Chapter 5

Evolution of double layers in rotating dusty plasmas with varying dust charged grains

5.1 Introduction:

In the earlier chapters, the formation of solitons in various plasma configurations has been discussed. Moreover, the dust particles were assumed to possess constant charge. In the present chapter, a further study has been made on double layers in rotating dusty plasma along with the consideration of the fluctuation of dust charge.

Dust dynamics has emerged as a pioneer area of research interest in many astrophysical problems ranging from the earth's atmosphere to the interstellar spaces. It plays a vital role in the various environments of planetary rings (Goertz and Linhua, 1988; Northrop *et al.*, 1989), cometary tails (Havnes, 1988; Goertz, 1989), earth's magnetosphere (Hoyle and Wickramsinghe, 1988) as well as in many applications generated in laboratory plasmas (Barkan *et al.*, 1995). The dynamics of dust grains in the vicinity of Mars has been exhaustively considered by Horanyi *et al.* (1990). In the backdrop of the Martian plasma environment, they showed the significance of electromagnetic force for the grain dynamics of Mars and revealed that the big grains form a narrow dust belt while the smaller ones occupy a much wider halo affected by the solar wind. Interaction of the dust charging process with plasma oscillation is very important (Nejoh, 1997; Cui and Goree, 1994) in the dynamics of dusty plasmas because of its common appearance in various plasma environments as mentioned.

Various investigations have been endeavoured to understand the characteristic features of nonlinear waves in plasmas contaminated by the dust charged grains (Rao *et al.*, 1990). The dust charged grains, under the effect of electro-magnetic field, leads to rich physical phenomena in astrophysical and space plasmas. The evolution of nonlinear waves in the form of solitary waves, low frequency shock waves as well as double layers has been highlighted in dusty plasma. The mechanism of charging on dust grains have been studied by many authors (Verheest *et al.*, 1997). In the continuous charging model, it is presumed that the charge on the dust grains is determined by the current collected from the plasma and this current is continuous with time. But there are two causes for the fluctuation of the charge on the dust grain: one of the causes is turbulence and the other is the spatial or temporal variations in the surrounding plasma properties. An analysis of the dust charge fluctuation effect was carried out by Zubia *et al.* (2007) for finding the damping of the electromagnetic kinetic Alfvén waves which revealed that the dust charge fluctuation damping is mainly due to the perpendicular motion of electrons and ions in the dusty magnetoplasma.

A double layer is a nonlinear electrostatic structure. Though from external viewpoint, the structure is charge neutral, yet internally the double layer is made up of two layers of opposite charges. They are usually thin structures embedded in undisturbed plasma. A vital trait of double layer is that particles entering in it are directly accelerated by the net potential structure which has kindled the study of double layer as particle accelerators. Thus it is pertinent to examine the possible existences of such double layer potential structures in dusty plasmas which are considered a thrust area in space and laboratory plasma studies.

In the present chapter, an attempt has been made to study the double layers in plasma rotating with a uniform angular velocity about an axis making an angle with the direction of plasma-acoustic wave propagation. The motivation here lies in the fact that the Coriolis force generated from rotation has a tendency to produce an equivalent magnetic field effect as and when the ionized medium rotates (Uberoi and Das, 1970). It needs mention here that although detailed theoretical studies of double layers associated with the plasma acoustic mode have been made in unmagnetized plasma, yet because of the complexity caused by the applied magnetic field, the double layers in magnetized plasmas have not been extensively studied so far. The variation of plasma constituents on the nature of dust acoustic wave and dust acoustic double layers in the presence of magnetic field has unearthed new findings contrary to the observation centred round unmagnetized bodies found in spaces. It is found that under certain plasma configuration, the effect of dust charge variation plays an important role to show the double layers in the dynamical system. Again the study on the dust acoustic wave or double layers through the augmentation of the Zakharov - Kuznetsov equation (El-Taibany and Sabry, 2005) reveals the existence of compressive and rarefactive solitary waves or double layers with the variation of plasma parameters.

This chapter, in sequel to the earlier observations in Chapters 2 and 3, adopts dusty plasma containing variable charge grains embedded in an external equivalent magnetic field produced due to slow rotation. A different approach has been followed for the derivation of Sagdeev potential-like equation (Sagdeev, 1966). Investigation for the existence of small amplitude dust acoustic double layers are being carried out in a slowly rotating plasma containing massive dust grains with variable negative charges in a background of hot electrons and ions. The study could generate interest for garnering advanced knowledge on rotating stars.

5.2 Basic equations and derivation of Sagdeev potential equation:

To study the evolution of double layers, simple plasma with electrons and ions contaminated with charge varying dust grains has been considered. The plasma is assumed to be rotating with a uniform angular velocity $\vec{\Omega}$ along an axis at an angle θ with the direction of wave propagation. The basic equations, governing the plasma dynamics, are the equations of continuity and motion. It is possible that other forces might play effective roles in the dynamical system, but they are considered to be negligible here because of the intention of solely studying the effect of Coriolis force. The basic equations, with respect to a rotating frame of reference (Uberoi and Das, 1970), can be written as:

$$\frac{\partial n_\alpha}{\partial t} + \vec{\nabla}.(n_\alpha \vec{v}_\alpha) = 0 \qquad (5.1)$$

$$\frac{\partial \vec{v}_\alpha}{\partial t}+\left(\vec{v}_\alpha.\vec{\nabla}\right)\vec{v}_\alpha=\frac{q_\alpha}{m_\alpha}\vec{E}+\frac{q_\alpha}{cm_\alpha}\left(\vec{v}_\alpha\times\vec{H}_T\right) \tag{5.2}$$

where the subscript $\alpha = i,e$ represents respectively for ions and electrons and $\vec{H}_T=\vec{H}_0+2\vec{\Omega}\left(\frac{cm_\alpha}{q_\alpha}\right)=\vec{H}_0+\vec{\bar{H}}$ is the modified magnetic field which represents the combination of an applied magnetic field $\vec{H}_0$ and the generation of a similar effect by the Coriolis force $\vec{\bar{H}}$. m_α is the mass of the α - charge moving with the velocity v_α having density n_α, and q_α is the charge. $\vec{E}$ is the electric field which derives the electrostatic potential ϕ from the assumption $\vec{\nabla}\times\vec{E}=0$, i.e. $\vec{E}=-\vec{\nabla}\phi$, which has been applied later on. The present study has been conducted in the absence of the applied magnetic field $\vec{H}_0$ with a view to scrutinizing the effect of Coriolis force in isolation.

To investigate the dust acoustic wave (DAW) in rotating plasma, the dynamics of electrons and ions can be approximated to the following Boltzmannian relations as (Kotsarenko *et al.*, 1998):

$$n_e = n_{e0}\exp\left(\frac{e\phi}{KT_e}\right) \tag{5.3}$$

$$n_i = n_{i0}\exp\left(-\frac{e\phi}{KT_i}\right) \tag{5.4}$$

where n_{e0} and n_{i0} represent respectively the electron and ion number densities at the equilibrium state. T_e and T_i are their corresponding temperatures with the assumption $T_\alpha\,(\alpha = e,i) >> T_d,\ T_d$ being the temperature of the dust particles.

It may be mentioned that the simplification to the Boltzmannian relations are based on the nature of thermal effects and wave propagation (Das *et al.*, 1995). As the consequence of the existences of low frequency waves, the phase velocity $\frac{\omega}{k}$ could be much less than the thermal velocity of electrons and ions, i.e. $\frac{\omega}{k} << v_{T_{e,i}}$, where $v_{T_{e,i}}$ are the thermal velocities of electrons and ions. Consequently, to know the importance of nonlinear waves in rotating plasma, the momentum equations of electrons and ions can, without any ambiguity, assume the Boltzmannian relations.

On the basis of the above mentioned simplifications, in order to study the dust acoustic double layers in rotating plasma, the governing equations, i.e. the equations of continuity and motion are simplified in the following normalized form:

$$\frac{\partial n_d}{\partial t} + \frac{\partial (n_d v_x)}{\partial x} = 0 \tag{5.5}$$

$$\frac{\partial v_x}{\partial t} + v_x \frac{\partial v_x}{\partial x} = -\frac{q_d}{m_d}\frac{\partial \phi}{\partial x} + \frac{q_d v_y \eta \sin\theta}{m_d c} \tag{5.6}$$

$$\frac{\partial v_y}{\partial t} + v_x \frac{\partial v_y}{\partial x} = -\frac{q_d v_x \eta \sin\theta}{m_d c} + \frac{q_d v_z \eta \cos\theta}{m_d c} \tag{5.7}$$

$$\frac{\partial v_z}{\partial t} + v_x \frac{\partial v_z}{\partial x} = -\frac{q_d v_y \eta \cos\theta}{m_d c} \tag{5.8}$$

which are supplemented by the Poisson's equation:

$$\frac{\partial^2 \phi}{\partial x^2} = 4\pi e\left[n_e - n_i - \frac{q_d n_d}{e}\right] \tag{5.9}$$

where m_d is the mass of the dust grain moving with the velocity $v_d(v_x, v_y, v_z)$. n_d and q_d denote respectively the number density and the charge of the dust grains and $\eta = 2\Omega$.

The dust grains may be charged due to the current produced from ions and electrons (Salimullah *et al.,* 2003; Tsytovich *et al.*, 2003) as discussed earlier, and accordingly the charge q_d of the dust grains changes according to:

$$\frac{dq_d}{dt} = I_i + I_e \tag{5.10}$$

where

$$I_i = \pi a^2 e\sqrt{\frac{8T_i}{\pi m_i}}n_i\left(1 - \frac{eq_d}{aT_i}\right) \tag{5.11}$$

and

$$I_e = -\pi a^2 e\sqrt{\frac{8T_e}{\pi m_e}}n_e \exp\left(\frac{eq_d}{aT_e}\right) \tag{5.12}$$

are the currents generated by ions and electrons to the grains.

This equation holds well under the condition $a << \lambda_d << \lambda_{mfp}$ (where λ_d is the plasma Debye length and λ_{mfp} is the mean free path for ion or electron collisions) and that too under the "orbital-motion-limited" (Bernstein and Rainowitz, 1959; Chen, 1965;

Barnes *et al.*, 1992) theory. This condition is satisfied by most of astrophysical and laboratory dusty plasmas. It is also true that for $\rho >> a$ (where ρ is the mean gyroradii), the effect of magnetic field on charging dust is unimportant (Hutchinson, 1987; Jana *et al.*, 1995; Das *et al.*, 2001). In this backdrop, the above charging model has been used in the magnetized dusty plasma.

All the plasma parameters in the basic equations are normalized in the following form:

$$\bar{x} = \frac{x}{\rho}, \quad \bar{t} = \frac{t}{\alpha\omega_d}, \quad \bar{\phi} = \frac{e\phi}{KT_e},$$

$$\bar{n}_d = \frac{n_d}{n_{d0}}, \quad \bar{z}_d = \frac{z_d}{z_{d0}}, \quad \bar{v}_d = \frac{v_d}{c_d}$$

The following notations:

$$\rho = \frac{c_d}{\alpha\omega_d}, \; \omega_d = \frac{ez_0\Omega}{cm_d}, \; \alpha^2 = \frac{1}{\gamma\delta_1 + \delta_2}, \; \delta_1 = \frac{n_{i0}}{z_{d0}n_{d0}}, \; \delta_2 = \frac{n_{e0}}{z_{d0}n_{d0}}, \; \delta_1 - \delta_2 = 1$$

$$\gamma = \frac{T_e}{T_i}, \; \lambda_d^2 = \frac{KT_e}{4\pi e^2 n_{d0} z_{d0}}, \; c_d^2 = \frac{n_{d0} z_{d0}^2 KT_e T_i}{m_d\left(n_{i0}T_e + n_{e0}T_i\right)}$$

have been used for further simplification.

Further relative electron concentration χ is defined as $\chi = \frac{n_{eo}}{n_{io}}$, which results in

$$\delta_1 = \frac{1}{1-\chi}.$$

This facilitates the writing of Eqs. (5.3)-(5.10) in the following form:

$$n_e = \exp(\phi) \tag{5.13}$$

$$n_i = \exp(-\gamma\phi) \tag{5.14}$$

$$\frac{\partial \bar{n}_d}{\partial \bar{t}} + \frac{\partial(\bar{n}_d \bar{v}_x)}{\partial \bar{x}} = 0 \tag{5.15}$$

$$\frac{\partial \bar{v}_x}{\partial \bar{t}} + \bar{v}_x \frac{\partial \bar{v}_x}{\partial \bar{x}} = \frac{\bar{z}_d}{\alpha^2} \frac{\partial \bar{\phi}}{\partial \bar{x}} - \frac{\bar{z}_d \bar{v}_y \sin\theta}{\alpha} \tag{5.16}$$

$$\frac{\partial \bar{v}_y}{\partial \bar{t}} + \bar{v}_x \frac{\partial \bar{v}_y}{\partial \bar{x}} = \frac{\bar{z}_d \bar{v}_x \sin\theta}{\alpha} - \frac{\bar{z}_d \bar{v}_z \cos\theta}{\alpha} \tag{5.17}$$

$$\frac{\partial \bar{v}_z}{\partial \bar{t}} + \bar{v}_x \frac{\partial \bar{v}_z}{\partial \bar{x}} = \frac{\bar{z}_d \bar{v}_y \cos\theta}{\alpha} \tag{5.18}$$

$$\frac{\lambda_d^2}{\rho^2} \frac{\partial^2 \bar{\phi}}{\partial \bar{x}^2} = \bar{z}_d \bar{n}_d + \delta_2 \bar{n}_e - \delta_1 \bar{n}_i \tag{5.19}$$

$$\frac{d\bar{z}_d}{d\bar{t}} = \frac{\alpha \omega_d \bar{n}_{d0}}{e\bar{z}_{d0}} [I_i + I_e] \tag{5.20}$$

The currents generated by ions and electrons to the dust grains take the following form:

$$I_i = \pi a^2 e \sqrt{\frac{8T_i}{\pi m_i}} \delta_1 \bar{n}_i \left(1 - \gamma \bar{z}_d\right) \tag{5.21}$$

and

$$I_e = -\pi a^2 e \sqrt{\frac{8T_e}{\pi m_e}} \delta_2 \bar{n}_e \exp\left(-\bar{z}_d\right) \tag{5.22}$$

In order to display the double layers through the derivation of the Sagdeev potential equation, the pseudopotential method is employed where the dependent variables are assumed to vary functionally as ξ where $\xi = x - Mt$ with respect to a moving frame with M being the wave velocity normalized by c_d. Use of this transformation reduces the basic equations, i.e. Eqs.(5.15)-(5.20) to the following form (omitting the bars hereafter):

$$-M \frac{dn_d}{d\xi} + \frac{d\left(n_d v_x\right)}{d\xi} = 0 \tag{5.23}$$

$$-M \frac{dv_x}{d\xi} + v_x \frac{dv_x}{d\xi} = \frac{z_d}{\alpha^2} \frac{d\phi}{d\xi} - \frac{z_d v_y \sin\theta}{\alpha} \tag{5.24}$$

$$-M \frac{dv_y}{d\xi} + v_x \frac{dv_y}{d\xi} = \frac{z_d v_x \sin\theta}{\alpha} - \frac{z_d v_z \cos\theta}{\alpha} \tag{5.25}$$

$$-M \frac{dv_z}{d\xi} + v_x \frac{dv_z}{d\xi} = \frac{z_d v_y \cos\theta}{\alpha} \tag{5.26}$$

$$\frac{\lambda_D^{\ 2}}{\rho^2}\frac{d^2\phi}{d\xi^2} = n_d z_d + \delta_2 \exp(\phi) - \delta_1 \exp(\gamma\phi) \tag{5.27}$$

$$-M\frac{dz_d}{d\xi} = \frac{\alpha\omega_d n_{d0}}{ez_{d0}}\left[I_i + I_e\right] \tag{5.28}$$

Now integration of Eqs.(5.23), (5.24), (5.26)) along with the use of appropriate boundary conditions as $v_d \to 0, \phi \to 0, n_d \to 1$ as $\xi \to \pm\infty$ yields:

$$v_x = M\left(1 - \frac{1}{n_d}\right) \tag{5.29}$$

$$v_y = \frac{M}{\alpha \sin\theta}\left(1 + \frac{\alpha^2 M^2}{z_d n_d^{\ 3}}\frac{dn_d}{d\phi}\right)\frac{d\phi}{d\xi} \tag{5.30}$$

$$v_z = M\cot\theta\left(\frac{1}{n_d} - 1\right) - \frac{\cot\theta}{M}\int_0^{\phi} n_d z_d d\phi \tag{5.31}$$

Using Eqs. (5.29)- (5.31), Eq. (5.25) is evaluated as:

$$\frac{d}{d\xi}\left[A(n_d, z_d)\frac{d\phi}{d\xi}\right] = z_d - n_d z_d - \frac{n_d z_d \cos^2\theta}{\alpha^2 M^2}\int_0^{\phi} n_d z_d d\phi \tag{5.32}$$

where $A(n_d, z_d) = 1 + \frac{\alpha^2 M^2}{z_d n_d^{\ 3}}\frac{dn_d}{d\phi}$.

Equation (32) is amended to the form:

$$\frac{1}{2}\frac{d}{d\phi}\left[A(n_d,z_d)\frac{d\phi}{d\xi}\right]^2 = A(n_d,z_d)\left[z_d - n_d z_d - \frac{n_d z_d \cos^2\theta}{\alpha^2 M^2}\int_0^{\phi} n_d z_d d\phi\right] \tag{5.33}$$

along with the simplification on $A(n_d,z_d)$ in eqn (5.33) as:

$$\begin{aligned} A(n_d,z_d) &= 1+\frac{\alpha^2 M^2}{z_d n_d^3}\left[\frac{d}{d\phi}\left(\frac{z_d n_d}{z_d}\right)\right] \\ &= 1+\frac{\alpha^2 M^2}{(z_d n_d)^3}\left[z_d\frac{d(z_d n_d)}{d\phi}-(z_d n_d)\frac{dz_d}{d\phi}\right] \end{aligned} \tag{5.34}$$

It is not possilble to simplify the above equation (5.34) to the standard form of the Sagdeev potential equation which is done in the case of a very simple model to study the double layers. The presence of $A(n_d,z_d)$ leads to complexity and the equation fails to derive the Sagdeev potential equation, even though some investigations have been made earlier with crude approximation (Das *et al.*, 1995; Das *et al.*, 2001). The present investigation makes an attempt to execute the observation without any simplification.

First, the parameter $\varepsilon = \frac{\lambda_d^2}{\rho^2}$, in Poisson equation Eq.(5.27), is considered to be very small, based on which the terms containing higher powers of ε are neglected, and then Eq.(5.27) can be written as:

$$z_d n_d = \delta_1 n_1 - \delta_2 n_e + \varepsilon\frac{d^2\phi}{d\xi^2} \tag{5.35}$$

which can be simplified further as

$$z_d n_d = \delta_1 n_1 - \delta_2 n_e + \frac{\varepsilon}{2}\frac{d}{d\phi}\left(\frac{d\phi}{d\xi}\right)^2 + C_1 \tag{5.36}$$

C_1 is the integration constant, which is calculated out to be zero followed from the use of boundary condition $\frac{d\phi}{d\xi} = 0$ at $\phi = 0$.

Thus Eq.(5.36) takes the form:

$$z_d n_d = \delta_1 n_1 - \delta_2 n_e + \frac{\varepsilon}{2}\frac{d}{d\phi}\left(\frac{d\phi}{d\xi}\right)^2 \tag{5.37}$$

The charging time is calculated roughly as $\tau_{ch} = \left(\frac{dq_d}{q_d dt}\right)^{-1} \equiv \frac{q_{d0}}{I_0}$, while the hydrodynamic time scale is of the order of $\tau_d \equiv (\alpha\omega_d)^{-1}$ where $I_0 = -\pi a^2 e\left(\frac{8T_e}{\pi m_e}\right)^{1/2} n_{e0}$ is the electron grain current evaluated at $q_d = 0$. It is found that $\tau_{ch} << \tau_d$, due to which $I_i + I_e \approx 0$ as an offshoot of the works done by Ma and Liu (1997). Consequently, from equations (5.20)-(5.22), we get

$$z_d = 1 - k\phi \quad \text{with} \quad k = \left(1 - \gamma^2\right) \tag{5.38}$$

Taking all the assumptions and modifications into consideration, Eq.(5.32) takes the following form:

$$\frac{d}{d\phi}\left[A(n_d,z_d)\frac{d\phi}{d\xi}\right]\frac{d\phi}{d\xi}=z_d-\left[\delta_1 n_i-\delta_2 n_e+\frac{\varepsilon}{2}\frac{d}{d\phi}\left(\frac{d\phi}{d\xi}\right)^2\right]-$$
$$\left[\delta_1 n_i-\delta_2 n_e+\frac{\varepsilon}{2}\frac{d}{d\phi}\left(\frac{d\phi}{d\xi}\right)^2\right]B\int_0^{\phi}\left[\delta_1 n_i-\delta_2 n_e+\frac{\varepsilon}{2}\frac{d}{d\phi}\left(\frac{d\phi}{d\xi}\right)^2\right]d\phi \tag{5.39}$$

which has been further simplified as:

$$\frac{dA}{d\phi}\left[\frac{d\phi}{d\xi}\right]^2+\frac{A}{2}\frac{d}{d\phi}\left[\frac{d\phi}{d\xi}\right]^2=z_d-\left[\delta_1 n_i-\delta_2 n_e+\frac{\varepsilon}{2}\frac{d}{d\phi}\left(\frac{d\phi}{d\xi}\right)^2\right]$$
$$-\left[\delta_1 n_i-\delta_2 n_e+\frac{\varepsilon}{2}\frac{d}{d\phi}\left(\frac{d\phi}{d\xi}\right)^2\right]B\int_0^{\phi}\left[\delta_1 n_i-\delta_2 n_e+\frac{\varepsilon}{2}\frac{d}{d\phi}\left(\frac{d\phi}{d\xi}\right)^2\right]d\phi \tag{5.40}$$

where $B=\dfrac{\cos^2\theta}{\alpha^2 M^2}$.

For evaluating the double layers in dusty plasmas by deriving the equivalent Sagdeev potential equation by adopting the Das *et al.* (2001) approach, $\left[\frac{d\phi}{d\xi}\right]^2$ in Eq.(5.40) is expanded up to the third term as:

$$\left[\frac{d\phi}{d\xi}\right]^2=A_1\phi^2-A_2\phi^3+A_3\phi^4=-V(\phi) \tag{5.41}$$

where A_1,A_2,A_3 are constants which can be computed in a self-consistent manner. Equation (5.41) is used in Equation (5.40) and thereafter the similar orders in ϕ are balanced, which leads to find some algebraic relations. These relations, after some mathematical manipulations, evaluates A_1,A_2,A_3 in the following forms:

$$A_1 = \frac{(\gamma\delta_1 + \delta_2 - k - B)}{\left[1 - \alpha^2 M^2 (\gamma\delta_1 + \delta_2 - k) + \varepsilon\right]} \tag{5.42}$$

$$A_2 = \frac{1}{3}\frac{(\gamma^2\delta_1 - \delta_2) - 4A_1\alpha^2 M^2 (\gamma\delta_1 + \delta_2)(\gamma\delta_1 + \delta_2 - k) - B(\gamma\delta_1 + \delta_2 - A_1\varepsilon)}{\left[1 - \alpha^2 M^2 (\gamma\delta_1 + \delta_2 - k) + \varepsilon\right]} \tag{5.43}$$

$$A_3 = \frac{(\gamma^3\delta_1 + \delta_2) - 15A_2\alpha^2 M^2 (\gamma\delta_1 + \delta_2)(\gamma\delta_1 + \delta_2 - k)}{12\left[1 - \alpha^2 M^2 (\gamma\delta_1 + \delta_2 - k) + \varepsilon\right]} - \frac{B\left[4(\gamma^2\delta_1 - \delta_2) + 3(\gamma\delta_1 + \delta_2)^2 - 6A_1\varepsilon(\gamma\delta_1 + \delta_2) - 15\varepsilon A_2\right]}{12\left[1 - \alpha^2 M^2 (\gamma\delta_1 + \delta_2 - k) + \varepsilon\right]} \tag{5.44}$$

From Eq.(5.41), it follows that the Sagdeev potential equation could be derived to know the characteristic features of double layers as:

$$V(\phi) = -\left[A_1\phi^2 - A_2\phi^3 + A_3\phi^4\right] \tag{5.45}$$

Finally, the condition for double layer, viz. $V(\phi) = 0$ and $\frac{\partial V}{\partial \phi} = 0$ at $\phi = \phi_m$ is imposed on Eq. (5.45), which enables to obtain

$$A_1 = A_2\phi_m - A_3\phi_m^2 \tag{5.46}$$

$$A_1 = \frac{3A_2}{2}\phi_m - 2A_3\phi_m^{\ 2} \tag{5.47}$$

Solutions of eqs. (5.46)- (5.47) give

$$A_2 = 2A_3\phi_m \tag{5.48}$$

$$A_1 = A_3\phi_m^2 \tag{5.49}$$

Ultimately Eq. (5.41) can be expressed as:

$$\left[\frac{d\phi}{d\xi}\right]^2 = A_3\phi_m^2\phi^2 - 2A_3\phi_m\phi^3 + A_3\phi^4 \tag{5.50}$$

which derives

$$\left[\frac{d\phi}{d\xi}\right]^2 = A_3\phi^2(\phi - \phi_m)^2 \tag{5.51}$$

along with the necessary conditions for the existence of double layer as $A_2^2 = 4A_1A_3$, $A_3 > 0$. Integration of Eq.(5.51) derives the profile of the double layer as:

$$\phi = \frac{1}{2}\phi_m\left(1 \pm \tanh\frac{\varepsilon}{\delta}\right) \tag{5.52}$$

where $\phi_m = \dfrac{A_2}{2A_3}$ is the amplitude and $\delta = \dfrac{2A_3}{A_2}$ is the width of double layer.

5.3 Results and Discussions:

From Eq. (5.51), it is evident that the existence of the double layer depends on the values of A_3 and that too, with $A_3 > 0$. Again from Eq. (5.49), $A_1 > 0$ is also implied. The compressive and rarefactive double layers are possible through the variation of the signs of A_2.

Fig 5.1 depicts the variation of A_1, A_2 and A_3 with relative electron concentration χ for parameters $\theta=60^o$, M=0.6, $\gamma=2$, $\varepsilon=0.001$ [curve (a)]; $\theta=50^o$, M=0.7, $\gamma=1.5$, $\varepsilon=0.001$ [curve (b)]; $\theta=60^o$, M=0.6, $\gamma=1.5$, $\varepsilon=0.001$ [curve (c)]. The figure shows that both A_1 and A_3 are positive conforming to the condition $A_3 > 0$ for the existence of double layers. From Fig. 5.1(i), it is seen that A_1 is always positive while Fig. 5.1(ii) shows that A_3 is also positive for different values of the plasma parameters. On the other hand, A_2 can assume negative values which are evident from Fig. 5.1(ii). Rarefactive solitary double layers are exhibited for the negative values of A_2. The study can be extended in the light of the fact that compressive solitary waves have been observed in the magnetosphere by Viking spacecraft (Ishihara and Sato, 2001) and Freja scientific satellite (Wu *et al.*, 1996).

For a more detailed study, in Fig. 5.2, the variation of A_1, A_2 and A_3 with temperature ratio γ is plotted for parameters $\theta=30^o$, M=0.6, $\chi=0.5$, $\varepsilon=0.001$ [curve (a)]; $\theta=30^o$, M=0.6, $\chi=0.7$, $\varepsilon=0.001$ [curve (b)]; $\theta=30^o$, M=0.6, $\chi=0.9$, $\varepsilon=0.001$ [curve (c)]. From Fig. 5.2(i) and 5.2(iii) respectively, it is clear that A_1 and A_3 are positive for small values of γ pointing further to the fact that for higher temperature ratio, double layers are not seen. The negative values of A_2 for small values of γ, as seen from Fig. 5.2 (ii), indicate the presence of rarefactive double layers.

In Fig. 5.3, the variation of $V(\phi)$ with ϕ for different values of χ is portrayed using plasma parameters $\theta=50^o$, M=0.6, $\gamma=1.2$, $\chi=0.1$, $\varepsilon=0.001$ [curve (a)]; $\theta=50^o$, M=0.6,

γ=1.2, χ=0.3, ε=0.001 [curve (b)]; θ=50°, M=0.6, γ=1.2, χ=0.5, ε=0.001 [curve (c)]. In addition to that, in Fig. 5.4, the variation of V(ϕ) with ϕ for different values of γ is sketched using plasma parameters θ=60°, M=0.6, γ=1.1, χ=0.5, ε=0.001 [curve (a)]; θ=60°, M=0.6, γ=1.2, χ=0.5, ε=0.001 [curve (b)]; θ=60°, M=0.6, γ=1.5, χ=0.5, ε=0.001 [curve (c)]. From both the figures, it is seen that rarefactive double layers are displayed.

A closer look at Fig. 5.1(iii) and 5.2 (iii) unfolds that it is possible that A_3 may be very small as a consequence of which the amplitude of the wave becomes very large which, in turn, makes it very narrow. This means that as $A_3 \rightarrow 0$, a very narrow wave packet is formed. This results in the continuous growth of high electric field and formation of enormous electric pressure in the soliton profile wherein density depression occurs. The generation of this high energy electric field accelerates the electrons to charge up the neutral or dust particles and gives high current to produce a radiation.

The present analysis has been successful in deriving the nonlinear wave equation and its solution yields double layers of various natures supported by plasma parameters which could be available in laboratory and space plasmas. Compressive as well as rarefactive double layers can co-exist in plasma by the control of dust charge fluctuation effect supplemented by the variation of plasma constituents. The effectiveness of slow rotation on the existence of double layers has also emanated from the study. The results obtained could be of interest with the possibility of its application in both laboratory and space.

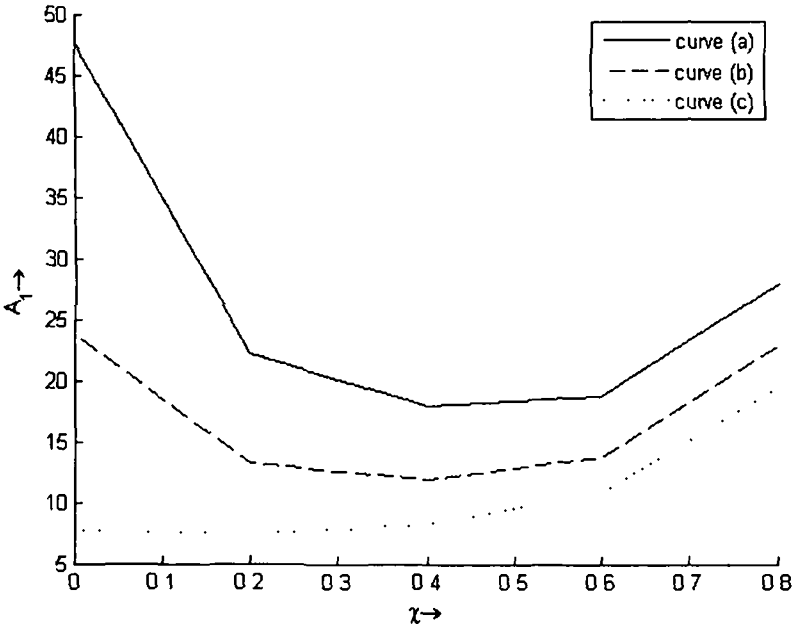

Fig. 5.1(i): Variation of A_1 with relative electron concentration χ for $\theta=60^o$, $M=0.6$, $\gamma=2$, $\varepsilon=0.001$ [curve (a)]; $\theta=50^o$, $M=0.7$, $\gamma=1.5$, $\varepsilon=0.001$ [curve (b)]; $\theta=60^o$, $M=0.6$, $\gamma=1.5$, $\varepsilon=0.001$ [curve (c)].

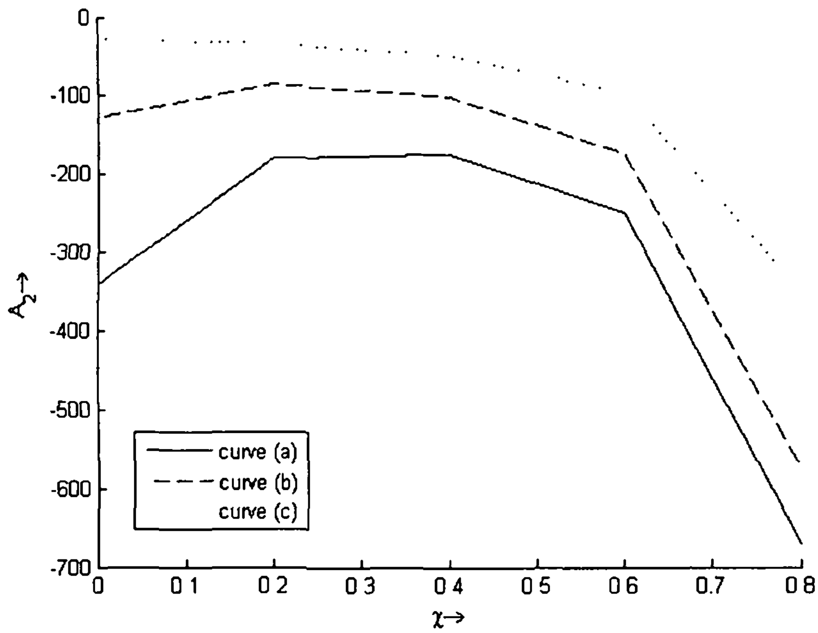

Fig. 5.1(ii): Variation of A_2 with relative electron concentration χ for $\theta=60^o$, $M=0.6$, $\gamma=2$, $\varepsilon=0.001$ [curve (a)]; $\theta=50^o$, $M=0.7$, $\gamma=1.5$, $\varepsilon=0.001$ [curve (b)]; $\theta=60^o$, $M=0.6$, $\gamma=1.5$, $\varepsilon=0.001$ [curve (c)].

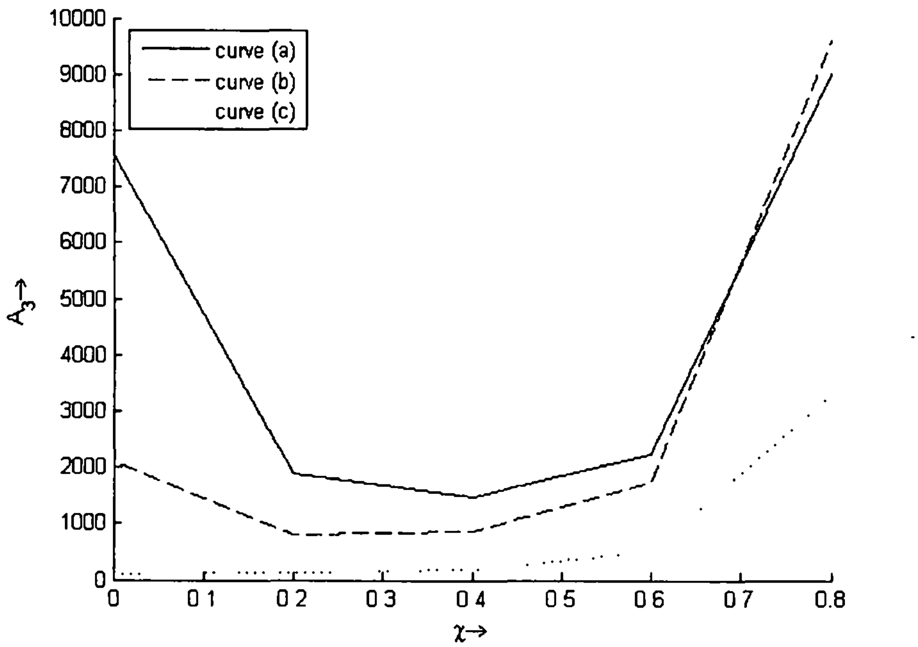

Fig. 5.1(iii): Variation of A_3 with relative electron concentration χ for $\theta=60^o$, M=0.6, $\gamma=2$, $\varepsilon=0.001$ [curve (a)]; $\theta=50^o$, M=0.7, $\gamma=1.5$, $\varepsilon=0.001$ [curve (b)]; $\theta=60^o$, M=0.6, $\gamma=1.5$, $\varepsilon=0.001$ [curve (c)].

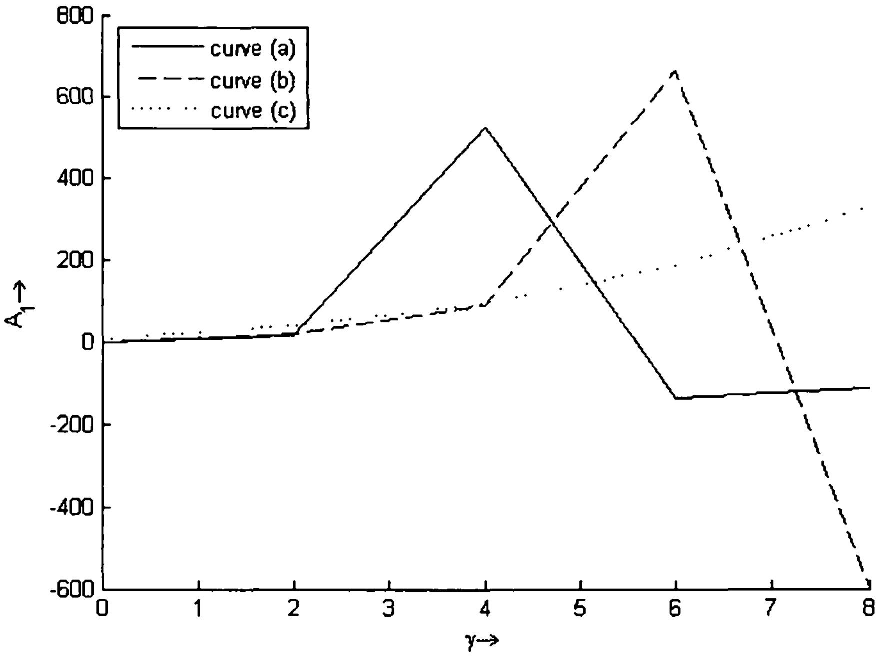

Fig. 5.2(i): Variation of A_1 with temperature ratio γ for $\theta=30^o$, M=0.6, $\chi=0.5$, $\varepsilon=0.001$ [curve (a)]; $\theta=30^o$, M=0.6, $\chi=0.7$, $\varepsilon=0.001$ [curve (b)]; $\theta=30^o$, M=0.6, $\chi=0.9$, $\varepsilon=0.001$ [curve (c)].

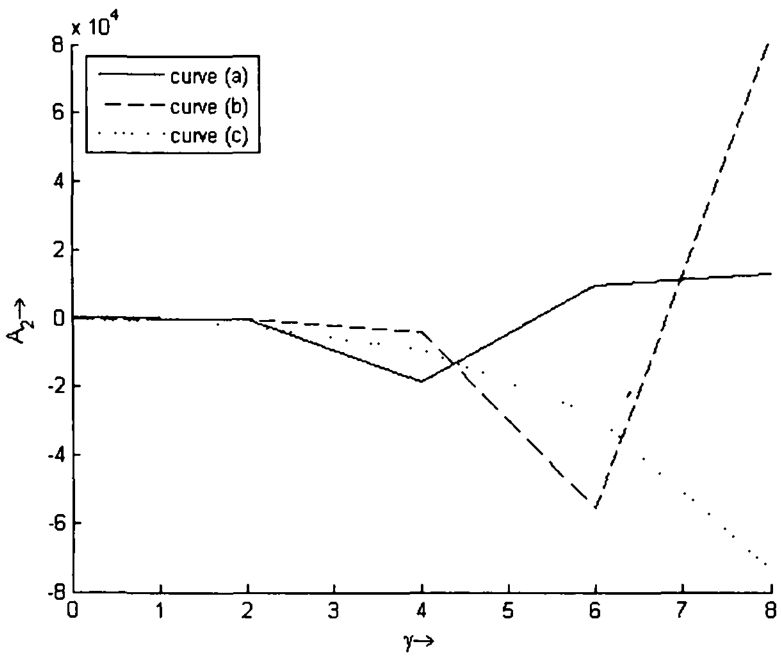

Fig. 5.2(ii): Variation of A_2 with temperature ratio γ for $\theta=30°$, $M=0.6$, $\chi=0.5$, $\varepsilon=0.001$ [curve (a)]; $\theta=30°$, $M=0.6$, $\chi=0.7$, $\varepsilon=0.001$ [curve (b)]; $\theta=30°$, $M=0.6$, $\chi=0.9$, $\varepsilon=0.001$ [curve (c)].

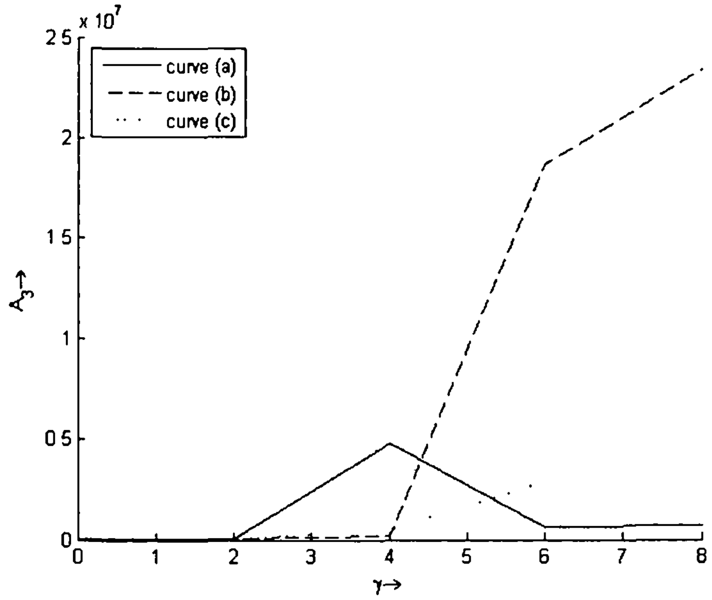

Fig. 5.2(iii): Variation of A_3 with temperature ratio γ for $\theta=30°$, $M=0.6$, $\chi=0.5$, $\varepsilon=0.001$ [curve (a)]; $\theta=30°$, $M=0.6$, $\chi=0.7$, $\varepsilon=0.001$ [curve (b)]; $\theta=30°$, $M=0.6$, $\chi=0.9$, $\varepsilon=0.001$ [curve (c)].

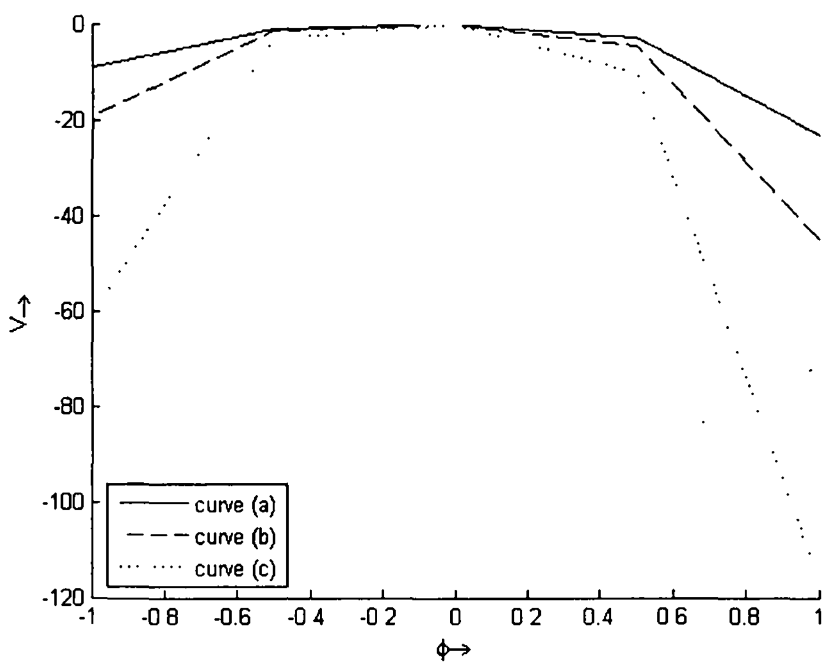

Fig. 5.3: Variation of V(ϕ) with ϕ for θ=50°, M=0.6, γ=1.2, χ=0.1, ε=0.001 [curve (a)]; θ=50°, M=0.6, γ=1.2, χ=0.3, ε=0.001 [curve (b)]; θ=50°, M=0.6, γ=1.2, χ=0.5, ε=0.001 [curve (c)].

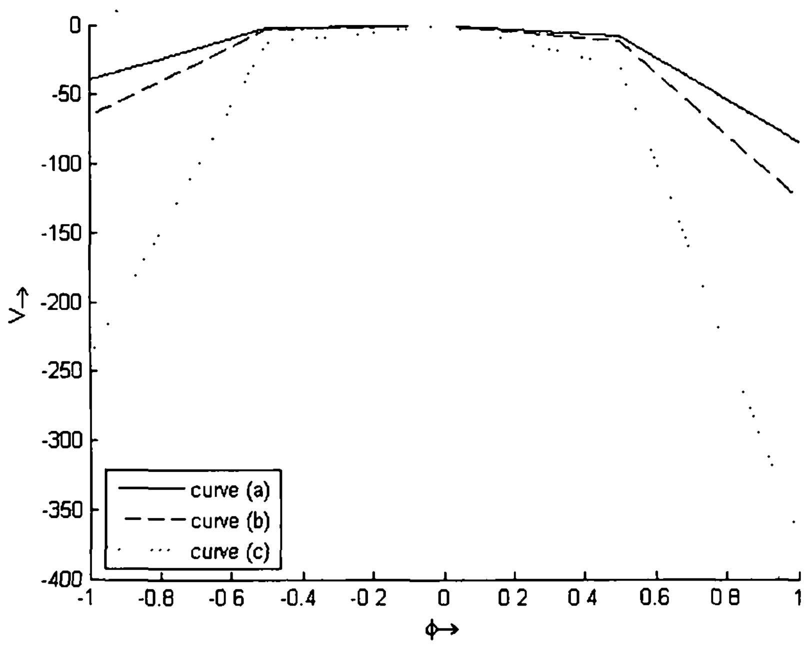

Fig. 5.4: Variation of V(ϕ) with ϕ for θ=60°, M=0.6, γ=1.1, χ=0.5, ε=0.001 [curve (a)]; θ=60°, M=0.6, γ=1.2, χ=0.5, ε=0.001 [curve (b)]; θ=60°, M=0.6, γ=1.5, χ=0.5, ε=0.001 [curve (c)].