Chapter 4

Transient behaviour of ion-acoustic waves in magnetized plasma with negative ions

4.1 Introduction:

In the earlier chapters, the formation of solitons in unmagnetized plasma consisting of isothermal electrons and singly charged positive cold ions has been discussed. In sequel to that, in this chapter, a study has been made on solitons in magnetized plasma containing an additional component, i.e. negative ions.

The study has been conducted in the light of the established fact that the observations in the space and the laboratory plasmas might be erroneous unless a proper existence of negative ions is taken into account, which in fact has a large effect on the features of wave propagation (Das and Uberoi, 1972) in the lower ionosphere. They have shown the appreciable effect of the presence of negative ions on the plasma diagnostics. Later Das (1972, 1975) and Das and Tagare (1975) studied the ion-acoustic solitary waves in multicomponent plasma with negative ions due to which compressive and rarefactive solitons were observed and achieved a milestone in soliton dynamics.

Subsequently such observations were proved experimentally in the laboratory (Watanabe, 1984; Nakamura and Tsykabayashi, 1985; Nakamura *et al.*, 1985). Some other studies have also been made (Duan and Zhao, 1999) but the actual results have been missed (Nishikawa and Kaw, 1975; Rao and Varma, 1978; Shivamoggi, 1988; Das and Singh, 1992) because of the adoption of mathematical simplifications. In the recent past, Das and Sarma (2000) derived the K-dV soliton and showed that when the solitary wave enters the inhomogeneous media, it grows quite fast by generating energy from the dynamical system.

4.2 Basic Equations and Derivation of Nonlinear Wave Equation:

For conducting a study on solitary wave propagation, a magnetized plasma consisting of positive (subscript α) and negative (subscript β) ions in the presence of isothermal electrons (under the assumption T_e>> T_i) has been considered.. Here it is assumed that the magnetic field $\vec{H}$ lies in the xz-plane, making an angle θ with the direction of wave propagation along x-direction. The basic equations governing the plasma dynamics are the equations of continuity and motion.

The basic normalised equations for the positively charged ions are:

$$\frac{\partial n_\alpha}{\partial t}+\frac{\partial}{\partial x}(n_\alpha u_{\alpha x})=0 \qquad (4.1)$$

$$\frac{\partial u_{\alpha x}}{\partial t}+u_{\alpha x}\frac{\partial u_{\alpha x}}{\partial x}=-\frac{\partial \phi}{\partial x}+u_{\alpha y}\sin\theta \qquad (4.2)$$

$$\frac{\partial u_{\alpha y}}{\partial t}+u_{\alpha x}\frac{\partial u_{\alpha y}}{\partial x}=-u_{\alpha x}\sin\theta+u_{\alpha z}\cos\theta \tag{4.3}$$

$$\frac{\partial u_{\alpha z}}{\partial t}+u_{\alpha x}\frac{\partial u_{\alpha z}}{\partial x}=-u_{\alpha y}\cos\theta \tag{4.4}$$

The basic normalised equations for the negatively charged ions are:

$$\frac{\partial n_{\beta}}{\partial t}+\frac{\partial}{\partial x}\left(n_{\beta}u_{\beta x}\right)=0 \tag{4.5}$$

$$\frac{\partial u_{\beta x}}{\partial t}+u_{\beta x}\frac{\partial u_{\beta x}}{\partial x}=\frac{1}{Q}\frac{\partial \phi}{\partial x}-\frac{1}{Q}u_{\beta y}\sin\theta \tag{4.6}$$

$$\frac{\partial u_{\beta y}}{\partial t}+u_{\beta x}\frac{\partial u_{\beta y}}{\partial x}=\frac{1}{Q}u_{\beta x}\sin\theta-\frac{1}{Q}u_{\beta z}\cos\theta \tag{4.7}$$

$$\frac{\partial u_{\beta z}}{\partial t}+u_{\beta x}\frac{\partial u_{\beta z}}{\partial x}=\frac{1}{Q}u_{\beta y}\cos\theta \tag{4.8}$$

where $Q=\dfrac{m_{\beta}}{m_{\alpha}}$.

The isothermality of the plasma, under the condition of neglecting the electron inertia, is assumed to be given by the Boltzmann relation as:

$$n_e=\exp(\Phi) \tag{4.9}$$

All the above equations are supplemented by the Poisson's equation:

$$\frac{\lambda_d^2}{\rho^2}\frac{\partial^2\phi}{\partial x^2} = n_e - n_\alpha + n_\beta \tag{4.10}$$

where λ_d is the Debye length.

n_e is the electron density normalized by n_0, the density at equilibrium position, $\Phi = \frac{e\phi}{kT_e}$ is the normalized electrostatic potential, where T_e is the electron temperature, $n_j\,(j=\alpha,\beta)$ is the number density of ions normalized by n_{j0}. The velocity u has been normalized by the ion acoustic speed $c_i = \sqrt{\left(\frac{kT_e}{m_i}\right)}$, where m_i is the mass of the ions. The space x and time t are respectively normalized by the ion gyro radius $\rho = \frac{c_i}{\omega_{ci}}$ and $(\omega_{ci})^{-1}$, where $\omega_{ci} = \frac{eH}{cm_i}$ is the ion-gyrofrequency.

To derive the K-dV equation, the following stretching co-ordinates ξ and τ as $\xi = \varepsilon^{\frac{1}{2}}(x - vt)$ and $\tau = \epsilon^{\frac{3}{2}}\, t$ have been used. Therefore,

$$\frac{\partial}{\partial x} = \varepsilon^{\frac{1}{2}}\frac{\partial}{\partial \xi} \tag{4.11a}$$

and

$$\frac{\partial}{\partial t} = -v\varepsilon^{\frac{1}{2}}\frac{\partial}{\partial \xi} + \varepsilon^{\frac{3}{2}}\frac{\partial}{\partial \tau} \tag{4.11b}$$

where v is the phase velocity of the ion-acoustic wave and ε measures the size of the perturbation. The dependent variables have been expanded as:

$$n_{\alpha,\beta} = n_{\alpha,\beta_0} + \varepsilon n_{\alpha,\beta_1} + \varepsilon^2 n_{\alpha,\beta_2} + ... \quad (4.12a)$$

$$u_{\alpha,\beta x} = \varepsilon u_{\alpha,\beta x_1} + \varepsilon^2 u_{\alpha,\beta x_2} + ... \quad (4.12b)$$

$$u_{\alpha,\beta y} = \varepsilon^{\frac{3}{2}} u_{\alpha,\beta y_1} + \varepsilon^{\frac{5}{2}} u_{\alpha,\beta y_2} + ... \quad (4.12c)$$

$$u_{\alpha,\beta z} = \varepsilon u_{\alpha,\beta z_1} + \varepsilon^2 u_{\alpha,\beta z_2} + ... \quad (4.12d)$$

$$\phi = \varepsilon\phi_1 + \varepsilon^2\phi_2 + ... \quad (4.12e)$$

where $n_{\alpha 0} = \alpha$ and $n_{\beta 0} = 1 - \alpha$.

Substitution of (4.11) and (4.12) into the system of equations (4.1) - (4.8) and (4.10) give:

$$\left(-v\varepsilon^{\frac{1}{2}}\frac{\partial}{\partial\xi} + \varepsilon^{\frac{3}{2}}\frac{\partial}{\partial\tau}\right)\left(\alpha + \varepsilon n_{\alpha_1} + \varepsilon^2 n_{\alpha_2} + ...\right) + \left(\alpha + \varepsilon n_{\alpha_1} + \varepsilon^2 n_{\alpha_2} + ...\right)\varepsilon^{\frac{1}{2}}\frac{\partial}{\partial\xi}\left(\varepsilon u_{\alpha x_1} + \varepsilon^2 u_{\alpha x_2} + ...\right) + \left(\varepsilon u_{\alpha x_1} + \varepsilon^2 u_{\alpha x_2} + ...\right)\varepsilon^{\frac{1}{2}}\frac{\partial}{\partial\xi}\left(\varepsilon n_{\alpha_1} + \varepsilon^2 n_{\alpha_2} + ...\right) = 0 \quad (4.13)$$

$$\left(-v\varepsilon^{\frac{1}{2}}\frac{\partial}{\partial\xi}+\varepsilon^{\frac{3}{2}}\frac{\partial}{\partial\tau}\right)\left(\varepsilon u_{\alpha x_1}+\varepsilon^2 u_{\alpha x_2}\ldots\ldots..\right)+$$
$$\left(\varepsilon u_{\alpha x_1}+\varepsilon^2 u_{\alpha x_2}\ldots\ldots..\right)\varepsilon^{\frac{1}{2}}\frac{\partial}{\partial\xi}\left(\varepsilon u_{\alpha x_1}+\varepsilon^2 u_{\alpha x_2}+\ldots\ldots\right)= \tag{4.14}$$
$$-\varepsilon^{\frac{1}{2}}\frac{\partial}{\partial\xi}\left(\varepsilon\phi_1+\varepsilon^2\phi_2+\ldots.\right)+\left(\varepsilon^{\frac{3}{2}}u_{\alpha y_1}+\varepsilon^{\frac{5}{2}}u_{\alpha y_2}+\ldots\ldots\right)\sin\theta$$

$$\left(-v\varepsilon^{\frac{1}{2}}\frac{\partial}{\partial\xi}+\varepsilon^{\frac{3}{2}}\frac{\partial}{\partial\tau}\right)\left(\varepsilon^{\frac{3}{2}}u_{\alpha y_1}+\varepsilon^{\frac{5}{2}}u_{\alpha y_2}\ldots\ldots..\right)+$$
$$\left(\varepsilon u_{\alpha x_1}+\varepsilon^2 u_{\alpha x_2}\ldots\ldots..\right)\varepsilon^{\frac{1}{2}}\frac{\partial}{\partial\xi}\left(\varepsilon^{\frac{3}{2}}u_{\alpha y_1}+\varepsilon^{\frac{5}{2}}u_{\alpha y_2}+\ldots\ldots\right)= \tag{4.15}$$
$$\left(\varepsilon u_{\alpha z_1}+\varepsilon^2 u_{\alpha z_2}\ldots\ldots..\right)\cos\theta-\left(\varepsilon u_{\alpha x_1}+\varepsilon^2 u_{\alpha x_2}\ldots\ldots..\right)\sin\theta$$

$$\left(-v\varepsilon^{\frac{1}{2}}\frac{\partial}{\partial\xi}+\varepsilon^{\frac{3}{2}}\frac{\partial}{\partial\tau}\right)\left(\varepsilon u_{\alpha z_1}+\varepsilon^2 u_{\alpha z_2}\ldots\ldots..\right)+$$
$$\left(\varepsilon u_{\alpha x_1}+\varepsilon^2 u_{\alpha x_2}\ldots\ldots..\right)\varepsilon^{\frac{1}{2}}\frac{\partial}{\partial\xi}\left(\varepsilon u_{\alpha z_1}+\varepsilon u_{\alpha z_2}+\ldots\ldots\right)= \tag{4.16}$$
$$-\left(\varepsilon^{\frac{3}{2}}u_{\alpha y_1}+\varepsilon^{\frac{5}{2}}u_{\alpha y_2}\ldots\ldots..\right)\cos\theta$$

and

$$\left(-v\varepsilon^{\frac{1}{2}}\frac{\partial}{\partial\xi}+\varepsilon^{\frac{3}{2}}\frac{\partial}{\partial\tau}\right)\left[(1-\alpha)+\varepsilon n_{\beta_1}+\varepsilon^2 n_{\beta_2}+\ldots\ldots..\right]+$$
$$\left[(1-\alpha)+\varepsilon n_{\beta_1}+\varepsilon^2 n_{\beta_2}+\ldots\ldots.\right]\varepsilon^{\frac{1}{2}}\frac{\partial}{\partial\xi}\left(\varepsilon u_{\beta x_1}+\varepsilon^2 u_{\beta x_2}+\ldots\ldots\right) \tag{4.17}$$
$$+\left(\varepsilon u_{\beta x_1}+\varepsilon^2 u_{\beta x_2}+\ldots\ldots.\right)\varepsilon^{\frac{1}{2}}\frac{\partial}{\partial\xi}\left(\varepsilon n_{\beta_1}+\varepsilon^2 n_{\beta_2}+\ldots..\right)=0$$

$$\left(-v\varepsilon^{\frac{1}{2}}\frac{\partial}{\partial\xi}+\varepsilon^{\frac{3}{2}}\frac{\partial}{\partial\tau}\right)\left(\varepsilon u_{\beta x_1}+\varepsilon^2 u_{\beta x_2}\ldots\ldots\ldots\right)+$$
$$\left(\varepsilon u_{\beta x_1}+\varepsilon^2 u_{\beta x_2}\ldots\ldots\ldots\right)\varepsilon^{\frac{1}{2}}\frac{\partial}{\partial\xi}\left(\varepsilon u_{\beta x_1}+\varepsilon^2 u_{\beta x_2}+\ldots\ldots\right)= \tag{4.18}$$
$$\varepsilon^{\frac{1}{2}}\frac{\partial}{\partial\xi}\left(\varepsilon\phi_1+\varepsilon^2\phi_2+\ldots.\right)-\left(\varepsilon^{\frac{3}{2}}u_{\beta y_1}+\varepsilon^{\frac{5}{2}}u_{\beta y_2}+\ldots\ldots\right)\sin\theta$$

$$\left(-v\varepsilon^{\frac{1}{2}}\frac{\partial}{\partial\xi}+\varepsilon^{\frac{3}{2}}\frac{\partial}{\partial\tau}\right)\left(\varepsilon^{\frac{3}{2}}u_{\beta y_1}+\varepsilon^{\frac{5}{2}}u_{\beta y_2}\ldots\ldots\ldots\right)+$$
$$\left(\varepsilon u_{\beta x_1}+\varepsilon^2 u_{\beta x_2}\ldots\ldots\ldots\right)\varepsilon^{\frac{1}{2}}\frac{\partial}{\partial\xi}\left(\varepsilon^{\frac{3}{2}}u_{\beta y_1}+\varepsilon^{\frac{5}{2}}u_{\beta y_2}+\ldots\ldots\right)= \tag{4.19}$$
$$\left(\varepsilon u_{\beta x_1}+\varepsilon^2 u_{\beta x_2}\ldots\ldots\ldots\right)\sin\theta-\left(\varepsilon u_{\beta z_1}+\varepsilon^2 u_{\beta z_2}\ldots\ldots\ldots\right)\cos\theta$$

$$\left(-v\varepsilon^{\frac{1}{2}}\frac{\partial}{\partial\xi}+\varepsilon^{\frac{3}{2}}\frac{\partial}{\partial\tau}\right)\left(\varepsilon u_{\beta z_1}+\varepsilon^2 u_{\beta z_2}\ldots\ldots\ldots\right)+$$
$$\left(\varepsilon u_{\beta x_1}+\varepsilon^2 u_{\beta x_2}\ldots\ldots\ldots\right)\varepsilon^{\frac{1}{2}}\frac{\partial}{\partial\xi}\left(\varepsilon u_{\beta z_1}+\varepsilon^2 u_{\beta z_2}+\ldots\ldots\right)= \tag{4.20}$$
$$\left(\varepsilon^{\frac{3}{2}}u_{\beta y_1}+\varepsilon^{\frac{5}{2}}u_{\beta y_2}\ldots\ldots\ldots\right)\cos\theta$$

$$K\varepsilon\frac{\partial^2}{\partial\xi^2}\left(\varepsilon\phi_1+\varepsilon^2\phi_2+\ldots\ldots.\right)$$
$$=\left[1+\left(\varepsilon\phi_1+\varepsilon^2\phi_2+\ldots\ldots\right)+\frac{1}{2}\left(\varepsilon\phi_1+\varepsilon^2\phi_2+\ldots\ldots\right)^2+\ldots..\right] \tag{4.21}$$
$$-\left(\alpha+\varepsilon n_{\alpha_1}+\varepsilon^2 n_{\alpha_2}+\ldots..\right)+\left[\left(1-\alpha\right)+\varepsilon n_{\beta_1}+\varepsilon^2 n_{\beta_2}+\ldots..\right]$$

where $K=\frac{\lambda_d^2}{\rho^2}$.

Equating the lowest orders of ε $\left(i.e.\ \varepsilon^{\frac{3}{2}}\right)$ from equations (4.13) to (4.16) evaluate:

$$n_{\alpha_1} = \frac{\alpha\cos^2\theta}{V^2}\phi_1 \tag{4.22}$$

$$u_{\alpha x_1} = \frac{\cos^2\theta}{V}\phi_1 \tag{4.23}$$

$$u_{\alpha y_1} = \sin\theta\frac{\partial\phi_1}{\partial\xi} \tag{4.24}$$

$$u_{\alpha z_1} = \frac{\sin\theta\cos\theta}{V}\phi_1 \tag{4.25}$$

Again equating the lowest orders of ε $\left(i.e.\ \varepsilon^{\frac{3}{2}}\right)$, the following expressions result from equations (4.17) to (4.20):

$$n_{\beta_1} = -\frac{(1-\alpha)\cos^2\theta}{V^2}\phi_1 \tag{4.26}$$

$$u_{\beta x_1} = -\frac{\cos^2\theta}{QV}\phi_1 \tag{4.27}$$

$$u_{\beta y_1} = \sin\theta\frac{\partial\phi_1}{\partial\xi} \tag{4.28}$$

$$u_{\beta z_1} = -\frac{\sin\theta\cos\theta}{QV}\phi_1 \tag{4.29}$$

Also equating the lowest orders of ε in equation (4.21) gives:

$$V^2 = \cos^2\theta\left[\alpha + \frac{1-\alpha}{Q}\right] \tag{4.30}$$

Now for the next higher order of ε $\left(i.e.\ \varepsilon^{\frac{5}{2}}\right)$, the following relations result from equations (4.13) to (4.16):

$$\frac{\partial}{\partial\tau}n_{\alpha_1} - V\frac{\partial}{\partial\xi}n_{\alpha_2} + \alpha\frac{\partial}{\partial\xi}u_{\alpha x_2} + \frac{\partial}{\partial\xi}\left(n_{\alpha_1}u_{\alpha x_1}\right) = 0 \tag{4.31}$$

$$\frac{\partial}{\partial\tau}u_{\alpha x_1} - V\frac{\partial}{\partial\xi}u_{\alpha x_2} + \frac{\partial\phi_2}{\partial\xi} + u_{\alpha x_1}\frac{\partial}{\partial\xi}u_{\alpha x_1} = u_{\alpha y_2}\sin\theta \tag{4.32}$$

$$V\frac{\partial^2}{\partial\xi^2}u_{\alpha y_1} + \frac{\partial}{\partial\xi}u_{\alpha z_2}\cos\theta - \frac{\partial}{\partial\xi}u_{\alpha x_2}\sin\theta = 0 \tag{4.33}$$

$$\frac{\partial}{\partial\tau}u_{\alpha z_1} - V\frac{\partial}{\partial\xi}u_{\alpha z_2} + u_{\alpha x_1}\frac{\partial}{\partial\xi}u_{\alpha z_1} = -u_{\alpha y_2}\cos\theta \tag{4.34}$$

Mathematical manipulations of equations (4.31) to (4.34) yield:

$$\frac{2\alpha}{V}\cos^2\theta\frac{\partial\phi_1}{\partial\tau}-V^2\frac{\partial}{\partial\xi}n_{\alpha_2}+\frac{3\alpha}{V^2}\cos^4\theta\phi_1\frac{\partial\phi_1}{\partial\xi}$$
$$+\alpha\cos^2\theta\frac{\partial\phi_2}{\partial\xi}+\alpha V^2\sin^2\theta\frac{\partial^3\phi_1}{\partial\xi^3}=0 \tag{4.35}$$

Still continuing with equating the next higher order of ε $\left(i.e.\ \varepsilon^{\frac{5}{2}}\right)$ from equations (4.17) to (4.20):

$$\frac{\partial}{\partial\tau}n_{\beta_1}-V\frac{\partial}{\partial\xi}n_{\beta_2}+(1-\alpha)\frac{\partial}{\partial\xi}u_{\beta x_2}+\frac{\partial}{\partial\xi}\left(n_{\beta_1}u_{\beta x_1}\right)=0 \tag{4.36}$$

$$\frac{\partial}{\partial\tau}u_{\beta x_1}-V\frac{\partial}{\partial\xi}u_{\beta x_2}+u_{\beta x_1}\frac{\partial}{\partial\xi}u_{\beta x_1}-\frac{1}{Q}\frac{\partial\phi_2}{\partial\xi}+\frac{1}{Q}u_{\beta y_2}\sin\theta=0 \tag{4.37}$$

$$V\frac{\partial^2}{\partial\xi^2}u_{\beta y_1}+\frac{1}{Q}\sin\theta\frac{\partial}{\partial\xi}u_{\beta x_2}-\frac{1}{Q}\cos\theta\frac{\partial}{\partial\xi}u_{\beta z_2}=0 \tag{4.38}$$

$$\frac{\partial}{\partial\tau}u_{\beta z_1}-V\frac{\partial}{\partial\xi}u_{\beta z_2}+u_{\beta x_1}\frac{\partial}{\partial\xi}u_{\beta z_1}-\frac{1}{Q}u_{\beta y_2}\cos\theta=0 \tag{4.39}$$

Mathematical manipulations of equations (4.36) - (4.39) give:

$$-2\frac{(1-\alpha)}{Q^2V}\cos^2\theta\frac{\partial\phi_1}{\partial\tau}-\frac{V^2}{Q}\frac{\partial n_{\beta_2}}{\partial\xi}-V^2(1-\alpha)\sin^2\theta$$
$$\frac{\partial^3\phi_1}{\partial\xi^3}+\frac{3(1-\alpha)}{Q^3V^2}\cos^4\theta\phi_1\frac{\partial\phi_1}{\partial\xi}-\frac{(1-\alpha)\cos^2\theta}{Q^2}\frac{\partial\phi_2}{\partial\xi}=0 \tag{4.40}$$

Division of equation (4.35) by Q and subtraction of equation (4.40) from it yields:

$$\frac{2V}{Q}\frac{\partial \phi_1}{\partial \tau} - V^2\left(1-\alpha+\frac{\alpha}{Q}\right)\sin^2\theta\frac{\partial^3 \phi_1}{\partial \xi^3} + \frac{V^2}{Q}\frac{\partial}{\partial \xi}\left(\phi_2 - n_{\alpha_2} + n_{\beta_2}\right) + \frac{3}{QV^2}\left(\alpha - \frac{1-\alpha}{Q^2}\right)\cos^4\theta\,\phi_1\frac{\partial \phi_1}{\partial \xi} = 0 \tag{4.41}$$

Again, equating the next higher order of ε in equation (4.21),

$$\frac{\partial}{\partial \xi}\left(\phi_2 - n_{\alpha_2} + n_{\beta_2}\right) = K\frac{\partial^3 \phi_1}{\partial \xi^3} - \phi_1\frac{\partial \phi_1}{\partial \xi} \tag{4.42}$$

Using equation (4.42) in Equation (4.41), the K-dV equation is derived in the following form:

$$\frac{\partial \phi_1}{\partial \tau} + \left[\frac{3}{2V^3}\left(\alpha - \frac{1-\alpha}{Q^2}\right)\cos^4\theta - \frac{V}{2}\right]\phi_1\frac{\partial \phi_1}{\partial \xi} + \left[\frac{KV}{2} - \frac{VQ}{2}\left(1-\alpha+\frac{\alpha}{Q}\right)\sin^2\theta\right]\frac{\partial^3 \phi_1}{\partial \xi^3} = 0 \tag{4.43}$$

which can be written as:

$$\frac{\partial \phi_1}{\partial \tau} + A\phi_1\frac{\partial \phi_1}{\partial \xi} + B\frac{\partial^3 \phi_1}{\partial \xi^3} = 0 \tag{4.44}$$

with

$$A = \left[\frac{3}{2V^3}\left(\alpha - \frac{1-\alpha}{Q^2}\right)\cos^4\theta - \frac{V}{2}\right] \tag{4.45}$$

and

$$B = \left[\frac{KV}{2} - \frac{VQ}{2}\left(1 - \alpha + \frac{\alpha}{Q}\right)\sin^2\theta\right] \tag{4.46}$$

The formation of the K-dV equation shows that the presence of the magnetic field does not have any physical change on the propagation of ion-acoustic waves in plasmas. But the effect of the magnetic field might change the actual nature of soliton dynamics.

4.3. Derivation of soliton solution:

In order to study the soliton solution, a well known method called hyperbolic method is employed to equation Eq. (4.44), for which a transformation $\chi = \nu(\xi - \lambda\tau)$ with $z = \tan h\ \xi$ is introduced. The specialty of hyperbolic method is that in this method, the equations have been made easier to yield soliton propagation.

The use of the transformation derives the following relations: $\frac{\partial}{\partial\tau} = -\nu\lambda\frac{\partial}{\partial\chi}$ and $\frac{\partial}{\partial\xi} = \nu\frac{\partial}{\partial\chi}$, and consequently equation (4.44) derives as:

$$Bv^2 \frac{\partial}{\partial \chi}\left(\frac{\partial^2 \phi_1}{\partial \chi^2}\right) + A\frac{\partial}{\partial \chi}\left(\frac{\phi_1^2}{2}\right) = \lambda \frac{\partial \phi_1}{\partial \chi} \tag{4.47}$$

Integrating once and applying the appropriate boundary conditions: $\phi_1 \to 0$, $\frac{\partial^2 \phi_1}{\partial \chi^2} \to 0$ as $|\lambda| \to \infty$, the following relation results:

$$Bv^2 \frac{\partial^2 \phi_1}{\partial \chi^2} + \frac{A}{2}\phi_1^2 = \lambda \phi_1 \tag{4.48}$$

Now $z = \tanh\ \xi$ is substituted which, in turn, gives:

$$\frac{\partial \phi_1}{\partial \chi} = \left(1 - z^2\right)\frac{\partial \phi}{\partial z} \tag{4.49}$$

and

$$\frac{\partial^2 \phi_1}{\partial \chi^2} = -2\left(1 - z^2\right) z \frac{\partial \phi_1}{\partial z} + \left(1 - z^2\right)^2 \frac{\partial^2 \phi_1}{\partial z^2} \tag{4.50}$$

Substitution of these values of $\frac{\partial \phi_1}{\partial \chi}$ and $\frac{\partial^2 \phi_1}{\partial \chi^2}$ in equation (4.48) gives:

$$Bv^2\left(1 - z^2\right)^2 \frac{\partial^2 \phi_1}{\partial z^2} - 2Bv^2 z\left(1 - z^2\right)\frac{\partial \phi_1}{\partial z} - \lambda \phi_1 + \frac{A}{2}\phi_1^2 = 0 \tag{4.51}$$

which is an ordinary differential equation having singularity at $z = \pm 1$, because of which the Frobenius method of having a series solution of the equation can be employed as:

$$\phi_1 = \sum_{r=0}^{\infty} a_r z^{\rho+r} \tag{4.52}$$

Thus,

$$\frac{\partial \phi_1}{\partial z} = \sum_{r=0}^{\infty} a_r (\rho + r) z^{\rho+r-1} \tag{4.53}$$

and

$$\frac{\partial^2 \phi_1}{\partial z^2} = \sum_{r=0}^{\infty} a_r (\rho + r)(\rho + r - 1) z^{\rho+r-2} \tag{4.54}$$

Substitution of these values in equation (4.51) gives:

$$\begin{aligned} &Bv^2 \left(1 - z^2\right)^2 \sum_{r=0}^{\infty} a_r (\rho + r)(\rho + r - 1) z^{\rho+r-2} - 2Bv^2 z \times \\ &\left(1 - z^2\right) \sum_{r=0}^{\infty} a_r (\rho + r) z^{\rho+r-1} - \lambda \sum_{r=0}^{\infty} a_r z^{\rho+r} + \frac{A}{2} \left[\sum_{r=0}^{\infty} a_r z^{\rho+r} \right]^2 = 0 \end{aligned} \tag{4.55}$$

The infinite series (4.52) is truncated to a finite one with (N+1) terms along with $\rho = 0$. Then actual number N of terms in the series is determined by equating the leading order of linear term with that of the nonlinear term, which enables to find N=2. Thus the series becomes:

$$\phi_1 = a_0 + a_1 z + a_2 z^2 \tag{4.56}$$

which, in turn, gives:

$$\frac{\partial \phi_1}{\partial x} = a_1 + 2a_2 z \tag{4.57}$$

and

$$\frac{\partial^2 \phi_1}{\partial x^2} = 2a_2 \tag{4.58}$$

Substitution of these values in equation (4.51) gives:

$$\begin{aligned} &Bv^2\left(1-2z^2+z^4\right)2a_2 - 2Bv^2 z\left(a_1+2a_2 z - a_1 z^2 - 2a_2 z^3\right) \\ &-\lambda\left(a_0+a_1 z + a_2 z^2\right) + \frac{A}{2}\left(a_0^2 + a_1^2 z^2 + a_2^2 z^4 + 2a_0 a_1 z + 2a_0 a_2 z^2 + 2a_1 a_2 z^3\right) = 0 \end{aligned} \tag{4.59}$$

Equating the co-efficients of z^3 on both sides of the above equation (4.59), derives:

$$a_1(2Bv^2 + Aa_2) = 0 \tag{4.60}$$

Similarly equating the co-efficients of z^4 from equation (4.59), gives:

$$2a_2 Bv^2 + 4a_2 Bv^2 + \frac{A}{2}a_2^2 = 0 \tag{4.61}$$

as a result of which

$$a_2 = -\frac{12Bv^2}{A} \tag{4.62}$$

is obtained.

Using equation (4.62), equation (4.60) can be written as:

$$a_1(2Bv^2 - 12Bv^2) = 0 \tag{4.63}$$

from which $a_1 = 0$ is evaluated.

Equating the co-efficients of z^2 on both sides of (4.59), gives:

$$-4Bv^2a_2 - 4Bv^2a_2 - \lambda a_2 + \frac{A}{2}(a_1^2 + 2a_0a_2) = 0 \tag{4.64}$$

which ultimately gives:

$$a_0 = \frac{8Bv^2 + \lambda}{A} \tag{4.65}$$

Balancing of the co-efficients of the constant term from both sides evaluates:

$$2a_2Bv^2 - \lambda a_0 + \frac{A}{2}a_0{}^2 = 0 \tag{4.66}$$

The substitution of the values of a_0 and a_2 in equation (4.66) gives:

$$v^2 = \frac{\lambda}{4B} \tag{4.67}$$

Thus, the co-efficients are obtained as $a_0 = \frac{3\lambda}{A}$, $a_1 = 0$ and $a_2 = -\frac{3\lambda}{A}$.

The determination of all the co-efficients finds ultimately the soliton solution in the following form:

$$\phi = \frac{3\lambda}{A}\sec h^2\left(\frac{\xi - \lambda\tau}{\delta}\right) \tag{4.68}$$

where $\delta = \nu^{-1}$ and $a_0 = \frac{3\lambda}{A}$ is the amplitude.

4.4 Results and Discussions:

The soliton solution depends on the variation of A which is a function of plasma parameters. To get more elaborate observations, A is plotted in Fig. 4.1 with ion concentration α for various values of θ. The plot shows that for small values of θ, A can be both positive and negative exhibiting the existence of both compressive and rarefactive solitons, but as the propagation angle increases, the solitons become of rarefactive nature. This suggests that when the propagation angle θ attains a critical value, it plays a vital role in the evolution of the soliton profile. In the vicinity of this critical point, the amplitude of the soliton becomes very large resulting in the decrease of width. Consequently a narrow wave packet is formed, wherein the electric field grows. As the amplitude grows large in the solitary wave profile, the soliton propagation collapses or explodes. Both the explosion and collapse solitons are apparently observed to have similar nature. Yet a basic difference between them is that collapse soliton conserves the energy whereas explosion soliton does not. Moreover, because of the narrow wave packet, there is a generation of high electric field in which depression of density occurs giving rise to soliton radiation.

The variation of A with propagation angle θ has been studied taking typical values of plasma parameters for various values of ion concentration α. The results are depicted in Fig. 4.2. From the figures, it is clear that there is a critical value of α below which the nonlinear term is always negative exhibiting rarefactive solitary profile. Above this critical value, the soliton shows both compressive and rarefactive nature. The point at which A equals zero bifurcates the entire region of propagation into two regions, demonstrating the existence of compressive as well as rarefactive solitons. The rarefactive region is seen to decrease with the increase of α resulting in the corresponding increase in the region of compressive solitons. Thus the presence of additional negative ions in plasma introduces a critical density around which nonlinearity undergoes a change and the soliton modifies into compressive and rarefactive nature.

Although the nonlinear variation due to the presence of the additional negative ions exhibits new findings, comparatively less attention has been paid to examine the findings of dispersive effect variation. It has been shown that the dispersive effect varies as and when the K-dV soliton formation is considered in magnetized plasma. We plot in Fig. 4.3 the variation of the co-efficient of the dispersive term B with ion concentration α for various values of θ. The plot shows that for small values ofθ, the soliton is entirely of compressive nature while for high values of θ, rarefactive solitons are observed. Thus there is a critical value of θ for which the dispersive effect vanishes and soliton structure cannot be exposed.

The variation of dispersiveness with θ for different values of α is shown in Fig. 4.4. The figures reveal that with increase in the value of α, compressive soliton gradually loses its dominance over the rarefactive soliton. The dispersive effect vanishes at a critical value of the propagation angle.

In the present chapter, the propagation of ion-acoustic waves in a multicomponent plasma with negative ions has been studied by deriving the K-dV equation. The roles of the various parameters in forming the compressive and rarefactive solitons propagating in space have been examined. It was found that due to the presence of the negative ions, the nonlinear co-efficient of the K-dV equation went to zero at some critical value of the plasma parameters and then turned negative resulting in the formation of rarefactive solitary waves. The K-dV equation was derived under the condition that the nonlinear co-efficient of the K-dV equation could vanish due to nonlinear interaction of the various plasma parameters. Moreover, due to the presence of magnetic field, the dispersive effect goes to zero and consequently, soliton formation fails in plasma acoustic wave. It is worth mentioning here that Kakutani *et al.* have shown that a soliton propagating at an angle θ with the magnetic field yields a critical value of θ at which dispersive effect disappears and soliton structure cannot be expected. The aim in this chapter is to give a theoretical analysis, in advance, of such types of solitary waves observed in multicomponent plasma with negative ions for the future applicability in laboratory as well as astrophysical plasmas.

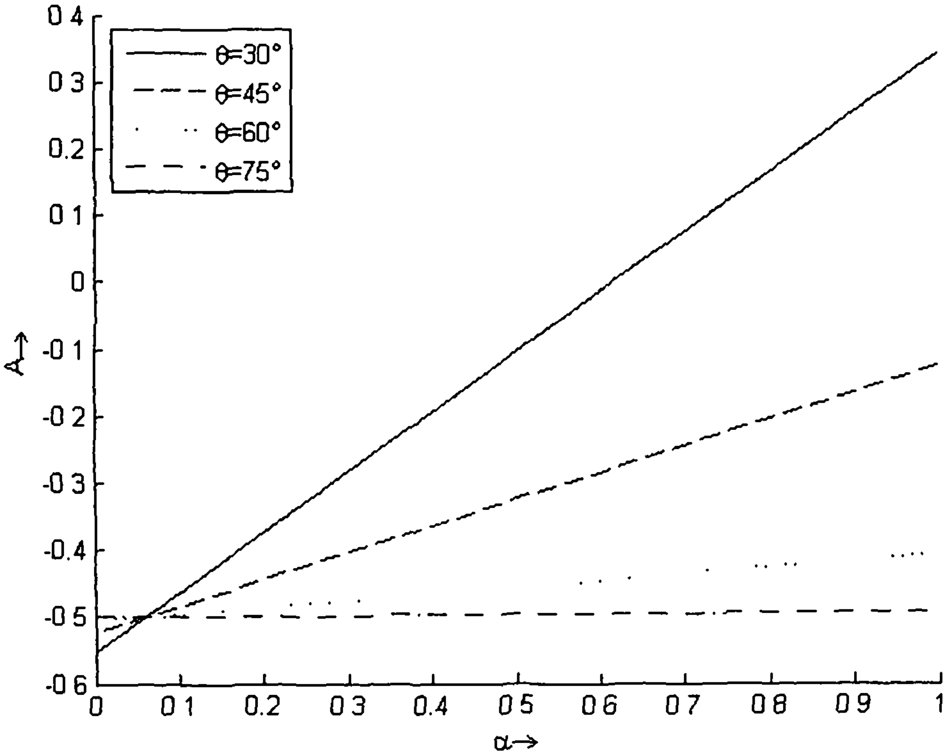

Fig 4.1: Variation of the co-efficient of the nonlinear term A with ion concentration α for various values of propagation angle θ

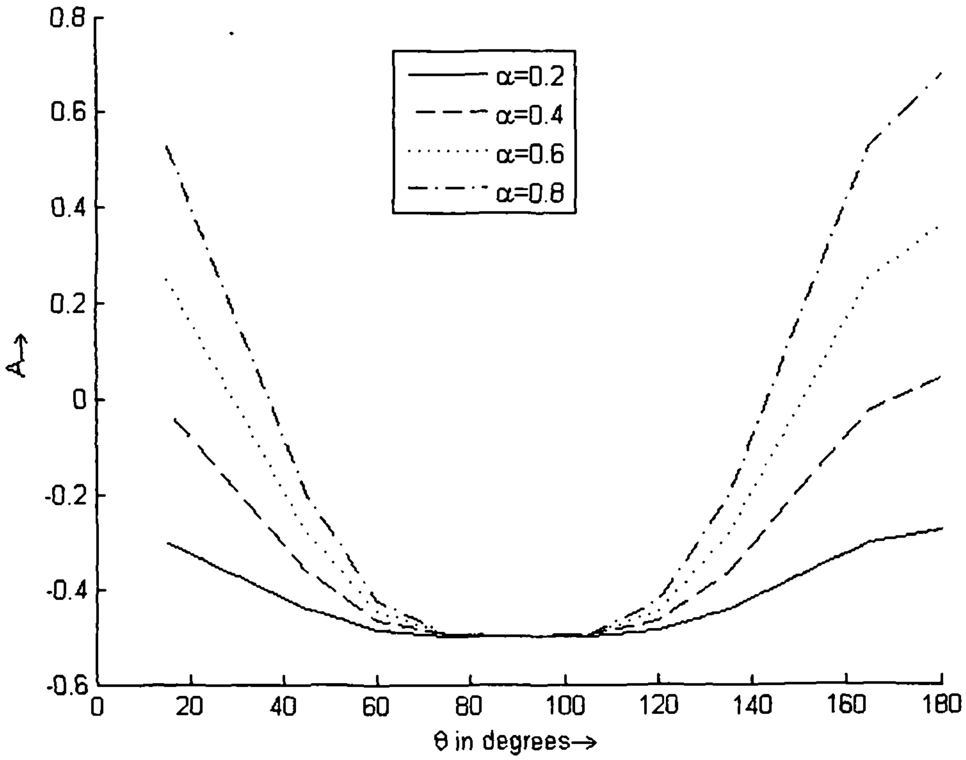

Fig 4.2: Variation of the co-efficient of the nonlinear term A with propagation angle θ for various values of ion concentration α

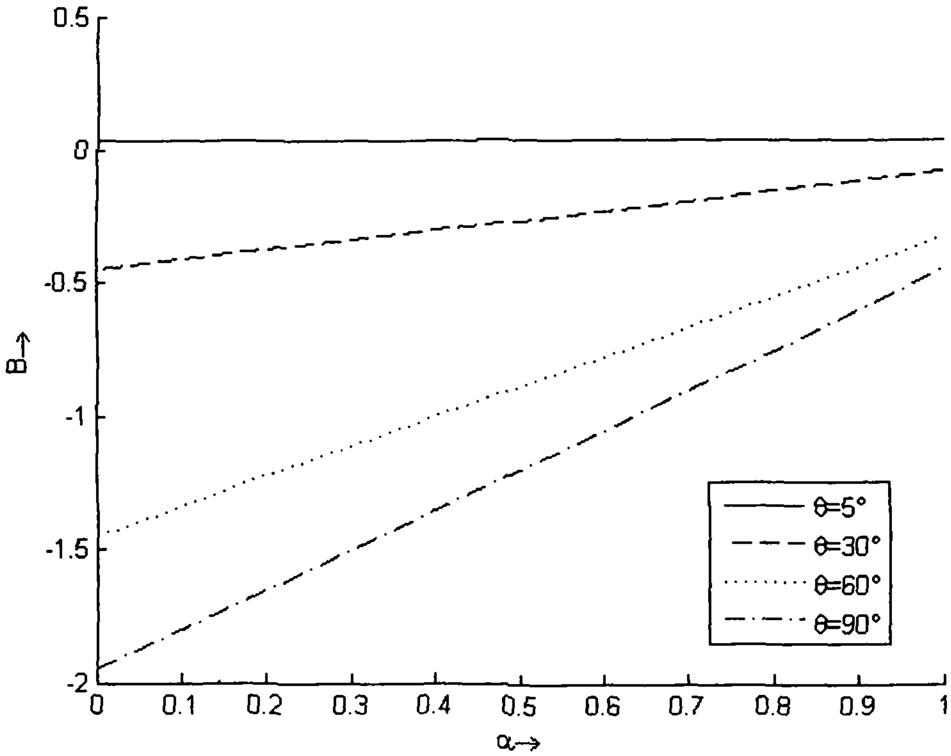

Fig 4.3: Variation of the co-efficient of the dispersive term B with ion concentration α for various values of propagation angle θ

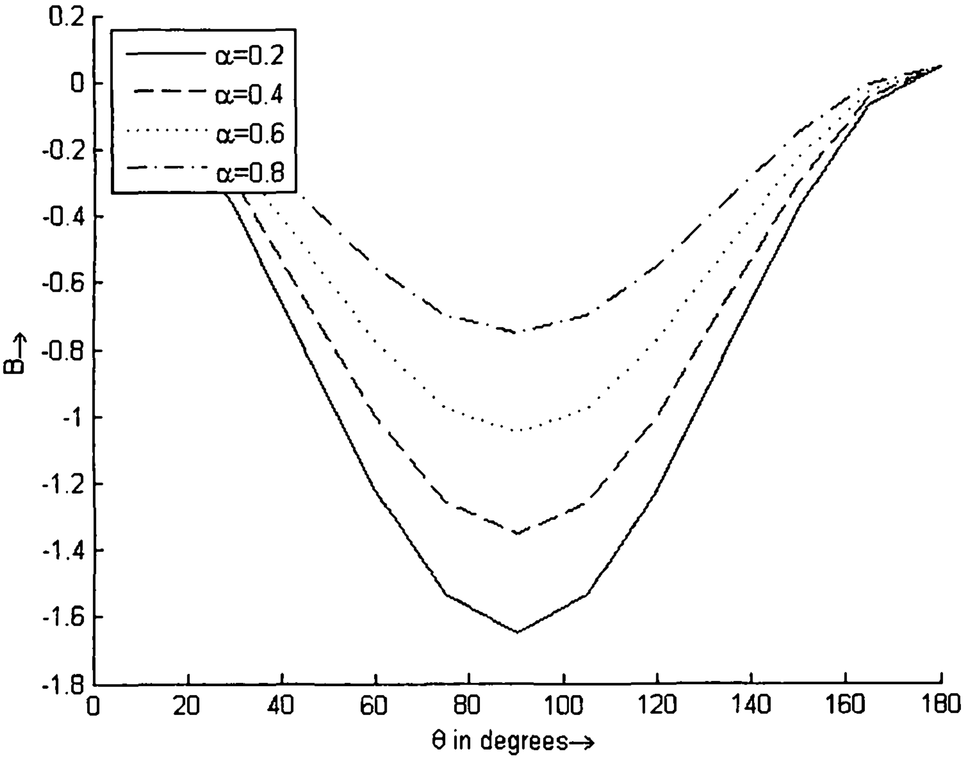

Fig 4.4: Variation of the co-efficient of the dispersive term B with propagation angle θ for various values of ion concentration α