Example 3.2.10

Then \((N, +, \cdot)\) is a nearring.

Let \( L = [0, 1] \). We define \( \hat{\mu} : N \to D(L) \) by \( \hat{\mu}(x) = \begin{cases} [0.9, 1] & \text{if } x = 0 \\ [0.5, 0.6] & \text{if } x = a \\ [0.2, 0.3] & \text{if } x \in \{b, c\}. \end{cases} \)

Then property of the i-v L-fuzzy ideal \( \hat{\mu} \) is given in Table 3.2 for different choice of t-norms, t-conorms and thresholds \( \hat{\alpha}, \hat{\beta} \).
3.2. Interval Valued Equiprime, 3-Prime and C-Prime L-Fuzzy Ideals

<table>
<thead>
<tr>
<th>Thresholds, t-norm and t-conorm</th>
<th>Property of i-v L-fuzzy ideal</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hat{\alpha} = [0.2, 0.3], \hat{\beta} = [0.5, 0.6]$, $T_f(g,h) = \begin{cases} g &amp; \text{if} \ g = 1 \ h &amp; \text{if} \ h = 1 \ 0 &amp; \text{otherwise}, \end{cases}$ $T_s(g,h) = \min(g,h)$, $C_f(g,h) = \max{g,h}$, $C_s(g,h) = \begin{cases} g &amp; \text{if} \ h = 0 \ h &amp; \text{if} \ g = 0 \ 1 &amp; \text{otherwise}. \end{cases}$</td>
<td>$\hat{\mu}$ is equiprime, 3-prime and c-prime.</td>
</tr>
</tbody>
</table>

| $\hat{\alpha} = [0.2, 0.3], \hat{\beta} = [0.5, 0.6]$. Let $P = [0, 0.6], F = [0.6, 1]$. $T_f(g,h) = T_s(g,h)$ $= \begin{cases} g \land_L h & \text{if} \ g \in F \text{ or } h \in F \\ m & \text{if} \ g \in P \text{ and } h \in P, \end{cases}$ $C_f(g,h) = \min\{g + h, 1\}$, $C_s(g,h) = \begin{cases} g & \text{if} \ h = 0 \\ h & \text{if} \ g = 0 \\ 1 & \text{otherwise} \end{cases}$ | $\hat{\mu}$ is not equiprime, however $\hat{\mu}$ is 3-prime and c-prime. |

| $\hat{\alpha} = [0.5, 0.6], \hat{\beta} = [0.9, 0.1]$. Let $P = [0, 0.5], F = [0.5, 1]$. $T_f(g,h) = T_s(g,h)$ $= \begin{cases} g \land_L h & \text{if} \ g \in F \text{ or } h \in F \\ m & \text{if} \ g \in P \text{ and } h \in P, \end{cases}$ $C_f(g,h) = C_s(g,h)$ $= \begin{cases} M & \text{if} \ g \in F \text{ and } h \in F. \end{cases}$ | $\hat{\mu}$ is not equiprime, not 3-prime and not c-prime. |

Table 3.2: Table for Example 3.2.10

Now we provide an example for an i-v 3-prime L-fuzzy ideal.

**Example 3.2.11.** Let $N = \{0, a, b, c\}$ be a set with binary operations $+$ and $\cdot$ defined as in Table 3.3.
3.2. Interval Valued Equiprime, 3-Prime and C-Prime L-Fuzzy Ideals

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Table 3.3: Nearring for Example 3.2.11

Then \((N, +, \cdot)\) is a nearring. Note that \((N, +)\) is Klein’s four group \(V_4\). The lattice \(L(V_4) = L(\text{Suzuki}[106])\) is shown in Figure 3.1, where subgroups of \(V_4\) are denoted by \(m = \{0\}, p = \{0, a\}, q = \{0, b\}, r = \{0, c\}, M = V_4\).

![Figure 3.1: Lattice \(L = \{m, p, q, r, M\}\) for Example 3.2.11](image)

Define \(\hat{\mu} : N \to D(L)\) by

\[
\hat{\mu}(x) = \begin{cases} 
[p, M] & \text{if } x = 0 \\
[m, r] & \text{if } x = a \\
[m, m] & \text{if } x \in \{b, c\}.
\end{cases}
\]

Consider the t-norm, t-conorm and thresholds given in (i) of Table 3.4. It can be verified that \(\hat{\mu}\) is an i-v 3-prime L-fuzzy ideal of \(N\). Note that \(\hat{\mu}\) is not an i-v equiprime L-fuzzy ideal of \(N\), because

\[
C_I(\hat{\alpha}, \hat{\mu}(a)) = C_I([m, r], [m, r]) = [m, M] \not\prec [p, M] = T_I([p, M], C_I([m, r], [p, M]))
\]

\[
= T_I(\hat{\beta}, C_I(\hat{\alpha}, \inf_{r \in N} \hat{\mu}(ara - arc))) \quad \text{and} \quad C_I(\hat{\alpha}, \hat{\mu}(a - c)) = C_I([m, r], [m, m]) = [m, r] \not\prec [p, M] = T_I([p, M], C_I([m, r], [p, M])) = T_I(\hat{\beta}, C_I(\hat{\alpha}, \inf_{r \in N} \hat{\mu}(ara - arc))).
\]
3.2. Interval Valued Equiprime, 3-Prime and C-Prime L-Fuzzy Ideals

Note that $\hat{\mu}$ is not an i-v c-prime L-fuzzy ideal of $N$, because $C_I(\hat{\alpha}, \hat{\mu}(c)) = C_I([m, r], [m, m]) = [m, r] \not\supset [p, M] = T_I([p, M], C_I([m, r], [p, M]))$

$= T_I(\hat{\beta}, C_I(\hat{\alpha}, \hat{\mu}(ca)))$ and $C_I(\hat{\alpha}, \hat{\mu}(a)) = C_I([m, r], [m, r]) = [m, M]$

$\not\supset [p, M] = T_I([p, M], C_I([m, r], [p, M])) = T_I(\hat{\beta}, C_I(\hat{\alpha}, \hat{\mu}(ca)))$.

Table 3.4 shows change in t-norm, t-conorm, thresholds and corresponding change in the property of the i-v L-fuzzy ideal $\hat{\mu}$.

<table>
<thead>
<tr>
<th>Thresholds, t-norm and t-conorm</th>
<th>Property of i-v L-fuzzy ideal</th>
</tr>
</thead>
<tbody>
<tr>
<td>(i) $\hat{\alpha} = [m, r]$, $\hat{\beta} = [p, M]$, $T_f(g, h) = g \land_L h$, $C_f(g, h) = g \lor_L h$, $C_s(g, h) = \begin{cases} g &amp; \text{if } h = m \ h &amp; \text{if } g = m \ M &amp; \text{otherwise.} \end{cases}$</td>
<td>$\hat{\mu}$ is 3-prime, however $\hat{\mu}$ is not c-prime and not equiprime.</td>
</tr>
<tr>
<td>(ii) $\hat{\alpha} = [m, m]$, $\hat{\beta} = [m, r]$, $T_f(g, h) = g \land_L h$, $C_f(g, h) = g \lor_L h$, $C_s(g, h) = C_s(g, h)$.</td>
<td>$\hat{\mu}$ is 3-prime and c-prime, however $\hat{\mu}$ is not equiprime.</td>
</tr>
</tbody>
</table>

Table 3.4: Table for Example 3.2.11

Now we provide an example to study properties of i-v L-fuzzy ideal by changing thresholds.
Example 3.2.12. Consider the nearring $N$ in Example 3.2.10. Let $L$ be the lattice shown in Figure 3.2.

Let $P = \{m, n, r, s\}, F = \{p, q, M\}$ or $P = \{m, n, p, q\}, F = \{r, s, M\}$.

We define

$$T_f(g,h) = T_s(g,h) = \begin{cases} g \land_L h & \text{if } g \in F \text{ or } h \in F \\ m & \text{if } g \in P \text{ and } h \in P, \end{cases}$$

$$C_f(g,h) = C_s(g,h) = \begin{cases} g \lor_L h & \text{if } g \in P \text{ or } h \in P \\ M & \text{if } g \in F \text{ and } h \in F. \end{cases}$$

Define $\hat{\mu} : N \to D(L)$ by

$$\hat{\mu}(x) = \begin{cases} [q, M] & \text{if } x = 0 \\ [n, s] & \text{if } x = a \\ [m, r] & \text{if } x \in \{b, c\}. \end{cases}$$

Then $\hat{\mu}$ is an i-v c-prime and 3-prime L-fuzzy ideal with thresholds $\hat{\alpha} = [m, r], \hat{\beta} = [n, s]$, however $\hat{\mu}$ is not an i-v equiprime L-fuzzy ideal.

Also $\hat{\mu}$ is an i-v L-fuzzy ideal with thresholds $\hat{\alpha} = [n, s], \hat{\beta} = [q, M]$, however $\hat{\mu}$ not i-v equiprime, 3-prime and c-prime L-fuzzy ideal.

Now we provide an example to show that level set of an i-v equiprime, 3-prime and c-prime L-fuzzy ideal need not be an ideal.

Example 3.2.13. Consider $R = \mathbb{Z}_6 = \{0, 1, 2, 3, 4, 5\}$ be the ring of integers modulo 6. Let $L$ be the lattice shown in the following Figure 3.3.
3.2. Interval Valued Equiprime, 3-Prime and C-Prime L-Fuzzy Ideals

For \( g, h \in L \) we define,

\[
T_f(g, h) = \begin{cases} 
  g & \text{if } h = M \\
  h & \text{if } g = M \\
  m & \text{otherwise},
\end{cases}
\]

\[
T_s(g, h) = g \wedge_L h,
\]

\[
C_f(g, h) = g \vee_L h \text{ and } C_s(g, h) = \begin{cases} 
  g & \text{if } h = m \\
  h & \text{if } g = m \\
  M & \text{otherwise.}
\end{cases}
\]

We define \( \hat{\mu} : R \to D(L) \) by

\[
\hat{\mu}(x) = \begin{cases} 
  [h, M] & \text{if } x = \emptyset \\
  [q, g] & \text{if } x \in \{2, 3, 4\} \\
  [m, c] & \text{if } x \in \{1, 5\}.
\end{cases}
\]

Take \( \hat{\alpha} = [m, c] \) and \( \hat{\beta} = [q, g] \). Then \( \hat{\mu} \) is an i-v equiprime, 3-prime and c-prime L-fuzzy ideal of \( N \). Note that \( [q, g] \in (\hat{\alpha}, \hat{\beta}) \) however \( \hat{\mu}_{[q, g]} = \{\emptyset, 2, 3, 4\} \) is not an ideal of \( N \).
Remark 3.2.14. By Theorem 1.2.7, there are 1440 i-v L-fuzzy ideals similar to i-v L-fuzzy ideal in Example 3.2.13, there are 6 i-v L-fuzzy ideals similar to i-v L-fuzzy ideal in Example 3.2.11, and 2 i-v L-fuzzy ideals similar to i-v L-fuzzy ideal in Example 3.2.12.

3.3 Interrelations between Interval Valued Prime L-fuzzy Ideals

Proposition 3.3.1. Let $N$ be a distributive nearring. Then $\hat{\mu}$ is an i-v equiprime L-fuzzy ideal of $N$ if and only if $\hat{\mu}$ is an i-v 3-prime L-fuzzy ideal of $N$.

Proof. ($\Rightarrow$) Let $a, b \in N$ and $\hat{\mu}$ be an i-v equiprime L-fuzzy ideal of $N$. Then

$C_I(\hat{\alpha}, \hat{\mu}(a)) \geq T_I(\hat{\beta}, C_I(\hat{\alpha}, \inf_{r \in N} \hat{\mu}(ar(b - 0))))$ or

$C_I(\hat{\alpha}, \hat{\mu}(b - 0)) \geq T_I(\hat{\beta}, C_I(\hat{\alpha}, \inf_{r \in N} \hat{\mu}(ar(b - 0))))$.

Hence $C_I(\hat{\alpha}, \hat{\mu}(ar(b))) \geq T_I(\hat{\beta}, C_I(\hat{\alpha}, \inf_{r \in N} \hat{\mu}(arb)))$ or $C_I(\hat{\alpha}, \hat{\mu}(arb)) \geq T_I(\hat{\beta}, C_I(\hat{\alpha}, \inf_{r \in N} \hat{\mu}(arb)))$.

Therefore $\hat{\mu}$ is an i-v 3-prime L-fuzzy ideal of $N$.

To prove the converse, let $a, x, y \in N$ and $\hat{\mu}$ be an i-v 3-prime L-fuzzy ideal of $N$. Then

$C_I(\hat{\alpha}, \hat{\mu}(a)) \geq T_I(\hat{\beta}, C_I(\hat{\alpha}, \inf_{r \in N} \hat{\mu}(ar(x - y))))$ or

$C_I(\hat{\alpha}, \hat{\mu}(x - y)) \geq T_I(\hat{\beta}, C_I(\hat{\alpha}, \inf_{r \in N} \hat{\mu}(ar(x - y)))) = T_I(\hat{\beta}, C_I(\hat{\alpha}, \inf_{r \in N} \hat{\mu}(arx - ary)))$.

Hence $C_I(\hat{\alpha}, \hat{\mu}(a)) \geq T_I(\hat{\beta}, C_I(\hat{\alpha}, \inf_{r \in N} \hat{\mu}(arx - ary)))$ or

$C_I(\hat{\alpha}, \hat{\mu}(x - y)) \geq T_I(\hat{\beta}, C_I(\hat{\alpha}, \inf_{r \in N} \hat{\mu}(arx - ary)))$.

Therefore $\hat{\mu}$ is an i-v equiprime L-fuzzy ideal of $N$. \qed

Proposition 3.3.2. If $\hat{\mu}$ is an i-v equiprime L-fuzzy ideal of $N$ with associated i-v t-norm idempotent then $\hat{\mu}$ is an i-v 3-prime L-fuzzy ideal of $N$. Converse holds if $N$ is a commutative ring.
3.3. Interrelations between Interval Valued Prime L-fuzzy Ideals

Proof. Let \( \hat{\mu} \) be an i-v equiprime L-fuzzy ideal of \( N \) with associated i-v t-norm idempotent. Then by Theorem 3.2.7, we get \( \hat{\mu}_k \) is an equiprime ideal of \( N \) for all \( \hat{k} \in (\hat{\alpha}, \hat{\beta}] \). Hence \( \hat{\mu}_k \) is a 3-prime ideal of \( N \) for all \( \hat{k} \in (\hat{\alpha}, \hat{\beta}] \).
By Theorem 3.2.8, \( \hat{\mu} \) is an i-v 3-prime ideal of \( N \).

To prove converse, let \( N \) be a commutative ring. Let \( \hat{\mu} \) be an i-v 3-prime L-fuzzy ideal of \( N \) with associated i-v t-norm idempotent. Then \( \hat{\mu}_k \) is a 3-prime ideal of \( N \) for all \( \hat{k} \in (\hat{\alpha}, \hat{\beta}] \). Then \( \hat{\mu}_k \) is an equiprime ideal of \( N \) for all \( \hat{k} \in (\hat{\alpha}, \hat{\beta}] \).
Therefore \( \hat{\mu} \) is an i-v equiprime L-fuzzy ideal of \( N \) (by Theorem 3.2.9). \( \square \)

**Proposition 3.3.3.** If \( \hat{\mu} \) is an i-v c-prime L-fuzzy ideal of \( N \) with associated i-v t-norm idempotent then \( \hat{\mu} \) is an i-v 3-prime L-fuzzy ideal of \( N \). Converse holds if \( N \) is a commutative ring.

Proof. Let \( \hat{\mu} \) be an i-v c-prime L-fuzzy ideal of \( N \) with associated i-v t-norm idempotent. Then \( \hat{\mu}_k \) is a c-prime ideal of \( N \) for all \( k \in (\hat{\alpha}, \hat{\beta}] \) \( \Rightarrow \) \( \hat{\mu}_k \) is a 3-prime ideal of \( N \) for all \( k \in (\hat{\alpha}, \hat{\beta}] \). Then \( \hat{\mu} \) is an i-v 3-prime L-fuzzy ideal of \( N \) (by Theorem 3.2.8).
To prove converse, let \( N \) be a commutative ring. Let \( \hat{\mu} \) be an i-v 3-prime L-fuzzy ideal of \( N \) with associated i-v t-norm idempotent. Then \( \hat{\mu}_k \) is a 3-prime ideal of \( N \) for all \( k \in (\hat{\alpha}, \hat{\beta}] \). Hence \( \hat{\mu}_k \) is a c-prime ideal of \( N \) for all \( k \in (\hat{\alpha}, \hat{\beta}] \).
Therefore \( \hat{\mu} \) is an i-v c-prime L-fuzzy ideal of \( N \) (by Theorem 3.2.9). \( \square \)

**Proposition 3.3.4.** Let \( \hat{\mu} \) be an i-v L-fuzzy subset of \( N \). For all \( \hat{k} \in (\hat{\alpha}, \hat{\beta}] \) if \( \hat{\mu}_k \) is an equiprime ideal of \( N \) then \( \hat{\mu} \) is an i-v 3-prime L-fuzzy ideal of \( N \).

Proof. Let \( \hat{\mu}_k \) be an equiprime ideal of \( N \) for all \( k \in (\hat{\alpha}, \hat{\beta}] \).
Then \( \hat{\mu}_k \) is a 3-prime ideal of \( N \) for all \( k \in (\hat{\alpha}, \hat{\beta}] \).
Therefore \( \hat{\mu} \) is an i-v 3-prime L-fuzzy ideal of \( N \) (by Theorem 3.2.8). \( \square \)

**Proposition 3.3.5.** Let \( \hat{\mu} \) be an i-v L-fuzzy subset of commutative ring \( R \). For all
3.3. Interrelations between Interval Valued Prime L-fuzzy Ideals

\[ \hat{k} \in (\hat{\alpha}, \hat{\beta}] \text{ if } \hat{\mu}_k \text{ is an equiprime ideal of } R \text{ then } \hat{\mu} \text{ is an i-v c-prime L-fuzzy ideal of } R. \]

**Proof.** Let \( \hat{\mu}_k \) be an equiprime ideal of commutative ring \( R \) for all \( \hat{k} \in (\hat{\alpha}, \hat{\beta}] \). Then \( \hat{\mu}_k \) is a c-prime ideal of \( R \) for all \( \hat{k} \in (\hat{\alpha}, \hat{\beta}] \). Therefore \( \hat{\mu} \) is an i-v c-prime L-fuzzy ideal of \( R \). \( \square \)

**Proposition 3.3.6.** Let \( \hat{\mu} \) be an i-v L-fuzzy subset of commutative ring \( R \). For all \( \hat{k} \in (\hat{\alpha}, \hat{\beta}] \) if \( \hat{\mu}_k \) is an 3-prime ideal of \( R \) then \( \hat{\mu} \) is an i-v c-prime L-fuzzy ideal of \( R \).

**Proof.** Let \( \hat{\mu}_k \) be an 3-prime ideal of commutative ring \( R \) for all \( \hat{k} \in (\hat{\alpha}, \hat{\beta}] \). Then \( \hat{\mu}_k \) is a c-prime ideal of \( R \) for all \( \hat{k} \in (\hat{\alpha}, \hat{\beta}] \). Therefore \( \hat{\mu} \) is an i-v c-prime L-fuzzy ideal of \( R \). \( \square \)

**Proposition 3.3.7.** Let \( \hat{\mu} \) be an i-v L-fuzzy subset of commutative ring \( R \). For all \( \hat{k} \in (\hat{\alpha}, \hat{\beta}] \) if \( \hat{\mu}_k \) is an 3-prime ideal of \( R \) then \( \hat{\mu} \) is an i-v equiprime L-fuzzy ideal of \( R \).

**Proof.** Let \( \hat{\mu}_k \) be an 3-prime ideal of commutative ring \( R \) for all \( \hat{k} \in (\hat{\alpha}, \hat{\beta}] \). Then \( \hat{\mu}_k \) is an equiprime ideal of \( R \) for all \( \hat{k} \in (\hat{\alpha}, \hat{\beta}] \). Therefore \( \hat{\mu} \) is an i-v equiprime L-fuzzy ideal of \( R \). \( \square \)

**Proposition 3.3.8.** Let \( \hat{\mu} \) be an i-v L-fuzzy subset of commutative ring \( R \). For all \( \hat{k} \in (\hat{\alpha}, \hat{\beta}] \) if \( \hat{\mu}_k \) is an c-prime ideal of \( R \) then \( \hat{\mu} \) is an i-v equiprime L-fuzzy ideal of \( R \).

**Proof.** Let \( \hat{\mu}_k \) be an c-prime ideal of commutative ring \( R \) for all \( \hat{k} \in (\hat{\alpha}, \hat{\beta}] \). Then \( \hat{\mu}_k \) is an equiprime ideal of \( R \) for all \( \hat{k} \in (\hat{\alpha}, \hat{\beta}] \). Therefore \( \hat{\mu} \) is an i-v equiprime L-fuzzy ideal of \( R \). \( \square \)

**Proposition 3.3.9.** Let \( \hat{\mu} \) be an i-v L-fuzzy subset of commutative ring \( R \). For all \( \hat{k} \in (\hat{\alpha}, \hat{\beta}] \) if \( \hat{\mu}_k \) is an c-prime ideal of \( R \) then \( \hat{\mu} \) is an i-v 3-prime L-fuzzy ideal of \( R \).
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Proof. Let $\hat{\mu}_k$ be an c-prime ideal of commutative ring $R$ for all $\hat{k} \in (\hat{\alpha}, \hat{\beta}]$. Then $\hat{\mu}_k$ is a 3-prime ideal of $R$ for all $\hat{k} \in (\hat{\alpha}, \hat{\beta}]$. Therefore $\hat{\mu}$ is an i-v 3-prime L-fuzzy ideal of $R$. $\square$

Proposition 3.3.10. Let $R$ be a commutative ring and $\hat{\mu}$ be an i-v L-fuzzy ideal of $N$ with associated i-v t-norm idempotent. Then following statements are equivalent.

1. $\hat{\mu}$ is an i-v equiprime L-fuzzy ideal of $R$.
2. $\hat{\mu}$ is an i-v 3-prime L-fuzzy ideal of $R$.
3. $\hat{\mu}$ is an i-v c-prime L-fuzzy ideal of $R$.

Proof. (1) $\Rightarrow$ (2). Let $\hat{\mu}$ be an i-v equiprime L-fuzzy ideal of $R$ with associated i-v t-norm idempotent. Then by Theorem 3.2.7, we get $\hat{\mu}_k$ is an equiprime ideal of $R$ for all $\hat{k} \in (\hat{\alpha}, \hat{\beta}]$. Then $\hat{\mu}_k$ is a 3-prime ideal of $R$ for all $\hat{k} \in (\hat{\alpha}, \hat{\beta}]$. By Theorem 3.2.8, $\hat{\mu}$ is an i-v 3-prime L-fuzzy ideal of $R$.

(2) $\Rightarrow$ (3). Let $\hat{\mu}$ be an i-v 3-prime L-fuzzy ideal of $R$ with associated i-v t-norm idempotent. Then by Theorem 3.2.8, we get $\hat{\mu}_k$ is a 3-prime ideal of $R$ for all $\hat{k} \in (\hat{\alpha}, \hat{\beta}]$. Then $\hat{\mu}_k$ is a c-prime ideal of $R$ for all $\hat{k} \in (\hat{\alpha}, \hat{\beta}]$. By Theorem 3.2.9, $\hat{\mu}$ is an i-v c-prime L-fuzzy ideal of $R$.

(3) $\Rightarrow$ (1). Let $\hat{\mu}$ be an i-v c-prime L-fuzzy ideal of $R$ with associated i-v t-norm idempotent. Then by Theorem 3.2.9, we get $\hat{\mu}_k$ is a c-prime ideal of $R$ for all $\hat{k} \in (\hat{\alpha}, \hat{\beta}]$. Then $\hat{\mu}_k$ is an equiprime ideal of $R$ for all $\hat{k} \in (\hat{\alpha}, \hat{\beta}]$. By Theorem 3.2.7, $\hat{\mu}$ is an i-v equiprime L-fuzzy ideal of $R$. $\square$

Proposition 3.3.11. Let $\hat{\mu}$ be an i-v equiprime L-fuzzy ideal of $N$ with associated i-v t-norm idempotent. If $\hat{\mu}$ has IFP then $\hat{\mu}$ is an i-v c-prime L-fuzzy ideal of $N$.

Proof. Let $\hat{\mu}$ be an i-v equiprime L-fuzzy ideal of $N$ with associated i-v t-norm idempotent. Then by Theorem 3.2.7, we get $\hat{\mu}_k$ is an equiprime ideal of $N$ for all $\hat{k} \in (\hat{\alpha}, \hat{\beta}]$. By Lemma 2.17 in Kedukodi, Kuncham and Bhavanri [75], we get $\hat{\mu}_k$ is a c-prime ideal of $N$ for all $\hat{k} \in (\hat{\alpha}, \hat{\beta}]$. Therefore $\hat{\mu}$ is an i-v c-prime L-fuzzy ideal of $N$. $\square$
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**Proposition 3.3.12.** Let \( \hat{\mu} \) be an i-v L-fuzzy subset of \( N \). If for all \( \hat{k} \in (\hat{\alpha}, \hat{\beta}] \) the level set \( \hat{\mu}_k \) is an equiprime ideal of \( N \) with IFP. Then \( \hat{\mu} \) is an i-v c-prime L-fuzzy ideal of \( N \).

**Proof.** Let \( a, b \in N \) and \( \hat{k} \in (\hat{\alpha}, \hat{\beta}] \) such that \( ab \in \hat{\mu}_k \). By IFP, we get \( anb \in \hat{\mu}_k \) for all \( n \in N \Rightarrow anb - an0 \in \hat{\mu}_k \) for all \( n \in N \Rightarrow a \in \hat{\mu}_k \) or \( b \in \hat{\mu}_k \). Hence \( \hat{\mu}_k \) is a c-prime ideal of \( N \). By Theorem 3.2.9, \( \hat{\mu} \) is an i-v c-prime L-fuzzy ideal of \( N \). \( \square \)

**Proposition 3.3.13.** Let \( \hat{\mu} \) be an i-v 3-prime L-fuzzy ideal of \( N \) with associated i-v t-norm idempotent. If \( \hat{\mu} \) has IFP then \( \hat{\mu} \) is an i-v c-prime L-fuzzy ideal of \( N \).

**Proof.** Let \( \hat{\mu} \) be an i-v 3-prime L-fuzzy ideal of \( N \) with associated i-v t-norm idempotent. By Theorem 3.2.8, we get \( \hat{\mu}_k \) is a 3-prime ideal of \( N \) for all \( \hat{k} \in (\hat{\alpha}, \hat{\beta}] \). Let \( \hat{k} \in (\hat{\alpha}, \hat{\beta}] \) and \( a, b \in N \) such that \( ab \in \hat{\mu}_k \). By Lemma 2.2.23 \( \hat{\mu}_k \) has IFP for all \( \hat{k} \in (\hat{\alpha}, \hat{\beta}] \). Then \( anb \in \hat{\mu}_k \) for all \( n \in N \Rightarrow a \in \hat{\mu}_k \) or \( b \in \hat{\mu}_k \). Hence \( \hat{\mu}_k \) is a c-prime ideal of \( N \). Therefore \( \hat{\mu} \) is an i-v c-prime L-fuzzy ideal of \( N \). (By Theorem 3.2.9). \( \square \)

**Proposition 3.3.14.** Let \( \hat{\mu} \) be an i-v L-fuzzy ideal of \( N \) with associated i-v t-norm idempotent. If \( \hat{\mu} \) has IFP then following are equivalent.

1. \( \hat{\mu} \) is an i-v c-prime L-fuzzy ideal of \( N \).
2. \( \hat{\mu} \) is an i-v 3-prime L-fuzzy ideal of \( N \).

**Proof.** (1) \( \Rightarrow \) (2) Let \( \hat{\mu} \) be an i-v c-prime L-fuzzy ideal of \( N \) with associated i-v t-norm idempotent. Then by Proposition 3.3.3, we get \( \hat{\mu} \) is i-v 3-prime L-fuzzy ideal of \( N \).

(2) \( \Rightarrow \) (1) Let \( \hat{\mu} \) be an i-v 3-prime L-fuzzy ideal of \( N \) with associated i-v t-norm idempotent. and \( \hat{\mu} \) has IFP. Then by Proposition 3.3.13, we get \( \hat{\mu} \) is an i-v c-prime ideal of \( N \). \( \square \)

**Corollary 3.3.15.** Let \( N \) be a distributive nearring and \( \hat{\mu} \) be an i-v L-fuzzy ideal of \( N \) with associated i-v t-norm idempotent. If \( \hat{\mu} \) has IFP then following statements are equivalent.
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1. \( \hat{\mu} \) is an i-v equiprime \( L \)-fuzzy ideal of \( N \).
2. \( \hat{\mu} \) is an i-v 3-prime \( L \)-fuzzy ideal of \( N \).
3. \( \hat{\mu} \) is an i-v c-prime \( L \)-fuzzy ideal of \( N \).

Proof. (1)\( \Rightarrow \) (2) Let \( \hat{\mu} \) be an i-v equiprime \( L \)-fuzzy ideal of \( N \) with associated i-v t-norm idempotent. Then by Proposition 3.3.2, we get \( \hat{\mu} \) is an i-v 3-prime \( L \)-fuzzy ideal of \( N \).

(2)\( \Rightarrow \) (3) Let \( \hat{\mu} \) be an i-v 3-prime \( L \)-fuzzy ideal of \( N \) with associated i-v t-norm idempotent and \( \hat{\mu} \) has IFP. Then by Proposition 3.3.13, we get \( \hat{\mu} \) is an i-v c-prime \( L \)-fuzzy ideal of \( N \).

(3)\( \Rightarrow \) (2) Let \( \hat{\mu} \) be an i-v c-prime \( L \)-fuzzy ideal of \( N \) with associated i-v t-norm idempotent. Then by Proposition 3.3.3, we get \( \hat{\mu} \) is an i-v 3-prime \( L \)-fuzzy ideal of \( N \).

(2)\( \Rightarrow \) (1) Let \( \hat{\mu} \) be an i-v 3-prime \( L \)-fuzzy ideal of \( N \) with associated i-v t-norm idempotent and \( \hat{\mu} \) has IFP. Then by Proposition 3.3.1 we get \( \hat{\mu} \) is an i-v equiprime \( L \)-fuzzy ideal of \( N \).

\[ \square \]

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**Theorem 3.4.1.** Let \( \hat{\mu} \) be an i-v fuzzy ideal of \( N \).

1. If \( \hat{\mu} \) is an i-v equiprime fuzzy ideal of \( N \) with associated i-v idempotent t-norm \( T_I \) then for every \( \hat{k} \in ([0,0],[1,1]) \), \( \hat{\mu}_{\hat{k} \vee q} \) is an equiprime ideal of \( N \).
2. If for every \( \hat{k} \in ([0,0],[1,1]) \), \( \hat{\mu}_{\hat{k} \vee q} \) is an equiprime ideal of \( N \) then \( \hat{\mu} \) is an i-v equiprime fuzzy ideal of \( N \).

Proof. To prove (1), let \( \hat{\mu} \) be an i-v equiprime fuzzy ideal of \( N \) with associated i-v idempotent t-norm \( T_I \). Then by Theorem 2.4.3(1), we get for every \( \hat{k} \in ([0,0],[1,1]) \), \( \hat{\mu}_{\hat{k} \vee q} \) is an ideal of \( N \). Let \( \hat{k} \in ([0,0],[1,1]) \). We will prove \( \hat{\mu}_{\hat{k} \vee q} \) is an equiprime ideal of \( N \). Let \( \hat{k} \in ([0,0],[1,1]) \). We will prove \( \hat{\mu}_{\hat{k} \vee q} \) is an equiprime ideal of \( N \). Let \( a, x, y \in N \) such that \( arx - ary \in \hat{\mu}_{\hat{k} \vee q} \) for all \( r \in N \Rightarrow C_I(\hat{a}, \hat{\mu}(arx - ary)) \geq \hat{k} \)
or \( C_I(\hat{\alpha}, \hat{\mu}(arx - ary)) + \hat{k} > 2\hat{\beta} \) for all \( r \in N \). Then \( C_I(\hat{\alpha}, \inf_{r \in N} \hat{\mu}(arx - ary)) \geq \hat{k} \) or \( C_I(\hat{\alpha}, \inf_{r \in N} \hat{\mu}(arx - ary)) + \hat{k} > 2\hat{\beta} \). As \( \hat{\mu} \) is an i-v equiprime fuzzy ideal of \( N \),

\[
\begin{align*}
C_I(\hat{\alpha}, \hat{\mu}(a)) & \geq T_I(\hat{\beta}, C_I(\hat{\alpha}, \inf_{r \in N} \hat{\mu}(arx - ary))) \quad \text{or} \quad C_I(\hat{\alpha}, \hat{\mu}(x - y)) \\
& \geq T_I(\hat{\beta}, C_I(\hat{\alpha}, \inf_{r \in N} \hat{\mu}(arx - ary))).
\end{align*}
\]

We get following cases.

Case (i), suppose \( C_I(\hat{\alpha}, \inf_{r \in N} \hat{\mu}(arx - ary)) \geq \hat{k} \). Then \( C_I(\hat{\alpha}, \hat{\mu}(a)) \geq T_I(\hat{\beta}, \hat{k}) \)

\[
\begin{align*}
T_I(\hat{k}, \hat{k}) = \hat{k} \quad \text{or} \quad C_I(\hat{\alpha}, \hat{\mu}(x - y)) \geq \hat{k} & \Rightarrow a \in \hat{\mu}_{k\forall q} \quad \text{or} \quad x - y \in \hat{\mu}_{k\forall q}. \quad \text{Therefore} \\
\hat{\mu}_{k\forall q} & \text{is an equiprime ideal of} \ N. \quad \text{Proof is similar for} \ \hat{k} = \hat{\beta}. \quad \text{Suppose} \ \hat{k} > \hat{\beta}. \quad \text{Then} \\
T_I(\hat{\beta}, \hat{k}) & \geq T_I(\hat{\beta}, \hat{\beta}) = \hat{\beta}. \quad \text{Hence} \ C_I(\hat{\alpha}, \hat{\mu}(a)) \geq \hat{\beta} \quad \text{and} \quad \hat{k} > \hat{\beta} \Rightarrow C_I(\hat{\alpha}, \hat{\mu}(a)) + \hat{k} \\
& \geq \hat{\beta} + \hat{k} > \hat{\beta} + \hat{\beta} = 2\hat{\beta} \Rightarrow C_I(\hat{\alpha}, \hat{\mu}(a)) + \hat{k} > 2\hat{\beta} \Rightarrow a \in \hat{\mu}_{k\forall q}. \quad \text{Similarly we can prove} \\
(x - y) & \in \hat{\mu}_{k\forall q}. \quad \text{Therefore} \ \hat{\mu}_{k\forall q} \text{is an equiprime ideal of} \ N.
\end{align*}
\]

Case (ii), suppose \( C_I(\hat{\alpha}, \inf_{r \in N} \hat{\mu}(arx - ary)) + \hat{k} > 2\hat{\beta} \). Then \( C_I(\hat{\alpha}, \inf_{r \in N} \hat{\mu}(arx - ary)) \)

\[
\begin{align*}
& > 2\hat{\beta} - \hat{k} \Rightarrow C_I(\hat{\alpha}, \hat{\mu}(a)) \geq T_I(\hat{\beta}, (2\hat{\beta} - \hat{k})) \quad \text{or} \quad C_I(\hat{\alpha}, \hat{\mu}(x - y)) \geq T_I(\hat{\beta}, (2\hat{\beta} - \hat{k})). \quad \text{Suppose} \\
\hat{\beta} & \geq \hat{k}. \quad \text{Then} \quad T_I(\hat{\beta}, (2\hat{\beta} - \hat{k})) \geq T_I(\hat{\beta}, \hat{k}) \geq T_I(\hat{k}, \hat{k}) = \hat{k}. \quad \text{Hence} \ C_I(\hat{\alpha}, \hat{\mu}(a)) \geq \hat{k} \quad \text{or} \\
C_I(\hat{\alpha}, \hat{\mu}(x - y)) & \geq \hat{k} \quad \Rightarrow a \in \hat{\mu}_{k\forall q} \quad \text{or} \quad (x - y) \in \hat{\mu}_{k\forall q}. \quad \text{Proof is similar for} \ \hat{k} = \hat{\beta}. \quad \text{Suppose} \ \hat{k} > \hat{\beta}. \quad \text{Then} \\
T_I(\hat{\beta}, (2\hat{\beta} - \hat{k})) & \geq T_I(2\hat{\beta} - \hat{k}, 2\hat{\beta} - \hat{k}) = 2\hat{\beta} - \hat{k}. \quad \text{Hence} \\
C_I(\hat{\alpha}, \hat{\mu}(a)) & \geq 2\hat{\beta} - \hat{k} \quad \text{and} \quad \hat{k} > (2\hat{\beta} - \hat{k}) \Rightarrow C_I(\hat{\alpha}, \hat{\mu}(a)) + \hat{k} > 2\hat{\beta} \Rightarrow a \in \hat{\mu}_{k\forall q}. \quad \text{Similarly} we can prove \ (x - y) \in \hat{\mu}_{k\forall q}. \quad \text{Therefore} \ \hat{\mu}_{k\forall q} \text{is an equiprime ideal of} \ N.
\end{align*}
\]

To prove (2), let for every \( \hat{k} \in ([0, 0], [1, 1]) \), \( \hat{\mu}_{k\forall q} \) is an equiprime ideal of \( N \). Then by Theorem 2.4.3(2), \( \hat{\mu} \) is an i-v fuzzy ideal of \( N \). We will prove \( \hat{\mu} \) is an i-v equiprime fuzzy ideal of \( N \). Suppose there exists \( a, x, y \in N \) such that

\[
\begin{align*}
C_I(\hat{\alpha}, \hat{\mu}(a)) & < T_I(\hat{\beta}, C_I(\hat{\alpha}, \inf_{r \in N} \hat{\mu}(arx - ary))) \quad \text{and} \\
C_I(\hat{\alpha}, \hat{\mu}(x - y)) & < T_I(\hat{\beta}, C_I(\hat{\alpha}, \inf_{r \in N} \hat{\mu}(arx - ary))). \quad \text{Choose} \ \hat{k} \in D([0, 1]) \text{ such that} \\
C_I(\hat{\alpha}, \hat{\mu}(a)) & < \hat{k} < T_I(\hat{\beta}, C_I(\hat{\alpha}, \inf_{r \in N} \hat{\mu}(arx - ary)) \quad \text{and} \\
C_I(\hat{\alpha}, \hat{\mu}(x - y)) & < \hat{k} < T_I(\hat{\beta}, C_I(\hat{\alpha}, \inf_{r \in N} \hat{\mu}(arx - ary))).
\end{align*}
\]

Then \( C_I(\hat{\alpha}, \hat{\mu}(a)) < \hat{k}, C_I(\hat{\alpha}, \hat{\mu}(x - y)) < \hat{k} \quad \text{and} \quad \hat{k} < T_I(\hat{\beta}, C_I(\hat{\alpha}, \inf_{r \in N} \hat{\mu}(arx - ary))) \)

\[
\begin{align*}
& \leq T_I(\hat{\beta}, C_I(\hat{\alpha}, \hat{\mu}(arx - ary))) \leq \hat{\beta} \quad \text{and} \\
& C_I(\hat{\alpha}, \hat{\mu}(arx - ary)) \quad \text{for all} \ r \in N. \quad \text{Then for} \ \hat{k} \in ([0, 0], [1, 1]) \text{, we get} \\
C_I(\hat{\alpha}, \hat{\mu}(arx - ary)) & \geq \hat{k} \quad \text{for all} \ r \in N \quad \text{and} \ C_I(\hat{\alpha}, \hat{\mu}(a)) < \hat{k}
\end{align*}
\]
and \( C_I(\hat{\alpha}, \hat{\mu}(x - y)) < \hat{k} \). Hence for \( \hat{k} \in ([0, 0], [1, 1]) \), we get \( arx - ary \in \hat{\mu}_{kqv} \) for all \( r \in N \) however \( a \notin \hat{\mu}_{kqv} \) and \( (x - y) \notin \hat{\mu}_{kqv} \). We get a contradiction to the fact that \( \hat{\mu}_{kqv} \) is an equiprime ideal of \( N \) for all \( \hat{k} \in ([0, 0], [1, 1]) \). Therefore \( \hat{\mu} \) is an i-v equiprime fuzzy ideal of \( N \).

\[ \square \]

**Theorem 3.4.2.** Let \( \hat{\mu} \) be an i-v fuzzy ideal of \( N \).

1. If \( \hat{\mu} \) is an i-v 3-prime fuzzy ideal of \( N \) with associated i-v idempotent t-norm \( T_I \) then for every \( \hat{k} \in ([0, 0], [1, 1]) \), \( \hat{\mu}_{kqv} \) is a 3-prime ideal of \( N \).

2. If for every \( \hat{k} \in ([0, 0], [1, 1]) \), \( \hat{\mu}_{kqv} \) is a 3-prime ideal of \( N \) then \( \hat{\mu} \) is an i-v 3-prime fuzzy ideal of \( N \).

**Proof.** To prove (1), let \( \hat{\mu} \) be an i-v 3-prime fuzzy ideal of \( N \) with associated i-v idempotent t-norm \( T_I \). Then by Theorem 2.4.3(1), we get for every \( \hat{k} \in ([0, 0], [1, 1]) \), \( \hat{\mu}_{kqv} \) is an ideal of \( N \). Let \( \hat{k} \in ([0, 0], [1, 1]) \). We will prove \( \hat{\mu}_{kqv} \) is a 3-prime ideal of \( N \). Let \( x, y \in N \) such that \( xry \in \hat{\mu}_{kqv} \) for all \( r \in N \) \( \Rightarrow C_I(\hat{\alpha}, \hat{\mu}(xry)) \geq \hat{k} \) or \( C_I(\hat{\alpha}, \hat{\mu}(xry)) + \hat{k} > 2\beta \) for all \( r \in N \). Then \( C_I(\hat{\alpha}, \inf_{r \in N} \hat{\mu}(xry)) \geq \hat{k} \) or \( C_I(\hat{\alpha}, \inf_{r \in N} \hat{\mu}(xry)) + \hat{k} > 2\beta \). As \( \hat{\mu} \) is an i-v 3-prime fuzzy ideal of \( N \), we get

\[
C_I(\hat{\alpha}, \hat{\mu}(x)) \geq T_I(\beta, C_I(\hat{\alpha}, \inf_{r \in N} \hat{\mu}(xry))) \quad \text{or} \quad C_I(\hat{\alpha}, \hat{\mu}(y)) \geq T_I(\beta, C_I(\hat{\alpha}, \inf_{r \in N} \hat{\mu}(xry)))
\]

We get following cases.

**Case (i):** Suppose \( C_I(\hat{\alpha}, \inf_{r \in N} \hat{\mu}(xry)) \geq \hat{k} \). Then \( C_I(\hat{\alpha}, \hat{\mu}(x)) \geq T_I(\beta, \hat{k}) \)

\[
\geq T_I(\beta, \hat{k}) = \hat{k} \quad \text{or} \quad C_I(\hat{\alpha}, \hat{\mu}(y)) \geq \hat{k} \Rightarrow x \in \hat{\mu}_{kqv} \quad \text{or} \quad y \in \hat{\mu}_{kqv}.
\]

Therefore \( \hat{\mu}_{kqv} \) is a 3-prime ideal of \( N \). Proof is similar for \( \hat{k} = \hat{\beta} \). Suppose \( \hat{k} > \hat{\beta} \). Then \( T_I(\hat{\beta}, \hat{k}) \)

\[
\geq T_I(\hat{\beta}, \hat{k}) = \hat{\beta} \quad \text{Hence} \quad C_I(\hat{\alpha}, \hat{\mu}(x)) \geq \hat{\beta} \quad \text{and} \quad \hat{k} > \hat{\beta} \Rightarrow C_I(\hat{\alpha}, \hat{\mu}(x)) + \hat{k} \geq \hat{\beta} + \hat{k} > \hat{\beta} + \hat{\beta} = 2\hat{\beta} \Rightarrow C_I(\hat{\alpha}, \hat{\mu}(x)) + \hat{k} > 2\hat{\beta} \Rightarrow x \in \hat{\mu}_{kqv}.
\]

Similarly we can prove \( y \in \hat{\mu}_{kqv} \) Therefore \( \hat{\mu}_{kqv} \) is a 3-prime ideal of \( N \).

**Case (ii):** Suppose \( C_I(\hat{\alpha}, \inf_{r \in N} \hat{\mu}(xry)) + \hat{k} > 2\hat{\beta} \). Then \( C_I(\hat{\alpha}, \inf_{r \in N} \hat{\mu}(xry)) \)

\[
> 2\hat{\beta} - \hat{k} \Rightarrow C_I(\hat{\alpha}, \hat{\mu}(x)) \geq T_I(\beta, (2\hat{\beta} - \hat{k})) \quad \text{or} \quad C_I(\hat{\alpha}, \hat{\mu}(y)) \geq T_I(\beta, (2\hat{\beta} - \hat{k})).
\]

Suppose \( \hat{\beta} \geq \hat{k} \). Then \( T_I(\beta, (2\hat{\beta} - \hat{k})) \geq T_I(\hat{\beta}, \hat{k}) = \hat{k} \). Hence \( C_I(\hat{\alpha}, \hat{\mu}(x)) \)

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If $c \geq d$ or $C_I(\hat{\alpha}, \hat{\mu}(y)) \geq \hat{k} \Rightarrow x \in \hat{\mu}_{k,vq}$ or $y \in \hat{\mu}_{k,vq}$. Proof is similar for $k = \hat{\beta}$. Suppose $k > \hat{\beta}$. Then $T_I(\hat{\beta}, (2\hat{\beta} - \hat{k})) \geq T_I(2\hat{\beta} - \hat{k}, 2\hat{\beta} - \hat{k}) = 2\hat{\beta} - \hat{k}$. Hence $C_I(\hat{\alpha}, \hat{\mu}(x)) \geq 2\hat{\beta} - \hat{k}$ and $k > (2\hat{\beta} - \hat{k}) \Rightarrow C_I(\hat{\alpha}, \hat{\mu}(x)) + \hat{k} > 2\hat{\beta} \Rightarrow x \in \hat{\mu}_{k,vq}$. Similarly we can prove $y \in \hat{\mu}_{k,vq}$. Therefore $\hat{\mu}_{k,vq}$ is a 3-prime ideal of $N$.

To prove (2), let for every $\hat{k} \in ([0, 0], [1, 1])$, $\hat{\mu}_{k,vq}$ is a 3-prime ideal of $N$. Then by Theorem 2.4.3(2), $\hat{\mu}$ is an i-v fuzzy ideal of $N$. We will prove $\hat{\mu}$ is an i-v 3-prime fuzzy ideal of $N$. Suppose there exists $x, y \in N$ such that $C_I(\hat{\alpha}, \hat{\mu}(x)) < T_I(\hat{\beta}, C_I(\hat{\alpha}, \inf \{\hat{\mu}(xry)\}_r) \leq r \in N)$ and $C_I(\hat{\alpha}, \hat{\mu}(y)) < \inf \{\hat{\mu}(xry)\}_r \leq r \in N)$. Then $C_I(\hat{\alpha}, \hat{\mu}(x)) < \hat{k}$, $C_I(\hat{\alpha}, \hat{\mu}(y)) < \hat{k}$, $C_I(\hat{\alpha}, \hat{\mu}(x)) < \hat{k}$ and $\hat{k} < T_I(\hat{\beta}, C_I(\hat{\alpha}, \inf \{\hat{\mu}(xry)\}_r) \leq r \in N) \leq T_I(\hat{\beta}, C_I(\hat{\alpha}, \hat{\mu}(xry))) \leq \hat{k} \wedge C_I(\hat{\alpha}, \hat{\mu}(xry))$ for all $r \in N$. Then $C_I(\hat{\alpha}, \hat{\mu}(xry)) \geq \hat{k}$ for all $r \in N$ and $C_I(\hat{\alpha}, \hat{\mu}(x)) < \hat{k}$, $C_I(\hat{\alpha}, \hat{\mu}(y)) < \hat{k}$. Hence for $\hat{k} \in ([0, 0], [1, 1])$, we get $xry \in \hat{\mu}_{k,vq}$ for all $r \in N$ however $x \notin \hat{\mu}_{k,vq}$ and $y \notin \hat{\mu}_{k,vq}$. We get a contradiction to the fact that $\hat{\mu}_{k,vq}$ is a 3-prime ideal of $N$ for all $\hat{k} \in ([0, 0], [1, 1])$. Therefore $\hat{\mu}$ is an i-v 3-prime fuzzy ideal of $N$.

\textbf{Theorem 3.4.3. Let $\hat{\mu}$ be an i-v fuzzy ideal of $N$.}

(1) If $\hat{\mu}$ is an i-v c-prime fuzzy ideal of $N$ with associated i-v idempotent t-norm $T_I$ then for every $\hat{k} \in ([0, 0], [1, 1])$, $\hat{\mu}_{k,vq}$ is a c-prime ideal of $N$.

(2) If for every $\hat{k} \in ([0, 0], [1, 1])$, $\hat{\mu}_{k,vq}$ is a c-prime ideal of $N$ then $\hat{\mu}$ is an i-v c-prime fuzzy ideal of $N$.

\textbf{Proof.} To prove (1), let $\hat{\mu}$ be an i-v c-prime fuzzy ideal of $N$ with associated i-v idempotent t-norm $T_I$. Then by Theorem 2.4.3(1), we get for every $\hat{k} \in ([0, 0], [1, 1])$, $\hat{\mu}_{k,vq}$ is an ideal of $N$. Let $\hat{k} \in ([0, 0], [1, 1])$. We will prove $\hat{\mu}_{k,vq}$ is a c-prime ideal of $N$.

Let $x, y \in N$ such that $xry \in \hat{\mu}_{k,vq}$ $\Rightarrow C_I(\hat{\alpha}, \hat{\mu}(x)) \geq \hat{k}$ or $C_I(\hat{\alpha}, \hat{\mu}(x)) + \hat{k} > 2\hat{\beta}$. As $\hat{\mu}$ is an i-v c-prime fuzzy ideal of $N$, $C_I(\hat{\alpha}, \hat{\mu}(x)) \geq T_I(\hat{\beta}, C_I(\hat{\alpha}, \hat{\mu}(xry))) \text{ or } C_I(\hat{\alpha}, \hat{\mu}(y))$
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\[ \geq T_I(\hat{\beta}, C_I(\hat{\alpha}, \hat{\mu}(xy))) \]. We get following cases.

Case (i): Suppose \( C_I(\hat{\alpha}, \hat{\mu}(xy)) \geq \hat{k} \). Then \( C_I(\hat{\alpha}, \hat{\mu}(x)) \geq T_I(\hat{\beta}, \hat{k}) \geq T_I(\hat{k}, \hat{k}) = \hat{k} \) or \( C_I(\hat{\alpha}, \hat{\mu}(y)) \geq \hat{k} \Rightarrow x \in \hat{\mu}_{k\wedge q} \) or \( y \in \hat{\mu}_{k\vee q} \). Therefore \( \hat{\mu}_{k\vee q} \) is a c-prime ideal of \( N \). Proof is similar for \( \hat{k} = \hat{\beta} \). Suppose \( \hat{k} > \hat{\beta} \). Then \( T_I(\hat{\beta}, \hat{k}) \geq T_I(\hat{\beta}, \hat{\beta}) = \hat{\beta} \). Hence \( C_I(\hat{\alpha}, \hat{\mu}(x)) \geq \hat{\beta} \) and \( \hat{k} > \hat{\beta} \Rightarrow C_I(\hat{\alpha}, \hat{\mu}(x)) + \hat{k} \geq \hat{\beta} + \hat{k} > \hat{\beta} + \hat{\beta} = 2\hat{\beta} \)

\[ \Rightarrow C_I(\hat{\alpha}, \hat{\mu}(x)) + \hat{k} > 2\hat{\beta} \Rightarrow x \in \hat{\mu}_{k\vee q} \). Similarly we can prove \( y \in \hat{\mu}_{k\vee q} \). Therefore \( \hat{\mu}_{k\vee q} \) is a c-prime ideal of \( N \).

Case (ii) Suppose \( C_I(\hat{\alpha}, \hat{\mu}(xy)) + \hat{k} > 2\hat{\beta} \). Then \( C_I(\hat{\alpha}, \hat{\mu}(xy)) > 2\hat{\beta} - \hat{k} \Rightarrow C_I(\hat{\alpha}, \hat{\mu}(x)) \geq T_I(\hat{\beta}, (2\hat{\beta} - \hat{k})) \) or \( C_I(\hat{\alpha}, \hat{\mu}(y)) \geq T_I(\hat{\beta}, (2\hat{\beta} - \hat{k})) \). Suppose \( \hat{\beta} \geq \hat{k} \). Then \( T_I(\hat{\beta}, (2\hat{\beta} - \hat{k})) \geq T_I(\hat{\beta}, \hat{k}) = \hat{k} \). Hence \( C_I(\hat{\alpha}, \hat{\mu}(x)) \geq \hat{k} \) or \( C_I(\hat{\alpha}, \hat{\mu}(y)) \geq \hat{k} \). Suppose \( \hat{k} > \hat{\beta} \). Then \( T_I(\hat{\beta}, (2\hat{\beta} - \hat{k})) \geq T_I(2\hat{\beta} - \hat{k}, 2\hat{\beta} - \hat{k}) = 2\hat{\beta} - \hat{k} \). Hence \( C_I(\hat{\alpha}, \hat{\mu}(x)) \geq 2\hat{\beta} - \hat{k} \) and \( \hat{k} > (2\hat{\beta} - \hat{k}) \Rightarrow C_I(\hat{\alpha}, \hat{\mu}(x)) + \hat{k} > 2\hat{\beta} \Rightarrow x \in \hat{\mu}_{k\vee q} \). Similarly we can prove \( y \in \hat{\mu}_{k\vee q} \). Therefore \( \hat{\mu}_{k\vee q} \) is a c-prime ideal of \( N \).

To prove (2), let for every \( \hat{k} \in ([0,0], [1,1]) \), \( \hat{\mu}_{k\vee q} \) is a c-prime ideal of \( N \). Then by Theorem 2.4.3(2), \( \hat{\mu} \) is an i-v fuzzy ideal of \( N \). We will prove \( \hat{\mu} \) is an i-v c-prime fuzzy ideal of \( N \). Suppose there exists \( x, y \in N \) such that \( C_I(\hat{\alpha}, \hat{\mu}(x)) \) lower than \( T_I(\hat{\beta}, C_I(\hat{\alpha}, \hat{\mu}(xy))) \) and \( C_I(\hat{\alpha}, \hat{\mu}(y)) < T_I(\hat{\beta}, C_I(\hat{\alpha}, \hat{\mu}(xy))) \). Choose \( \hat{k} \in D([0,1]) \) such that \( C_I(\hat{\alpha}, \hat{\mu}(x)) < \hat{k} \) lower than \( T_I(\hat{\beta}, C_I(\hat{\alpha}, \hat{\mu}(xy))) \) and \( C_I(\hat{\alpha}, \hat{\mu}(y)) < \hat{k} \) lower than \( T_I(\hat{\beta}, C_I(\hat{\alpha}, \hat{\mu}(xy))) \). Then \( C_I(\hat{\alpha}, \hat{\mu}(x)) < \hat{k} \), \( C_I(\hat{\alpha}, \hat{\mu}(y)) < \hat{k} \) and \( \hat{k} < T_I(\hat{\beta}, C_I(\hat{\alpha}, \hat{\mu}(xy))) \) lower than \( T_I(\hat{\beta}, C_I(\hat{\alpha}, \hat{\mu}(xy))) \) lower than \( \hat{\beta} \wedge C_I(\hat{\alpha}, \hat{\mu}(xy)) \). Then \( C_I(\hat{\alpha}, \hat{\mu}(xy)) \geq \hat{k} \) and \( C_I(\hat{\alpha}, \hat{\mu}(x)) < \hat{k} \), \( C_I(\hat{\alpha}, \hat{\mu}(y)) < \hat{k} \). Hence for \( \hat{k} \in ([0,0], [1,1]) \), we get \( xy \in \hat{\mu}_{k\vee q} \) however \( x \notin \hat{\mu}_{k\wedge q} \) and \( y \notin \hat{\mu}_{k\wedge q} \). We get a contradiction to the fact that \( \hat{\mu}_{k\vee q} \) is a c-prime ideal of \( N \) for all \( \hat{k} \in ([0,0], [1,1]) \). Therefore \( \hat{\mu} \) is an i-v c-prime fuzzy ideal of \( N \).

\[ \square \]

**Theorem 3.4.4.** Let \( \hat{k}, \hat{n} \in D([0,1]) \).

(1) Let \( \hat{\mu} \) be an i-v equiprime fuzzy ideal of \( N \) with associated i-v idempotent t-norm
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$T_1$. Then for each $a, x, y \in N$, $(arx - ary)_k \in \hat{\mu}_q$ for all $r \in N \Rightarrow a_k \in \hat{\mu}_q$ or $(x - y)_k \in \hat{\mu}_q$.

Conversely, if $\hat{\mu}$ is an i-v fuzzy subset of $N$ and i-v fuzzy points satisfy the following properties:

(i) $x_k, y_h \in \hat{\mu} \Rightarrow (x + y)_{T(j, k)} \in \hat{\mu}_q$, where $T_j$ is an idempotent i-v t-norm,

(ii) $x_k \in \hat{\mu} \Rightarrow (-x)_k \in \hat{\mu}_q$, (iii) $x_k \in \hat{\mu}, y \in N \Rightarrow (y + x - y)_k \in \hat{\mu}_q$,

(iv) $x_k \in \hat{\mu}, y \in N \Rightarrow (xy)_k \in \hat{\mu}_q$, (v) $i_k \in \hat{\mu}, x, y \in N \Rightarrow (x(y + i) - xy)_k \in \hat{\mu}_q$

then $\hat{\mu}$ is an i-v fuzzy ideal of $N$. Further

(vi) if $\hat{\mu}$ is an i-v fuzzy subset of $N$ and i-v fuzzy points satisfy properties (i),(ii),(iii),(iv), (v) and for each $a, x, y \in N$, $(arx - ary)_k \in \hat{\mu}_q$ for all $r \in N$

$\Rightarrow a_k \in \hat{\mu}_q$ or $(x - y)_k \in \hat{\mu}_q$ then $\hat{\mu}$ is an i-v equiprime fuzzy ideal of $N$.

Proof. To prove (1), let $\hat{k} \in D([0, 1])$ and $a, x, y \in N$ such that $(arx - ary)_k \in \hat{\mu}$ for all $r \in N$. Then $C_1(\hat{\alpha}, \hat{\mu}(arx - ary)) \geq \hat{k}$ for all $r \in N$

$\Rightarrow C_1(\hat{\alpha}, \inf_{r \in N} \hat{\mu}(arx - ary)) \geq \hat{k}$. As $\hat{\mu}$ is an i-v equiprime fuzzy ideal of $N$, we get

$C_1(\hat{\alpha}, \hat{\mu}(a)) \geq T_1(\hat{\beta}, C_1(\hat{\alpha}, \inf_{r \in N} \hat{\mu}(arx - ary)))$ or

$C_1(\hat{\alpha}, \hat{\mu}(x - y)) \geq T_1(\hat{\beta}, C_1(\hat{\alpha}, \inf_{r \in N} \hat{\mu}(arx - ary)))$.

Hence $C_1(\hat{\alpha}, \hat{\mu}(a)) \geq T_1(\hat{\beta}, \hat{k})$ or $C_1(\hat{\alpha}, \hat{\mu}(x - y)) \geq T_1(\hat{\beta}, \hat{k})$. Suppose $\hat{\beta} \geq \hat{k}$. Then $C_1(\hat{\alpha}, \hat{\mu}(a)) \geq T_1(\hat{\beta}, \hat{k}) \geq T_1(\hat{k}, \hat{k}) = k$ or $C_1(\hat{\alpha}, \hat{\mu}(x - y)) \geq T_1(\hat{\beta}, \hat{k}) \geq T_1(\hat{k}, \hat{k})$

$= \hat{k}$. Hence $a \in \hat{\mu}_q$ or $(x - y) \in \hat{\mu}_q$. Proof is similar for $\hat{\beta} < \hat{k}$. Then $C_1(\hat{\alpha}, \hat{\mu}(a)) \geq T_1(\hat{\beta}, \hat{k}) \geq T_1(\hat{\beta}, \hat{\beta}) = \hat{\beta}$ or $C_1(\hat{\alpha}, \hat{\mu}(x - y)) \geq T_1(\hat{\beta}, \hat{k}) \geq T_1(\hat{\beta}, \hat{\beta}) = \hat{\beta}$.

Hence $C_1(\hat{\alpha}, \hat{\mu}(a)) + \hat{k} > \hat{\beta} + \hat{\beta} = 2\hat{\beta}$ or $C_1(\hat{\alpha}, \hat{\mu}(x - y)) + \hat{k} > \hat{\beta} + \hat{\beta} = 2\hat{\beta}$.

Therefore $a \in \hat{\mu}_q$ or $(x - y) \in \hat{\mu}_q$.

Conversely, let $\hat{\mu}$ be an i-v fuzzy subset of $N$ and i-v fuzzy points satisfy (i),(ii),(iii),(iv) and (v). Then by Theorem 2.4.4, we get $\hat{\mu}$ is an i-v fuzzy ideal of $N$. Now suppose i-v fuzzy points satisfy (vi) and $\hat{\mu}$ is not an i-v equiprime fuzzy ideal of $N$. Then there exists $a, x, y \in N$ such that

$C_1(\hat{\alpha}, \hat{\mu}(a)) < T_1(\hat{\beta}, C_1(\hat{\alpha}, \inf_{r \in N} \hat{\mu}(arx - ary)))$ and
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\[ C_I(\hat{\alpha}, \hat{\mu}(x - y)) < T_I(\hat{\beta}, C_I(\hat{\alpha}, \inf_{r \in N} \hat{\mu}(arx - ary))). \]

Choose \( \hat{k} \in (\hat{\alpha}, \hat{\beta}) \) such that
\[ C_I(\hat{\alpha}, \hat{\mu}(a)) < \hat{k} < T_I(\hat{\beta}, C_I(\hat{\alpha}, \inf_{r \in N} \hat{\mu}(arx - ary))) \]
and
\[ C_I(\hat{\alpha}, \hat{\mu}(x - y)) < \hat{k} < T_I(\hat{\beta}, C_I(\hat{\alpha}, \inf_{r \in N} \hat{\mu}(arx - ary))). \]

Then
\[ C_I(\hat{\alpha}, \hat{\mu}(a)) < k < T_I(\hat{\beta}, C_I(\hat{\alpha}, \inf_{r \in N} \hat{\mu}(arx - ary))) > \hat{k} \]
\[ \Rightarrow C_I(\hat{\alpha}, \inf_{r \in N} \hat{\mu}(arx - ary))) > \hat{k} \Rightarrow C_I(\hat{\alpha}, \hat{\mu}(arx - ary))) \geq \hat{k} \text{ for all } r \in N. \]
Hence
\[ (arx - ary)_{\hat{k}} \in \hat{\mu}_q \text{ for all } r \in N, \text{ however } a_{\hat{k}} \notin \hat{\mu}_q \text{ and } (x - y)_{\hat{k}} \notin \hat{\mu}_q. \]
We get a contradiction to the assumption (vi). Therefore \( \hat{\mu} \) is an i-v equiprime fuzzy ideal of \( N \).

**Theorem 3.4.5.** Let \( \hat{k}, \hat{n} \in D([0, 1]) \).

(1) Let \( \hat{\mu} \) be an i-v 3-prime fuzzy ideal of \( N \) with associated i-v idempotent t-norm \( T_I \).
Then for each \( x, y \in N, (xry)_{\hat{k}} \in \hat{\mu}_q \) for all \( r \in N \Rightarrow x_{\hat{k}} \in \hat{\mu}_q \) or \( y_{\hat{k}} \in \hat{\mu}_q \).
Conversely, if \( \hat{\mu} \) be an i-v fuzzy subset of \( N \) and i-v fuzzy points satisfy the following properties:

(i) \( x_{\hat{k}}, y_{\hat{n}} \in \hat{\mu} \Rightarrow (x + y)_{T_I(\hat{k}, \hat{n})} \in \hat{\mu}_q \), where \( T_I \) is an idempotent i-v t-norm,
(ii) \( x_{\hat{k}} \in \hat{\mu} \Rightarrow (-x)_{\hat{k}} \in \hat{\mu}_q \), (iii) \( x_{\hat{k}} \in \hat{\mu}, y \in N \Rightarrow (y + x)_{\hat{k}} \in \hat{\mu}_q \),
(iv) \( x_{\hat{k}} \in \hat{\mu}, y \in N \Rightarrow (xy)_{\hat{k}} \in \hat{\mu}_q \), (v) \( i_{\hat{k}} \in \hat{\mu}, x, y \in N \Rightarrow (xy + i - xy)_{\hat{k}} \in \hat{\mu}_q \)
then \( \hat{\mu} \) is an i-v fuzzy ideal of \( N \). Further

(vi) if \( \hat{\mu} \) is an i-v fuzzy subset of \( N \) and i-v fuzzy points satisfy properties (i), (ii), (iii), (iv),
(v) and for each \( x, y \in N, (xry)_{\hat{k}} \in \hat{\mu}_q \) for all \( r \in N \)
\[ \Rightarrow x_{\hat{k}} \in \hat{\mu}_q \text{ or } y_{\hat{k}} \in \hat{\mu}_q \text{ then } \hat{\mu} \text{ is an i-v 3-prime fuzzy ideal of } N. \]

**Proof.** To prove (1), let \( \hat{k} \in D([0, 1]) \) and \( x, y \in N \) such that \( (xry)_{\hat{k}} \in \hat{\mu} \) for all \( r \in N \). Then
\[ C_I(\hat{\alpha}, \hat{\mu}(xry)) \geq \hat{k} \text{ for all } r \in N \Rightarrow C_I(\hat{\alpha}, \inf_{r \in N} \hat{\mu}(xry)) \geq \hat{k}. \]
As \( \hat{\mu} \) is an i-v 3-prime fuzzy ideal of \( N \), we get
\[ C_I(\hat{\alpha}, \hat{\mu}(x)) \geq T_I(\hat{\beta}, C_I(\hat{\alpha}, \inf_{r \in N} \hat{\mu}(xry))) \text{ or } C_I(\hat{\alpha}, \hat{\mu}(y)) \geq T_I(\hat{\beta}, \inf_{r \in N} \hat{\mu}(xry))). \]
Hence
\[ C_I(\hat{\alpha}, \hat{\mu}(x)) \geq T_I(\hat{\beta}, \hat{k}) \text{ or } C_I(\hat{\alpha}, \hat{\mu}(y)) \geq T_I(\hat{\beta}, \hat{k}). \]
Suppose \( \beta \geq \hat{k} \). Then
\[ C_I(\hat{\alpha}, \hat{\mu}(x)) \geq T_I(\hat{\beta}, \hat{k}) \geq T_I(\hat{k}, \hat{k}) = \hat{k} \text{ or } C_I(\hat{\alpha}, \hat{\mu}(y)) \geq T_I(\hat{\beta}, \hat{k}) \geq T_I(\hat{k}, \hat{k}) = \hat{k}. \]
Hence \(x \in \hat{\mu}_q\) or \(y \in \hat{\mu}_q\). Proof is similar for \(\hat{\beta} = \hat{k}\). Suppose \(\hat{\beta} < \hat{k}\). Then \(C_1(\hat{\alpha}, \hat{\mu}(x)) \geq T_I(\hat{\beta}, \hat{k}) \geq T_I(\hat{\beta}, \hat{\beta}) = \hat{\beta}\) or \(C_1(\hat{\alpha}, \hat{\mu}(y)) \geq T_I(\hat{\beta}, \hat{k}) \geq T_I(\hat{\beta}, \hat{\beta}) = \hat{\beta}\). Hence \(C_1(\hat{\alpha}, \hat{\mu}(x)) + \hat{k} > \hat{\beta} + \hat{\beta} = 2\hat{\beta}\) or \(C_1(\hat{\alpha}, \hat{\mu}(y)) + \hat{k} > \hat{\beta} + \hat{\beta} = 2\hat{\beta}\). Hence \(x \in \hat{\mu}_q\) or \(y \in \hat{\mu}_q\).

Conversely, let \(\hat{\mu}\) be an i-v fuzzy subset of \(N\) and i-v fuzzy points satisfy (i),(ii),(iii),(iv) and (v). Then by Theorem 2.4.4, we get \(\hat{\mu}\) is an i-v fuzzy ideal of \(N\). Now suppose i-v fuzzy points satisfy (vi) and \(\hat{\mu}\) is not an i-v 3-prime fuzzy ideal of \(N\). Then there exists \(x, y \in N\) such that

\[
C_1(\hat{\alpha}, \hat{\mu}(x)) < T_I(\hat{\beta}, C_1(\hat{\alpha}, \inf_{\mu(xry)})) \quad \text{and} \quad C_1(\hat{\alpha}, \hat{\mu}(y)) < T_I(\hat{\beta}, C_1(\hat{\alpha}, \inf_{\mu(xry)})).
\]

Choose \(\hat{k} \in (\hat{\alpha}, \hat{\beta})\) such that

\[
C_1(\hat{\alpha}, \hat{\mu}(x)) < \hat{k} < T_I(\hat{\beta}, C_1(\hat{\alpha}, \inf_{\mu(xry)})) \quad \text{and} \quad C_1(\hat{\alpha}, \hat{\mu}(y)) < \hat{k} < T_I(\hat{\beta}, C_1(\hat{\alpha}, \inf_{\mu(xry)})).
\]

Then

\[
C_1(\hat{\alpha}, \hat{\mu}(x)) < \hat{k}, C_1(\hat{\alpha}, \hat{\mu}(y)) < \hat{k} \quad \text{and} \quad T_I(\hat{\beta}, C_1(\hat{\alpha}, \inf_{\mu(xry)})) > \hat{k}
\]

\[
\Rightarrow C_1(\hat{\alpha}, \inf_{\mu(xry)}) > \hat{k} \Rightarrow C_1(\hat{\alpha}, \hat{\mu}(xry))) \geq \hat{k} \quad \text{for all} \ r \in N. \quad \text{Hence} \ (xry)_k \in \hat{\mu}_q \quad \text{for all} \ r \in N, \quad \text{however} \ x_k \notin \hat{\mu}_q \quad \text{and} \ y_k \notin \hat{\mu}_q. \quad \text{We get a contradiction to the assumption (vi). Therefore} \ \hat{\mu} \ \text{is an i-v 3-prime fuzzy ideal of} \ N.
\]

**Theorem 3.4.6.** Let \(\hat{k}, \hat{n} \in D([0,1])\).

(1) Let \(\hat{\mu}\) be an i-v c-prime fuzzy ideal of \(N\) with associated i-v idempotent t-norm \(T_I\). Then for each \(x, y \in N, (xry)_k \in \hat{\mu}_q\) for all \(r \in N \Rightarrow x_k \in \hat{\mu}_q \) or \(y_k \in \hat{\mu}_q\).

Conversely, if \(\hat{\mu}\) be an i-v fuzzy subset of \(N\) and i-v fuzzy points satisfy the following properties:

(i) \(x_k, y_h \in \hat{\mu} \Rightarrow (x + y)_{T_J(k, h)} \in \hat{\mu}_q, \) where \(T_J\) is an idempotent i-v t-norm,

(ii) \(x_k \in \hat{\mu} \Rightarrow (-x)_k \in \hat{\mu}_q, \) (iii) \(x_k \in \hat{\mu}, y \in N \Rightarrow (y + x - y)_k \in \hat{\mu}_q,\)

(iv) \(x_k \in \hat{\mu}, y \in N \Rightarrow (x)_k \in \hat{\mu}_q, \) (vi) \(i_k \in \hat{\mu}, x, y \in N \Rightarrow (x + i - xy)_k \in \hat{\mu}_q\)

then \(\hat{\mu}\) is an i-v fuzzy ideal of \(N\). Further

(vi) if \(\hat{\mu}\) is an i-v fuzzy subset of \(N\) and i-v fuzzy points satisfy properties (i),(ii),(iii),(iv), (v) and for each \(x, y \in N, (xry)_k \in \hat{\mu}_q \Rightarrow x_k \in \hat{\mu}_q \) or \(y_k \in \hat{\mu}_q\) then \(\hat{\mu}\) is an i-v c-prime
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fuzzy ideal of $N$.

Proof. To prove (1), let $\hat{k} \in D([0,1])$ and $x, y \in N$ such that $(xy)_{\hat{k}} \in \hat{\mu}$ . Then $C_I(\hat{\alpha}, \hat{\mu}(xy)) \geq \hat{k}$. As $\hat{\mu}$ is an i-v c-prime fuzzy ideal of $N$, we get $C_I(\hat{\alpha}, \hat{\mu}(x)) \geq T_I(\hat{\beta}, C_I(\hat{\alpha}, \hat{\mu}(xy)))$ or $C_I(\hat{\alpha}, \hat{\mu}(y)) \geq T_I(\hat{\beta}, C_I(\hat{\alpha}, \hat{\mu}(xy)))$.

Hence $C_I(\hat{\alpha}, \hat{\mu}(x)) \geq T_I(\hat{\beta}, \hat{k})$ or $C_I(\hat{\alpha}, \hat{\mu}(y)) \geq T_I(\hat{\beta}, \hat{k})$. Suppose $\hat{\beta} \geq \hat{k}$. Then $C_I(\hat{\alpha}, \hat{\mu}(x)) \geq T_I(\hat{\beta}, \hat{k}) \geq T_I(\hat{k}, \hat{k}) = \hat{k}$ or $C_I(\hat{\alpha}, \hat{\mu}(y)) \geq T_I(\hat{\beta}, \hat{k}) \geq T_I(\hat{k}, \hat{k}) = \hat{k}$.

Hence $x \in \hat{\mu}_q$ or $y \in \hat{\mu}_q$. Proof is similar for $\hat{\beta} < \hat{k}$. Suppose $\hat{\beta} < \hat{k}$. Then $C_I(\hat{\alpha}, \hat{\mu}(x)) \geq T_I(\hat{\beta}, \hat{k}) \geq T_I(\hat{\beta}, \hat{\beta}) = \hat{\beta}$ or $C_I(\hat{\alpha}, \hat{\mu}(y)) \geq T_I(\hat{\beta}, \hat{k}) \geq T_I(\hat{\beta}, \hat{\beta}) = \hat{\beta}$.

Hence $C_I(\hat{\alpha}, \hat{\mu}(x)) + \hat{k} > \hat{\beta} + \hat{\beta} = 2\hat{\beta}$ or $C_I(\hat{\alpha}, \hat{\mu}(y)) + \hat{k} > \hat{\beta} + \hat{\beta} = 2\hat{\beta}$. Hence $x \in \hat{\mu}_q$ or $y \in \hat{\mu}_q$.

Conversely, let $\hat{\mu}$ be an i-v fuzzy subset of $N$ and i-v fuzzy points satisfy (i),(ii),(iii),(iv) and (v). Then by Theorem 2.4.4, we get $\hat{\mu}$ is an i-v fuzzy ideal of $N$. Now suppose i-v fuzzy points satisfy (vi) and $\hat{\mu}$ is not an i-v c-prime fuzzy ideal of $N$. Then there exists $x, y \in N$ such that $C_I(\hat{\alpha}, \hat{\mu}(x)) < T_I(\hat{\beta}, C_I(\hat{\alpha}, \hat{\mu}(xy)))$ and $C_I(\hat{\alpha}, \hat{\mu}(y)) < T_I(\hat{\beta}, C_I(\hat{\alpha}, \hat{\mu}(xy)))$. Choose $\hat{k} \in (\hat{\alpha}, \hat{\beta})$ such that $C_I(\hat{\alpha}, \hat{\mu}(x)) < \hat{k} < T_I(\hat{\beta}, C_I(\hat{\alpha}, \hat{\mu}(xy)))$ and $C_I(\hat{\alpha}, \hat{\mu}(y)) < \hat{k} < T_I(\hat{\beta}, C_I(\hat{\alpha}, \hat{\mu}(xy)))$. Then $C_I(\hat{\alpha}, \hat{\mu}(x)) < \hat{k}$, $C_I(\hat{\alpha}, \hat{\mu}(y)) < \hat{k}$ and $T_I(\hat{\beta}, C_I(\hat{\alpha}, \hat{\mu}(xy))) > \hat{k} \Rightarrow C_I(\hat{\alpha}, \hat{\mu}(xy))) > \hat{k} \Rightarrow C_I(\hat{\alpha}, \hat{\mu}(xy))) \geq \hat{k}$ . Hence $(xy)_{\hat{k}} \in \hat{\mu}_q$, however $x_{\hat{k}} \notin \hat{\mu}_q$ and $y_{\hat{k}} \notin \hat{\mu}_q$. We get a contradiction to the assumption (vi). Therefore $\hat{\mu}$ is an i-v c-prime fuzzy ideal of $N$. 

□