Chapter 1

General Introduction

1.1 A cursory look at basic concept of plasma:

Plasma is envisaged to be a distinct phase of matter in contrast to solids, liquids and gases because of its unique properties. It is defined as a collection of neutral and charged particles wherein the particles interact collectively and maintain quasineutrality (Chen, 1983). In 1928, the term 'plasma' was used by an American chemist Irving Langmuir to describe the positive column region in the discharge tube containing almost equal number of ions and electrons so that the resultant space charge was small (Uberoi, 1988). Langmuir, his colleague Lewi Tonks and their collaborators in their quest for developing vacuum tubes that could carry large currents, developed the theory of plasma sheaths - the boundary layers which form between ionized layers and solid surfaces (Chen, 1983). Langmuir also found that certain regions of plasma discharge tube exhibit periodic variation of the electron density, which is referred to as 'Langmuir waves'. In fact, his work formed the basis of plasma research in diverse directions.

The man-made plasmas generated in the gas discharge are found in mercury vapour rectifiers, in electric arcs, and in neon and fluorescent lamps (Uberoi, 1988). Apart from these, man-made plasmas are also seen in explosions, strong shock-wave and in fire flame. The major reason which has attracted many towards research in plasma physics is the fact that it combines many disciplines of physics into a single study (Kunkel and Rosenbluth, 1966). Although it had its beginning in the study of electrical discharges through gases, over the years, interest in plasma has increased manifold chiefly due to its applications in electrical and aeronautical engineering, in geophysics, space science and thermonuclear power generation.

Plasma can be studied by a variety of ways based on the infrastructural constituent. Moreover, they can be dealt with by microscopic as well as macroscopic approaches. The concept of waves in plasmas has attracted a great deal of attention in plasmas because of its exciting features and capability of supporting a rich variety of diagnosis. In the nascent stages, in studying the wave propagation, linear perturbation technique has been used for the evolution of waves and has shown potential interest of being employed in space as well as laboratory plasma (Hartree, 1931; Appleton, 1932). However, the linear theory works as long as the wave amplitude is small and the linear equations are valid, and thus has very limited applicability in many practical situations. As an instance, when the waves are linearly unstable, the wave amplitude grows exponentially according to the prediction of the linear theory. With the growth of amplitude, the linear theories no longer hold good. Furthermore, other processes like particle trapping, resonant wave particle effects, wave decay and higher harmonic generations, which are not taken into consideration in linear theories, may be of importance. In fact, some of these processes are actually behind the saturation of wave amplitude as observed in experiments on unstable modes. Thus the intense study on nonlinear waves and instabilities forms an essential part of the study of plasma systems. The initial stride in overcoming the drawbacks of the linear theories is the study of weakly nonlinear wave propagation, which is indispensable for a thorough understanding of the nonlinear plasma dynamics.

The first documented observation of the nonlinear wave was made by the Scottish scientist Scott Russell (Russell, 1838) while observing 'a heap of water' propagating down a canal. He christened it as 'solitary wave' perhaps because of the fact that such type of wave motion stands alone and apart from other types of oscillatory wave motions. Half a century later, Korteweg and de Vries (Korteweg and de Vries, 1895) gave a mathematical formulation of such nonlinear wave propagation through the derivation of a nonlinear wave equation, which was later coined as K-dV equation. A real breakthrough was achieved from the computer study of the K-dV equation by Zabusky and Kruskal (1965), who found that the individual identity of the steady solitary wave solution of the K-dV equation is preserved notwithstanding the mutual interactions among waves, and they decided to call it 'soliton'. Soliton has been defined as a stationary localized nonlinear wave whose profile can be formed by the balance of weak dispersion and nonlinearity. A reasonably large category of nonlinear waves have been subsequently studied by nonlinear equations like Burger's equation, nonlinear Schrödinger equation (NLS), KdV-ZK equations (Bhatnagar, 1979). These equations have received significant attention in the dynamics with a view to studying the different heuristic features. Consequently numerous methods for obtaining the nonlinear wave solutions have been developed. Out of them, the inverse scattering transforms (Gardner *et al.*, 1967) and a cluster of methods like the steady state method (Davidson, 1972), tanh-method (Das and Sarma, 1998) and sine-cosine method (Yan, 1996) deserve to be mentioned.

As far as plasma is concerned, Washimi and Tanuiti (1966) were probably the pioneers in deriving a nonlinear wave equation in the form of the well-known K-dV equation and that too by the application of a special mathematical formulation known as the reductive perturbation technique. The solution of this equation displays the nature of solitons in plasma which was later confirmed in experiments by Ikezi *et al.* (1970) and Ikezi (1973).

The steady state solution of the K-dV equation portrays the solitary wave formation in hump-shaped profile as a hyperbolic function in plasma and demonstrated that the nonlinear wave propagation in a weak dispersive medium resulted in soliton propagation. In 1966, Sagdeev developed a parallel approach to study the nonlinear waves of arbitrary amplitude which was termed as the Sagdeev potential equation. This equation also yields solitary wave for small amplitude approximation.

1.2 Mathematical formulations for nonlinear wave propagation in plasmas:

With the intention of studying the solitary wave, approaches called reductive perturbation technique (Washimi and Taniuti, 1966) and Sagdeev potential analysis (Sagdeev, 1966) have been made use of. These methods have been derived for simple fully ionized plasma composed of electrons and ions and later on were instrumental in bridging a good agreement between laboratory and space plasmas.

1.2.1 Reductive perturbation approach:

The key object of this approach is to reduce the system of plasma equations into a solvable nonlinear wave equation, which in the long run, might reproduce the salient features of the different kinds of wave propagation. The application of this technique trims down the system of equations to simplified form which successfully derives the characteristic behaviour of the waves. To illustrate this, we consider the basic equations in simplified plasma with electrons and ions as constituents (regarded as two component plasma) as follows:

Equation of Continuity: $\frac{\partial n_\alpha}{\partial t}+\vec{\nabla}.(n_\alpha \vec{v}_\alpha)=0$ (1.1)

Equation of Motion: $m_\alpha n_\alpha\left[\frac{\partial \vec{v}_\alpha}{\partial t}+(\vec{v}_\alpha.\vec{\nabla})\vec{v}_\alpha\right]+\vec{\nabla}P_\alpha=n_\alpha q_\alpha\left[\vec{E}+\frac{\vec{v}_\alpha\times\vec{B}}{c}\right]$ (1.2)

and the Maxwell's equations

$$\vec{\nabla}\times\vec{E}=-\frac{1}{c}\frac{\partial \vec{B}}{\partial t} \tag{1.3}$$

$$\vec{\nabla}\times\vec{B}=\frac{4\pi}{c}\sum q_\alpha n_\alpha \vec{v}_\alpha+\frac{1}{c}\frac{\partial \vec{E}}{\partial t} \tag{1.4}$$

$$\vec{\nabla}.\vec{E} = 4\pi\sum q_\alpha n_\alpha \tag{1.5}$$

$$\vec{\nabla}.\vec{B} = 0 \tag{1.6}$$

where m_α is the mass of the α^{th} (α = i,e) charge moving with velocity $\vec{v}_\alpha$, and having charge q_α and density n_α. $\vec{E}$ and $\vec{B}$ represent the electric and the magnetic fields respectively.

To facilitate the derivation of the K-dV equation in a uniform plasma background free from magnetic field effects, the ions are supposed to be cold and non-drifting relative to the electrons ($T_i << T_e$). Furthermore, the electron inertia effect has been ignored and the isothermal equation of state $P_e = n_e K_B T_e$ (K_B being the Boltzmann constant and T_e being constant) is adopted for the electrons. Based on these basic assumptions, the equation of motion for the electrons, viz. equation (1.2) becomes:

$$e\frac{\partial\phi}{\partial x} - \frac{K_B T_e}{n_e}\frac{\partial n_e}{\partial x} = 0 \tag{1.7}$$

where ϕ is the electrostatic potential which can be derived from $\vec{\nabla}\times\vec{E} = 0 \Rightarrow \vec{E} = -\vec{\nabla}\phi$.

A straightforward integration of equation (1.7) derives the electron density n_e in the form of Boltzmann relation as:

$$n_e = n_0 \exp\left(\frac{e\phi}{K_B T_e}\right) \tag{1.8}$$

with $n_0 = n_{e0} = n_{i0}$.

Equation (1.5) now becomes:

$$\frac{\partial^2\phi}{\partial x^2} = 4\pi e\left[n_0 \exp\frac{e\phi}{K_B T_e} - n_i\right] \tag{1.9}$$

The equations for ions are of the following form:

$$\frac{\partial n_i}{\partial t} + \frac{\partial(n_i v_i)}{\partial x} = 0 \tag{1.10}$$

$$\frac{\partial v_i}{\partial t} + v_i\frac{\partial v_i}{\partial x} = -\frac{e}{m_i}\frac{\partial\phi}{\partial x} \tag{1.11}$$

The plasma parameters (n_α, v_α, ϕ) and the space-time variables are normalized in the following form:

$$\bar{n} = \frac{n_i}{n_0} \tag{1.12a}$$

$$\bar{v} = \frac{v_i}{\left(K_B T_e / m_i\right)^{\frac{1}{2}}} \tag{1.12b}$$

$$\bar{\phi} = \frac{e\phi}{K_B T_e} \tag{1.12c}$$

$$\bar{x} = \frac{x}{\left(K_B T_e / 4\pi n_0 e^2\right)^{\frac{1}{2}}} \tag{1.12d}$$

$$\bar{t} = t\left(4\pi n_0 e^2 \big/ m_i\right)^{\frac{1}{2}} \tag{1.12e}$$

Use of these normalized parameters reduces the basic equations (1.8) - (1.11) to the following dimensionless forms (omitting the bars hereafter):

$$n_e = \exp(\phi) \tag{1.13}$$

$$\frac{\partial^2 \phi}{\partial x^2} = \exp(\phi) - n \tag{1.14}$$

$$\frac{\partial n}{\partial t} + \frac{\partial (nv)}{\partial x} = 0 \tag{1.15}$$

$$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x} = -\frac{\partial \phi}{\partial x} \tag{1.16}$$

In order to obtain the desired nonlinear wave equation by employing the reductive perturbation technique from the governing equations, we assume wave propagation of small but finite amplitude along x-direction. For this purpose, a scale transformation of the following form is used:

$$\xi = \varepsilon^{\alpha}(x - \lambda t) \tag{1.17}$$

$$\eta = \varepsilon^{\beta} t \tag{1.18}$$

Here λ is the unknown phase velocity of the ion-acoustic wave to be determined by mathematical analysis in a self-consistent manner. This scale transformation is referred to as Gardner-Morikawa transformation (Gardner and Morikawa, 1960). Similarly there is a range of space-time co-ordinates depending on the choice of α and β. Later on, different stretching co-ordinates have been used to generate an extensive class of nonlinear wave equations whose solutions highlight various nature of plasma propagation. Correspondingly the plasma parameters have been perturbed as:

$$p=\sum_{\beta}\varepsilon^{\beta}p^{(\beta)} \tag{1.19a}$$

with

$$p^{(\beta)}\equiv\left(n^{(\beta)},v^{(\beta)},\phi^{(\beta)}\right) \tag{1.19b}$$

Thereafter stretching co-ordinates (1.17)-(1.18) and the perturbed series (1.19) have been employed in the equations (1.13)-(1.16). A system of equations have been obtained after balancing the leading orders of ε (taking $\alpha=\frac{1}{2}$ and $\beta=\frac{3}{2}$). First order in ε i.e. $0(\varepsilon)$ evaluates:

$$\phi^{(1)}=n^{(1)} \tag{1.20a}$$

$$\frac{\partial n^{(1)}}{\partial\xi}=\frac{\partial v^{(1)}}{\partial\xi} \tag{1.20b}$$

and $$\frac{\partial v^{(1)}}{\partial \xi}=\frac{\partial \phi^{(1)}}{\partial \xi} \tag{1.20c}$$

Consequently the following relations have been derived:

$$\phi^{(1)}=n^{(1)}=v^{(1)} \tag{1.21a}$$

and $$\lambda=1 \tag{1.21b}$$

The next higher order in ε i.e. $0(\varepsilon^2)$ evaluates:

$$\frac{\partial n^{(1)}}{\partial \eta}-\frac{\partial n^{(2)}}{\partial \xi}+\frac{\partial\left(n^{(1)}v^{(1)}\right)}{\partial \xi}+\frac{\partial v^{(2)}}{\partial \xi}=0 \tag{1.22}$$

$$\frac{\partial v^{(1)}}{\partial \eta}-\frac{\partial v^{(2)}}{\partial \xi}+v^{(1)}\frac{\partial v^{(1)}}{\partial \xi}=-\frac{\partial \phi^{(2)}}{\partial \xi} \tag{1.23}$$

$$\frac{\partial^2 \phi^{(1)}}{\partial \xi^2}=\phi^{(2)}+\frac{\left(\phi^{(1)}\right)^2}{2}-n^{(2)} \tag{1.24}$$

Elimination of $n^{(2)}$ and $v^{(2)}$ from equations (1.22)-(1.24) and subsequent mathematical manipulation along with the results of O (ε), results in the derivation of the desired K-dV equation as:

$$\frac{\partial \phi^{(1)}}{\partial \eta}+A\phi^{(1)}\frac{\partial \phi^{(1)}}{\partial \xi}+B\frac{\partial^3 \phi^{(1)}}{\partial \xi^3}=0 \tag{1.25}$$

with A=1 and B=$\frac{1}{2}$.

By the use of the transformation $\zeta = \xi - U\eta$ (Davidson, 1972; Das, 1975), the solution of the K-dV equation by the steady state method has been found to be:

$$\phi^{(1)} = \frac{3U}{A} \sec h^2 \left[\frac{\zeta}{\delta}\right] \tag{1.26}$$

with $\delta = \sqrt{\frac{4B}{U}}$.

Equation (1.26) displays the profile of a solitary wave with amplitude $\frac{3U}{A}$ and width $\sqrt{\frac{4B}{U}}$.

1.2.2 Nonperturbative approach:

In order to depict arbitrary amplitude wave phenomena, nonperturbative approach has been applied to derive the Sagdeev potential equation. This procedure is founded on the assumption that all the plasma parameters functionally depend on $\xi = x - Mt$, M being the Mach number. We consider the normalized basic equations (1.13) - (1.16) in simplified plasma composed of electrons and ions (regarded as two component plasma).

Using the transformation $\xi = x - Mt$, the equations (1.14)-(1.16) reduce to the following form:

$$\frac{\partial^2\phi}{\partial\xi^2} = \exp\phi - n \tag{1.27}$$

$$-M\frac{dn}{d\xi} + \frac{d}{d\xi}(nv) = 0 \tag{1.28}$$

$$-M\frac{dv}{d\xi} + v\frac{dv}{d\xi} = -\frac{d\phi}{d\xi} \tag{1.29}$$

Integrating equations (1.28)-(1.29) and making use of the boundary conditions $v \to 0, n \to 1$, $\phi \to 0$ as $\xi \to \pm\infty$, the following equations have been derived:

$$v = M\left(1 - \frac{1}{n}\right) \tag{1.30}$$

$$n = \left(1 - \frac{2\phi}{M^2}\right)^{-\frac{1}{2}} \tag{1.31}$$

Elimination of the parameter n from equations (1.27) and (1.31) and subsequent integration derives the Sagdeev potential equation in the following form:

$$\frac{1}{2}\left(\frac{d\phi}{d\xi}\right)^2 + V(\phi, M) = 0 \tag{1.32}$$

where $V(\phi, M) = 1 - \exp\phi + M^2\left[1 - (1 - 2\phi)^{\frac{1}{2}}\right]$ (1.33)

This equation designates an analogue particle motion which was detected initially by Davis *et al.* (1958) in fluid dynamics and later by Sagdeev (1966) in plasma dynamics and as such, was coined in the latter's name as Sagdeev potential equation.

It is evident that the equation will admit a solution only when $V(\phi,M)<0$. The essential boundary conditions for the derivation of the solitary wave profile are:

(i) $\left.\dfrac{dV(\phi,M)}{d\phi}\right|_{\phi=0}<0$

(ii) There exists a non-zero ϕ_m, the maximum (or minimum) of ϕ at which $V(\phi_m,M)=0$.

(iii) $V(\phi,M)$ must be negative in the region $(0,\phi_m)$. The schematic variation of $V(\phi,M)$ has been depicted in Figure 1.1(a).

The soliton solution of the Sagdeev potential equation (1.32) is obtained as:

$$\xi=\int_0^{\phi}\frac{d\phi}{\sqrt{-2V(\phi,M)}} \qquad (1.34)$$

Equation (1.27) can be numerically solved to find the potential profile of a soliton, which is illustrated in Figure 1.1(b).

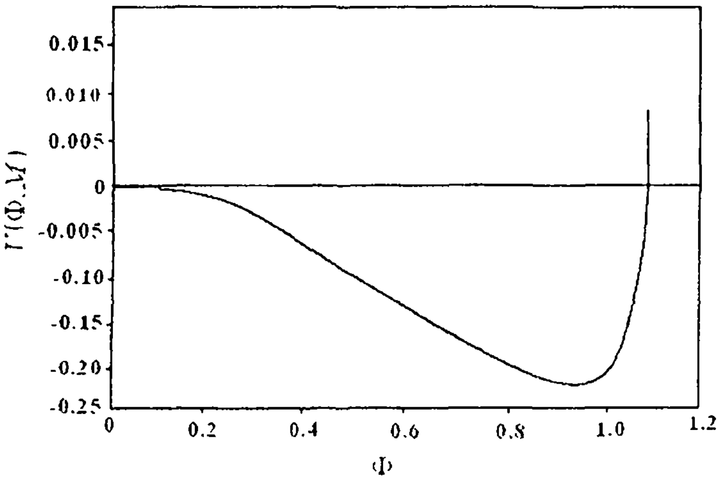

Fig 1.1(a): Variation of V (ϕ, M) with ϕ for M=1.5

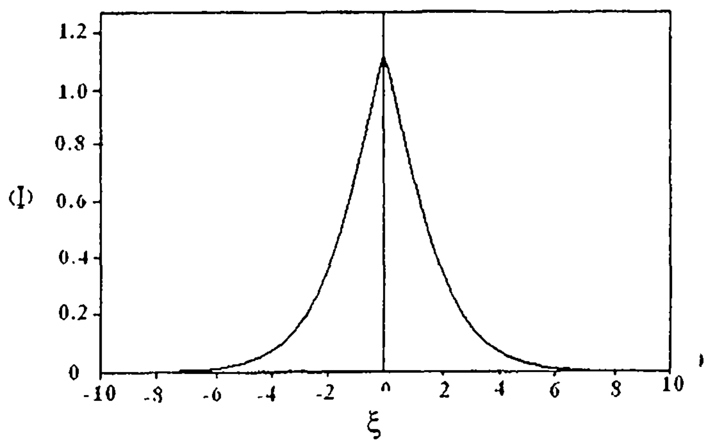

Fig 1.1(b): Variation of soliton profile ϕ with ξ

1.3 Multicomponent Plasma:

The study on nonlinear wave phenomena in plasma was initially confined to simple plasma comprising simply of electrons and a single species of positively charged ions. But as a matter of fact, plasma cannot be simple but consists of multiple charged species, i.e. ions of various kinds along with electrons. Studies based on linear theory to be acquainted with the low frequency wave in the lower ionosphere have revealed the valuable role played by multiple plasma, i.e. plasma with multiple charges and draws conclusion that serious error might creep in the diagnostic procedure if the negative ions are not taken into consideration in plasma dynamics (Das, 1975; Das and Singh, 1992; Das and Sarma, 2000). The presence of negative ions in plasma has unfolded some stirring observations in soliton (Lonngren, 1983; Chauhan *et al.*,1997) which, in turn, became a milestone in the study of nonlinear solitary waves in plasmas - theoretical (Das, 1975, 1976, 1977a, 1977b; Das and Tagare, 1975) as well as experimental (Watanabe, 1984). The study revealed the existence of compressive and rarefactive solitary waves which have been generated because of the interaction of the additional negative ions in plasmas. It further found that there is a critical concentration at which the nonlinearity in the K-dV equation as well as the soilton observation disappears. In the vicinity of this critical density, the amplitude of the ion-acoustic wave becomes infinitely large resulting in the decrease in width. These fascinating observations yielded by the interference of the negative ions (Ludwig *et al.*, 1984; Nakamura and Tsykabayashi, 1984; Nakamura *et al.*, 1985; Nakamura, 1987; Cooney *et al.*, 1991; Williams *et al.*, 1992) have found solid support from laboratory observations over and above the occurrences of such features in space proved by satellite observations (Torven, 1986; Raadu, 1989). Extension of such work has later been performed in plasma with multi-temperature electrons by the trapping of electrons in the potential well in plasma both theoretically (Schamel, 1972, 1973; Goswami and Buti, 1976; Singh and Das, 1989; Das and Sen, 1994) and experimentally (Jones *et al.*, 1975; Nishida and Nagasawa, 1986). The study has also been encompassed in multi-temperature electron plasma embedded by an applied magnetic field (Roychoudhury and Bhattacharya, 1989; Nejoh, 1987; Bharuthram and Shukla, 1992).

1.4 Dusty Plasma:

Dusty plasma is unique multicomponent plasma which comprises of dispersed macroscopic dust charged grains that form a colloidal-type suspension (Sodha and Guha, 1971) in any parent plasma background. The finite size of the dust grains (usually of the order of micron and submicron sizes) effects a qualitative alteration over the conventional charging mechanism of the plasma particles. The dust grains are charged through surface-plasma interaction processes on a macroscopic scale size of the order of the dust surface area. Devoid of any radiative environment, the typically micron-size dust grains normally acquire negative charges (Whipple, 1981; Meyer-Vernet, 1982; Havnes, 1984) to a high order of magnitude with respect to normal electronic charge ($q_d \sim 10^3 - 10^4 e$) (Goertz, 1989; Goertz and Ip, 1984; Barkan *et al.*, 1994; Walch *et al.*, 1995). The mass of the dust grains is also quite high, up to 10^6 - 10^8 times of the mass of a proton.

Even though the shape of the interplanetary dust particles at the instant of formation can be arbitrary, they would have a propensity for being spherical in shape because of phenomena like proton erosion and vaporization. The grain can be deemed as being 'isolated' or 'dust in plasma' which is conditionally defined as $a << \lambda_D < d$, where 'a' and'd' denote the dust grain radius and intermolecular distance respectively, while λ_D is the Debye shielding distance. Besides, when the dust grains in plasma are closely packed, the plasma is called 'dusty plasma' (Whipple, 1981; Goertz and Ip., 1984; Whipple *et al.*, 1985; Goertz, 1989; Mendis and Rosenberg, 1992).

Dusty plasma is ubiquitous and occurs in many natural conditions of astrophysical environments (Mendis and Rosenberg, 1994), including the environments of the earth and other planets as well as in laboratory experiments of basic practical interests. The dynamics of dusty plasma in the solar system was in all probability studied first by Goertz (1989). Afterwards, interest was kindled in laboratory plasma wherein dust grains were found near the container-wall region of fusion plasma (Lipschulz, 1987), plasma discharges (Sheehan *et al.*, 1990; Carlile *et al.*, 1991), dusty crystals, etc.

It is possible to disperse dust grains into plasmas (Sheehan *et al.*, 1990). Numerous experimental studies were carried out on the effects of dust grains on various plasma waves and the low frequency dust acoustic modes (Chu *et al.*, 1994; Barkan *et al.*, 1995; Barkan and Marlino, 1995; D'Angelo, 1995).

The charge on a dust grain in a plasma environment can be computed by the equation:

$$\frac{dQ}{dt} = I_i + I_e + I_{sec} + I_{ph} \tag{1.35}$$

where Q is the grain charge, I_i, I_e, I_{sec} and I_{ph} are the currents generated on the dust by the ions, electrons, secondary and photoelectric emissions respectively. Under the presumption of the Maxwellian plasma and also dust velocity << ion velocity,

$$I_e = -4\pi a^2 n_e \left(\frac{2\pi m_e}{K_B T_e}\right)^{-\frac{1}{2}} \exp\left(\frac{e\phi}{K_B T_e}\right) \tag{1.36}$$

$$I_i = -4\pi a^2 n_i Z_e \left(\frac{2\pi m_i}{K_B T_i}\right)^{-\frac{1}{2}} \left(1 - \frac{Ze\phi}{K_B T_i}\right) \tag{1.37}$$

$$I_{ph} = 4a^2 \Gamma \left(1 + \frac{\max(0, q_d)}{E_{s_{\max}}}\right) \exp\left(-\frac{\max(0, q)}{E_{s_{\max}}}\right) \tag{1.38}$$

$$I_{sec} = 3.7\delta_m I_{e0} n_e \left(\frac{K_B T_e}{2\pi m_e}\right)^{\frac{1}{2}} \exp\left(\frac{e\phi}{K_B T_e}\right) F_5\left(\frac{E_m}{4K_B T_e}\right) \tag{1.39}$$

where $F_5(x) = x^2 \int_0^\infty \left[u^5 \exp\left\{-(xu^2 + u)\right\}\right] du$ (1.40)

Γ designates the current density, E_{max} denotes the maximum energy in the energy distribution of the photoelectrons, ϕ is the grain surface potential, δ_m is a secondary yield parameter (material parameter) in the range 0.5 - 0.3 and E_m (in the range 0.1 - 2.0 KeV) is the energy of the impacting electrons for maximum electron yield $\delta(E)$.

Usually, the dust charge behaves as a dynamical variable (Jana *et al.*, 1993; Varma *et al.*, 1993; Jana *et al.*, 1995), particularly on longer time scale phenomena. The fluctuation in dust charge arises as a result of oscillations on plasma currents flowing into them. Estimation of the relative scaling of the wave time scale (τ_ω) and the dust charging time scale (τ_c) for the plasma gives:

$$\frac{\tau_\omega}{\tau_c} \approx \sqrt{\frac{m_d}{m_i}\frac{n_{i0}}{n_{d0}Z_{d0}}\frac{1}{Z_d}}\left(\frac{a}{\lambda_{di}}\right)\frac{1}{k\lambda_{di}} \approx p\left(\frac{a}{\lambda_{di}}\right)\frac{1}{k\lambda_{di}}, \qquad (1.41)$$

for $k^2\lambda_{di}^2 << 1$.

Here λ_{di} stands for the ion Debye length and k denotes the wave number. The above scaling has been obtained for dust acoustic wave (DAW). $\tau_\omega >> \tau_c$ necessitates p>1, in which case the effect of the steady state charged dust could be insignificant within the quasi-neutrality limit of the parent plasma fluctuations. In the other extreme dust ($\tau_\omega << \tau_c$) also, the average dust charge fluctuation effect could be nominal and this can occur for p<1.

In the intermediate cases, when $\tau_\omega \approx \tau_c$, non-collisional damping crops up due to non-steady state where dust charge fluctuation dynamics are unavoidable. Consequently, for comparable wave time scale and dust charging time scale, noteworthy damping or growth of the plasma mode under consideration should take place. So the theoretical plasma model of the dusty plasmas for collective plasma dynamics and its instabilities entails, in general, the consideration of dust charge fluctuation dynamics. Nevertheless, under certain circumstances, a constant dust charge model could be acceptable for selective dust-plasma parameter domain (Melandso, 1992; Tsytovish and Havnes, 1993; Dwivedi *et al.*, 1996). On the basis of such perception, Rao *et al.* (1990) was the first to derive the dust acoustic wave (DAW) in dusty plasma devoid of the dust charging fluctuations and later was found to be consistent with the experimental work by Barkan *et al.* (1996). Afterwards, many other workers widened the study with such model (Shukla *et al.*, 1991; Shukla and Silin, 1992; D'Angelo, 1993; Mirza *et al.*, 2007). Supplementary works were carried out by several others (Jana *et al.*, 1993; Jana *et al.*, 1995; Rao and Shukla, 1994; Dwivedi and Pandey, 1995) in carrying out studies on the nonlinear coherent structures of the dust acoustic mode in plasmas.

Collective modes of dusty plasma have been investigated by a number of authors (Verheest, 1967; Bliokh and Yaraskenko, 1985; de Angelis *et al.*, 1988). Goertz (1989) treated linear electrostatic waves in planetary environments in a brief manner. Rao *et al.* (1990) took up dusty plasma where dust particles are considered as plasma species with constant negative dust charge in the company of Boltzmannian electrons and ions. D'Angelo (1990) studied low frequency electrostatic modes in magnetized dusty plasma. Shukla and Silin (1992) revealed a new dust-ion-acoustic (DIA) mode, which is the typical ion -acoustic mode, altered by the presence of static negatively charged dust grains. Pilipp *et al.* (1987), Shukla *et al.* (1991), Shukla (1992), Rao, (1993a, 1993b, 1993c, 1995) have extensively explored the electromagnetic modes in dusty plasma.

The fact that dust charge may fluctuate in dusty plasma was first discussed by Melandso *et al.* (1993). In addition, the outcome of such fluctuation in plasma dynamics has been considered by Varma *et al.* (1993), Jana *et al.* (1995), D'Angelo (1994), Bhatt and Pandey (1994), Ma and Yu (1994) and Shukla and Resendes (2000).

1.5 Rotating Plasma:

On the basis of several investigations on the effect of Coriolis force on the problems of hydrodynamic and hydro magnetic instability, Chandrasekhar (1953) opined that this force, however small it may be in magnitude, may play a dominant role in cosmic phenomena. Ever since, a number of workers have tried to examine the nature of wave propagation in rotating plasmas with the inclusion of the Coriolis force. Chandrasekhar had pointed out that the superposition of the Coriolis force and magnetic force does not yield just the superposed results; instead the interaction of these two forces gives rise to various new phenomena. Later, Lehnert (1954) and Hide (1966) numerically estimated the ratio of the Coriolis force to the magnetic force for the plasmas in the Sun's interior and for conducting fluids in the Earth's core respectively, and showed that the Coriolis force can play a dominant role, particularly in the presence of low magnetic fields. In fact, these studies were able to generate a lot of interest in the study of wave propagation in space plasmas in the presence of Coriolis force. Side by side, even in laboratory plasmas, the role of rotation in wave propagation became important from the viewpoint of comprehending some fundamental traits of the rotating plasmas.

An exhaustive study on the effect of Coriolis force on the Alfvén waves has been made by using the Magneto Hydro Dynamic (MHD) model of description for plasmas. In a study of low frequency Alfvén waves, Lehnert (1954) found that the inclusion of Coriolis force entails modification of certain aspects of Alfvén's theory of sunspots. The Alfvén wave was found to be divided into left and right circularly polarized waves - a finding of great significance in solar physics for disturbances of sunspot size. Moreover, the stabilization of flute disturbances by the Coriolis force in rotating cylindrical plasma was scrutinized by Lehnert (1962). Tandon and Bajaj (1966) conducted a study on the effect of Hall current, compressibility and the displacement current on wave propagation in a rarefied rotating plasma including the Coriolis force by considering the two-fluid plasma model.

Interest of the scientific community was kindled for conducting theoretical and experimental investigations on rotating plasmas because of its great importance in problems encountered in rotating plasma devices in laboratory as well as in space plasmas. Uberoi and Das (1970) showed that the Coriolis force has a tendency to produce an equivalent magnetic field effect as and when the ionized medium rotates. It is worth mentioning here that studies were also conducted by Mamun (1994), Mofiz and Podder (1987) and Mushtaq and Shah (2005) wherein a multispecies rotating magnetised plasma has been considered. Again, based on the observation on rotating star, especially with high rotation neutron star or pulsar, Mamun (1994) has studied the evolution of small amplitude waves showing the formation of narrow wave packet with the increase of rotation which causes the soliton radiation termed as pulsar radiation. The effect of immobile dust on stability of magnetized rotating plasma has been analyzed by Mikhailovskii *et al.* (2008). They have shown that the electric field which appears in the presence of dust, leads to instability of the magnetized rotating plasma called the dust-induced rotational instability (DRI). Thus, as a whole, the rotation has been found to play an effective role in wave propagation.

1.6 Presentation of the thesis:

On the basis of the mathematical formulations for nonlinear wave propagation in plasmas as mentioned, the thesis encompasses some theoretical studies in ideal plasmas with a view to discerning their salient features, and that too, through the derivation of the K-dV and Sagdeev potential equations. The thesis has been arranged with some problems which have been dealt with in the various chapters.

Chapter 2 encounters a unique problem in conformity with ideal plasma. A simple unmagnetized plasma consisting of isothermal electrons and singly charged positive cold ions, rotating around an axis at an angle with the propagation direction of the acoustic mode has been considered. The nonlinear wave mode has been derived as an equivalent Sagdeev potential equation. The key in this chapter lies in the application of a special method, known as the hyperbolic method, for the study of nonlinear wave propagation in plasma dynamics. Moreover, under small amplitude approximation, the nonlinear plasma acoustic mode has been utilized to study the evolution of soliton propagation. The main stress here has been laid on the interaction of the Coriolis force on the changes of the coherent soliton structures. The solitary wave solution is found to yield the different natures of solitons, viz. compressive and rarefactive solitons as well as its collapses or explosions along with soliton dynamics. The study unfolds some exciting observations in exhibiting a narrow wave packet with the simultaneous generation of high electric pressure and the growth of high energy which ultimately demonstrates the phenomena of radiating soliton. The results obtained are interesting in the light of the fact that it shows that the rotation, however small in magnitude in plasma, plays an effective role in exhibiting the radiative soliton even in simple plasma.

This chapter is based on the work published in Physics of Plasmas **13**, 082303 (2006) [G. C. Das and A. **Nag**].

In **Chapter 3**, in sequel to the earlier work, isothermal plasma contaminated with micron-sized dust charges, has been taken into consideration. This has been done with the intention of studying the nonlinear acoustic wave in rotating dusty plasma augmented through the derivation of a modified Sagdeev potential equation. The fact that small interaction causes the interaction of the Coriolis force leads to complexity in the derivation of the nonlinear wave equation. As a result, in contrast to the steady state method, the nonlinear wave equation has been successfully solved by the use of the hyperbolic method. Primary attention has been paid to the changes on the evolution and propagation of soliton, and the variation caused by the dusty plasma constituents as well as by the Coriolis force has been highlighted. The overall studies exhibited soliton propagation in a small rotating plasma device and showed the effective role of their existence through the variation of amplitude and Mach number. In order to lend support to the theoretical investigations, numerical plasma parameters have been taken for yielding the inherent features of solitons. The results obtained could be of interest as an advanced theoretical knowledge for explaining the salient features of soliton radiation.

This chapter is based on the work published in Physics of Plasmas **14**, 083705 (2007) [G. C. Das and **A. Nag**].

In **Chapter 4**, a study based on reductive perturbation technique of ion-acoustic wave propagation through the derivation of Korteweg-de Vries(K-dV) equation has been conducted in magnetized plasma with negative ions. In the previous chapters, the ions were taken to be positively charged. But the observations in the space and the laboratory plasmas might be erroneous unless a proper existence of negative ions is taken into account. The findings of the study indicate that the presence of negative ions changes the nonlinearity whereas the applied magnetic field causes variation in dispersiveness, as a result of which the formation of soliton exhibits different nature on soliton propagation. To derive the soliton solution, the hyperbolic method has been employed successfully to solve the modified K-dV equation and have shown the characteristics variation on the formation of soliton profile with intensification of high energy region coined as soliton radiation region in plasmas. The observations have then been evaluated with some symbolic computation relying on future experiments to detect those effects which appear due to the parametric variation.

In the previous chapters, under small amplitude approximation, charge neutrality condition has been made use of in deriving the solitary waves. In **Chapter 5**, which is the penultimate chapter of the thesis, a general approach has been pursued where Sagdeev-like potential equation has been derived without taking recourse to the charge neutrality condition. In contrast to the earlier observations on double layers in simple plasma, this chapter considers the interaction of slowly rotating dusty plasma contaminated with the dust charging effect. Because of complexity of the ideal plasma model, the derivation of Sagdeev potential equation fails by usual pseudopotential analysis. Therefore, anchored in a modified mathematical approach, the nonlinear wave equation has been derived. Later, under small amplitude approximations, the modified Sagdeev potential equation of different forms has been derived to exhibit salient features of double layers. Further, parametric analysis has been carried out which shows the various natures of double layers which purely depends on the concentrations as well as on the thermal effects along with slow rotation.

Chapter 6, being the concluding chapter, deals with the essence of the overall views emanating from the works carried out in the thesis. It also points out the limitations of the present work and highlights some areas in which related further research work may be undertaken.