CHAPTER IV

FRIEDMANN COSMOLOGICAL VISCOUS FLUID UNIVERSE IN A SCALE COVARIANT THEORY

1. INTRODUCTION

Several new theories of gravitation have been formulated which are considered to be alternatives to Einstein’s theory of gravitation. Among the theories, a scale covariant theory of gravitation which is formulated by Canuto et al. (1977a) admits a variable G and which is a viable alternative in general relativity (Wesson, 1980 and Will, 1984). The field equations in the scale covariant theory can be derived by taking the general relativistic equations (Ellis, 1971), writing the tensors in co-tensor form and replacing covariant differentiation (Canuto et al., 1977a). A scale covariant theory provides the necessary theoretical framework in which it becomes sensible to discuss the possible variation of the gravitational constant G. Beesham (1986) studies the Bianchi type I cosmological models in the scale covariant theory. Venkateswarlu and Pavan Kumar (2005) have studied higher dimensional string cosmologies in a scale covariant theory of gravitation. Reddy et al (2002) studied the Friedmann universe with bulk viscosity in a scale covariant theory. Rahman and Banerji (1985) have given a related condition in the scale covariant theory for the absence of a singularity of zero volume or infinite density for FRW universes. In order to study the evolution of the Universe,
many authors constructed cosmological models containing a viscous fluid. The presence of viscosity in fluid introduces many interesting features in the dynamics of homogenous cosmological models. The effect of bulk viscosity on the evolution of Friedmann models in general relativity have been discussed by Canuto and Goldman (1983a, 1983b); Johri and Sudharsan (1988), while the same in Brans and Dicke (1961) scalar tensor theory have studied by Pimental (1994). Ibotombi and Anita (2007) have studied cosmological models in a scale covariant theory of gravitation with the help of the special law of variation for Hubble’s parameter.

In this work, we investigate FRW cosmological models with bulk viscosity with equation of state, $p = \gamma \rho$ where the adiabatic parameter $\gamma$ varies with cosmic time in the scale covariant theory. The purpose of the present work is to study cosmological models in a scale covariant theory of gravitation by considering the power law expansion of the Universe. In section 2, we have presented the field equation. In section 3, solutions of the field equations are presented. In section 4, we have presented the conclusion.

2. FIELD EQUATIONS

Einstein’s field equations are valid in gravitational units whereas physical quantities are measured in atomic units in the case of scale covariant theory. The metric tensors in the two systems of units are related by a conformal transformation
where latin indices take values 1,2,3,4, bars denote gravitational units and unbar denotes atomic quantities. The gauge function '\( \Phi \)' is a function of all space time co-ordinates. The possibilities that have been considered for gauge function \( \Phi \) are (Canuto et al., 1977)

\[
\Phi(t) = \left( \frac{t_0}{t} \right)^\varepsilon, \quad \varepsilon = \pm 1, \pm \frac{1}{2}
\]

(4.2)

where \( t_0 \) is constant.

In scale covariant theory, the field equations are

\[
R_{ab} - \frac{R}{2} g_{ab} + f_{ab}(\Phi) = G T_{ab},
\]

(4.3)

where \( f_{ab}(\Phi) \) is given by

\[
\Phi^2 f_{ab} = -2\Phi \Phi_{a;b} + 4\Phi_a \Phi_b + g_{ab} \left( 2\Phi \Phi^{k;k} - \Phi^{b} \Phi_{b;k} \right).
\]

(4.4)

In these equations \( R_{ab} \) is the Ricci tensor and \( T_{ab} \) is the energy momentum tensor. A semicolon denotes covariant derivative and \( \Phi_a \) denotes ordinary derivative with respect to \( x^a \).

We consider the spatially homogenous and isotropic space time given by the Robertson-Walker metric.
\[
\frac{ds^2}{d^2} = dt^2 - R^2(t) \left[ \frac{dr^2}{1 - kr^2} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \right],
\]
(4.5)

where \( R(t) \) and \( k = 0, \pm 1 \) stand for scale factor and curvature parameter respectively.

The energy momentum tensor of the cosmic fluid is used,

\[ T_{ab} = (\bar{p} + \rho)u_a u_b - \bar{p} g_{ab} \]
(4.6)

together with

\[ u^a u_a = 1 \]
(4.7)

and

\[ \bar{p} = p - \xi u^a \dot{u}_a \]
(4.8)

where \( u^a \) is the 4-velocity vector of the fluid, \( p \) and \( \rho \) are the proper pressure and energy density respectively and \( \xi \) is the coefficient of the bulk viscosity.

\[ u^a \dot{u}_a = 3 \frac{\dot{R}}{R} \]
(4.9)

For the metric equation (4.5) and the energy-momentum tensor equation (4.6) along with equations (4.7), (4.8), (4.9), the field equation (4.3) yields
\[ \frac{2 \ddot{R}}{R} + \frac{\dot{R}^2 + k}{R^2} = -G\bar{\rho} + 4 \left( \frac{\Phi \dot{R}}{R \Phi} \right) - \left( \frac{\Phi}{\Phi} \right)^2 + 2 \frac{\ddot{\Phi}}{\Phi}, \]  \hspace{1cm} (4.10)

\[ 3 \left( \frac{\dot{R}^2 + k}{R^2} \right) = G\rho + 6 \left( \frac{\Phi \dot{R}}{R \Phi} \right) + 3 \left( \frac{\Phi}{\Phi} \right)^2, \]  \hspace{1cm} (4.11)

and \[ \dot{\rho} + 3(\bar{\rho} + \rho) \frac{\dot{R}}{R} = 0, \]  \hspace{1cm} (4.12)

where over dot indicates differentiation with respect to ‘t’.

3. SOLUTION OF THE FIELD EQUATIONS

In order to solve the field equations (4.10)-(4.12), we assume that both the scale factor and the scalar field evolve as the power function of time.

\[ R = R_0 \left( \frac{t}{t_0} \right)^\alpha, \]  \hspace{1cm} (4.13)

and

\[ \Phi = \Phi_0 \left( \frac{t}{t_0} \right)^\beta, \]  \hspace{1cm} (4.14)

where the subscript o refers to the values of the parameters at the present epoch and \( t_0 \) is the present epoch, i.e. the age of the universe and \( \alpha \) and \( \beta \) are constants.

The deceleration parameter is defined by
\[ q = -\frac{\dot{R}R}{R^2}. \] \hspace{1cm} (4.15)

Substituting equation (4.13) into equation (4.15), we get

\[ q = \frac{1}{\alpha} - 1. \] \hspace{1cm} (4.16)

Using equations (4.13) and (4.14) into equations (4.10)-(4.12), we get

\[ \bar{p} = -\frac{1}{G} \left[ 2\alpha(\alpha - 1) - 2\beta(\beta - 1) + (\alpha^2 - 4\alpha\beta + \beta^2)t^{-2} + \frac{k}{R^2} \right], \] \hspace{1cm} (4.17)

\[ \rho = \frac{1}{G} \left[ 3\alpha^2 - 3\beta^2 - 6\alpha\beta t^{-2} + \frac{3k}{R^2} \right]. \] \hspace{1cm} (4.18)

Now restricting the distribution with the barotropic equation of state i.e.

\[ p = \gamma \rho, \hspace{0.5cm} -1 \leq \gamma \leq 1, \] \hspace{1cm} (4.19)

and using (4.8), we obtain the explicit form of the physical quantities 'p' and '\xi' as

\[ p = \gamma \frac{1}{G} \left[ 3\alpha^2 - 3\beta^2 - 6\alpha\beta t^{-2} + \frac{3k}{R^2} \right], \] \hspace{1cm} (4.20)

\[ \xi = \frac{1}{3GH} \left[ 3\alpha^2(\gamma + 1) - \beta^2(3\gamma + 1) - \alpha\beta(6\gamma + 4) - 2(\alpha - \beta) t^{-2} + \frac{k}{R^2} (3\gamma + 1) \right]. \] \hspace{1cm} (4.21)

Using equations (4.17) and (4.18) into equation (4.12), we have
\[ \beta(\alpha + \beta)(\alpha - 1) = 0. \]

Now we discuss for cosmological solutions in two different cases:

Case I: \( \beta \neq 0, \alpha = -\beta, \alpha \neq 1 \)

In this case FRW model for bulk viscous fluid in scale covariant theory is obtained as

\[
ds^2 = dt^2 - R_0^2 \left( \frac{t}{t_0} \right)^{-2\beta} \left[ \frac{dr^2}{1 - kr^2} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \right],
\]

\[
\rho = \frac{3}{G} \left[ \frac{kt_0^{-2\beta}}{R_0^2 t^{-2\beta}} + 3\beta^2 \frac{1}{t^2} \right],
\]

\[
p = \frac{3\gamma}{G} \left[ \frac{kt_0^{-2\beta}}{R_0^2 t^{-2\beta}} + 3\beta^2 \frac{1}{t^2} \right],
\]

\[
\xi = \frac{1}{3GH} \left[ \frac{kt_0^{-2\beta}}{R_0^2 t^{-2\beta}} (3\gamma + 1) + \{6\beta^2 (\gamma + 1) + 4\beta \} \frac{1}{t^2} \right],
\]

\[
q = -\frac{(\beta + 1)}{\beta}.
\]

As an example if we take \( \beta = -\frac{1}{2} \), the above equations can be written as
\[ \rho = \frac{3}{G} \left[ \frac{kt_0}{R_0^2} \frac{1}{t} + \frac{3}{4} \frac{1}{t^2} \right], \quad (4.27) \]

\[ p = \frac{3\gamma}{G} \left[ \frac{kt_0}{R_0^2} \frac{1}{t} + \frac{3}{4} \frac{1}{t^2} \right], \quad (4.28) \]

\[ \xi = \frac{1}{3GH} \left[ \frac{kt_0}{R_0^2} (3\gamma + 1) \frac{1}{t} + \frac{3}{2} (\gamma + 1) - 2 \left( \frac{1}{t} \right) - \frac{1}{t^2} \right], \quad (4.29) \]

\[ q = 1. \quad (4.30) \]

We now look three interesting physical cosmological models.

1. False vacuum Model \((\gamma = -1)\)

In this case the physical quantities take the explicit forms

\[ \rho = -p = \frac{3}{G} \left[ \frac{kt_0}{R_0^2} \frac{1}{t} + \frac{3}{4} \frac{1}{t^2} \right], \quad (4.31) \]

\[ \xi = - \frac{2}{3GH} \left[ \frac{kt_0}{R_0^2} \frac{1}{t} + \frac{1}{t^2} \right]. \quad (4.32) \]

2. Stiff fluid model \((\gamma = 1)\)

\[ \rho = p = \frac{3}{G} \left[ \frac{kt_0}{R_0^2} \frac{1}{t} + \frac{3}{4} \frac{1}{t^2} \right], \quad (4.33) \]

and
\[ \xi = \frac{1}{3GH} \left[ 4 \frac{kt_0}{R_0^2} \frac{1}{t} + \frac{1}{t^2} \right]. \tag{4.34} \]

3. Radiation dominated Model \((\gamma = \frac{4}{3})\)

\[ \rho = \frac{3}{G} \left[ \frac{kt_0}{R_0^2} \frac{1}{t} + \frac{3}{4} \frac{1}{t^2} \right], \tag{4.35} \]

\[ p = \frac{1}{G} \left[ \frac{kt_0}{R_0^2} \frac{1}{t} + \frac{3}{4} \frac{1}{t^2} \right], \tag{4.36} \]

\[ \xi = \frac{1}{3GH} \left[ 2 \frac{kt_0}{R_0^2} \frac{1}{t} \right]. \tag{4.37} \]

Case II: \(\beta \neq 0, \alpha = 1, \alpha \neq -\beta\).

In this case FRW model for bulk viscous fluid in scale covariant theory is obtained as

\[ ds^2 = dt^2 - R_0^2 \left( \frac{t}{t_0} \right)^2 \left[ \frac{d\theta^2}{1 - kr^2} + r^2 d\theta^2 + r^2 \sin^2 \theta d\varphi^2 \right], \tag{4.38} \]

\[ \rho = \frac{3}{G} \left[ \frac{kt_0^2}{R_0^2} + (1 - \beta^2 - 2\beta) \right] \frac{1}{t^2}, \tag{4.39} \]

\[ p = \frac{3\gamma}{G} \left[ \frac{kt_0^2}{R_0^2} + (1 - \beta^2 - 2\beta) \right] \frac{1}{t^2}, \tag{4.40} \]
\( \xi = \frac{3\gamma + 1}{3GH} \left[ \frac{kt_o^2}{R_0^2} + (1 - \beta^2 - 2\beta) \right] \frac{1}{t^2} , \) \hspace{1cm} (4.41)

\( q = 0. \) \hspace{1cm} (4.42)

As an example if we take \( \beta = 1, \) the above equations can be written as

\[ \rho = \frac{3}{Gt^2} \left[ \frac{kt_o^2}{R_0^2} - 2 \right], \] \hspace{1cm} (4.43)

\[ p = \frac{3\gamma}{Gt^2} \left[ \frac{kt_o^2}{R_0^2} - 2 \right], \] \hspace{1cm} (4.44)

\[ \xi = \frac{3\gamma + 1}{3Gt^2} \left[ \frac{kt_o^2}{R_0^2} - 2 \right]. \] \hspace{1cm} (4.45)

Again, we discuss the three interesting physical cosmological models:

1. False vacuum model \((\gamma = -1).\)

\[ \rho = -p = \frac{3}{Gt^2} \left[ \frac{kt_o^2}{R_0^2} - 2 \right], \] \hspace{1cm} (4.46)

\[ \xi = \frac{-2}{3Gt^2} \left[ \frac{kt_o^2}{R_0^2} - 2 \right]. \] \hspace{1cm} (4.47)

2. Stiff fluid model \((\gamma = 1).\)
\[ \rho = p = \frac{3}{Gt^2} \left[ \frac{k t_0^2}{R_0^2} - 2 \right], \]  
\[ (4.48) \]

\[ \xi = \frac{4}{3Ght^2} \left[ \frac{k t_0^2}{R_0^2} - 2 \right]. \]  
\[ (4.49) \]

3. Radiation dominated model \((\gamma = \frac{1}{3})\).

\[ \rho = \frac{3}{Gt^2} \left[ \frac{k t_0^2}{R_0^2} - 2 \right], \]  
\[ (4.50) \]

\[ p = \frac{1}{Gt^2} \left[ \frac{k t_0^2}{R_0^2} - 2 \right], \]  
\[ (4.51) \]

\[ \xi = \frac{2}{3Ght^2} \left[ \frac{k t_0^2}{R_0^2} - 2 \right]. \]  
\[ (4.52) \]

4. CONCLUSION

Many authors have been discussed cosmological models with a bulk viscosity. In general relativity, Mohanty and Pradhan (1991) have discussed FRW viscous fluid model. Reddy and Venkateswara Rao (2001) have obtained cosmological models with bulk viscosity in the scalar tensor theory of gravitation proposed by Saez and Ballester (1986). The effect of the bulk viscosity is expected to play an important role in the early evolution of the Universe and is to produce a change in the perfect fluid model. In this work we have obtained FRW models with bulk viscosity in a scale covariant theory of
gravitation by considering power-law function of time for both scale factor and scalar field. We have discussed important models like false vacuum model, stiff fluid model and radiating models for case I: $\beta \neq 0, \alpha = -\beta, \alpha \neq 1$ and case II: $\beta \neq 0, \alpha = 1, \alpha = \beta$. In each case, the energy density, pressure and coefficient of the bulk viscosity vary inversely with time and hence tend to infinity as time tends to zero. When $t \rightarrow \infty$, the model essentially gives an empty universe. The model has a singular origin at time $t = 0$. The model represents an expanding universe with a Big Bang Start, and the radius of universe increases as the age of universe increases. We observed that in case I deceleration parameter $q = 1$ and in case II the model is marginal inflates since $q = 0$. 