CHAPTER-V

RESULT AND DISCUSSION

Electrorheological fluid is interesting not only because of its applicability in industry and technology but also for mathematics involves in it. In one hand, the precise control over viscosity of ERF under electric field gives wide range of uses in technology; on the other hand, the controlling of different terms in the set of equations for ERF gives a challenge in mathematics. The later is compulsory before one goes to use ERF in technology where precise control over viscosity is required. Keeping this in view, this research attempted to address some of the mathematical challenges involve in the study of ERF.

5.1 Analysis of Results

The main work of the research carried out are presented in chapter-2, chapter-3 and chapter-4. The second chapter shows the model for
ERF which captures different features. The following model for the extra stress tensor $S$ was finally derived.

$$
S = \alpha_2 \{ (1 + |D|^2)^{(p-1)/2} - 1 \} E \otimes E + (\alpha_{31} + \alpha_{33}|E|^2)
(1 + |D|^2)^{(p-2)/2} D + \alpha_{51}(1 + |D|^2)^{(p-2)/2}(DE \otimes E + E \otimes DE),
$$

(193)

where $\alpha_{ij}$ are constants and $p = p(|E|^2)$ is a function such that

$$
1 < p_{\infty} \leq p(|E|^2) \leq p_0.
$$

(194)

To ensure the validity of the Clausius-Duhem inequality we further require that the constant coefficients $\alpha_{ij}$ and the function $p$ are such that

$$
\alpha_{31} > 0, \quad \alpha_{33} > 0, \quad \alpha_{33} + \frac{4}{3} \alpha_{51} > 0,
$$

(195)

$$
k(p_0)|\alpha_{21}| < 2\sqrt{\alpha_{33} \sqrt{2\alpha_{51}}} \quad \text{if} \quad \alpha_{33} \leq \frac{4}{3} \alpha_{51},
$$

\[ \quad < \sqrt{\frac{3}{2}} \left( \alpha_{33} + \frac{4}{3} \alpha_{51} \right) \quad \text{if} \quad \frac{4}{3} |\alpha_{51}| \leq \alpha_{33} \]

(196)

where $k(p_0) = 1$ if $p_0 \leq 3$ and $k(p_0) > 1$ is a computable constant for $p_0 > 3$. It is to be noted here that these requirements ensure that the operator induced by $-\nabla.S(D,E)$ is coercive. The coefficients $\alpha_{ij}$ are to be determined from experiment. As reflected from (195)
and (196), it is obvious that the effect of these coefficients in the expression for extra stress is always positive and hence the viscosity increases with the applied electric field.

The third chapter contains the proof of a theorem which confirms the existence of strong solution for velocity of the system under consideration (ERF under electric field). The theorem specify a time range when this velocity exists:

Assume that the extra stress tensor $S$ satisfies (65)-(67) and $S(0, E) = 0$. Let $v_0 \in W^{2,2}(\Omega) \cap V_p$ be a given initial velocity, $f \in C(I; W^{1,2}(\Omega))$, $\partial_t f \in C(I; L^2(\Omega))$ be a given force, $E \in W^{1,\infty}(I; W^{1,\infty}(\Omega))$ be a given electric field and let $p = p(|E|^2)$ be a $C^1$-function with $p_\infty \leq p(|E|^2) \leq p_0$. If

$$\frac{3}{2} < p_\infty \leq p_0 \leq 2$$

(197)

then there exists a time $T^* > 0$, such that a strong solution $v$ of the system (62) exists on $I' = [0, T^*]$. This solution satisfies

$$\text{ess sup} \|\partial_t v(s)\|_2^2 + \int_0^{T^*} \mathcal{I}(t, v) \, dt + J(t, v) \leq C(f, v_0, E)$$

(198)

In particular we have that for $1 < r < 6(p_\infty - 1)$

$$v \in L^{p_\infty} \mathcal{S} \left( I', W^{2, \frac{3p_\infty}{p_\infty - 6}} \cap C(I'; V_p) \right)$$

(199)
Theorem confirms that for a variable electric field, the system has got a strong solution as a function of electric field through a parameter $p = p(|E|^2)$ and the system shows a $p$-structure where $p$ is not a constant but a function of the electric field.

The approximate solution for velocity using Galerkin approximation is derived as:

$$v^N(t, x) = \sum_{i=1}^{N} c_i^N \omega^r(x)$$  \hspace{1cm} (200)

The fourth chapter gives a detail comparison of the strong solution and the weak solution derived with the additional condition that $p =$ constant. Here we have again established an another theorem:

Assume that the extra stress tensor $S$ satisfies (133)-(135) and $S(0, E) = 0$. Let $v_0 \in W^{2,2}(\Omega) \cap V_p$ be a given initial velocity, $f \in C(I; W^{1,2}(\Omega))$, $\partial_t f \in C(I, L^2(\Omega))$ be a given force, $E \in C^1(I; C^1(\Omega))$ be a given electric field. Let $v$ be a strong solution of the problem (62) on the interval $I' = [0, T']$ for $p \in [\frac{5}{3}, 2]$ satisfying (74) and
Suppose that $v^m$ is a weak solution of the problem

$$\max_{1 \leq m \leq M} \|v^m\|_2^2 + k \sum_{m=1}^{M} \|Dv^m\|_p^p \leq C(f, v_0, E), \quad (201)$$

satisfying (136) and $t_M \leq T'$. Then for all

$$\alpha < \alpha_0(p) = \frac{5p - 6}{4(p - 1)} \quad (202)$$

there exists a constant $C$ that only depends on $v_0, f, \Omega, T'$ and $\alpha$ but not on the time step size $k$, such that the following error estimate is valid, provided that the time step size is chosen sufficiently small, i.e. $k \leq k_0(p, T')$,

$$\max_{1 \leq m \leq M} \|v(t_m) - v^m\|_2^2 + k \sum_{m=1}^{M} \|D\{v(t_m) - v^m\}\|_p^2 \leq Ck^{2\alpha}. \quad (203)$$

In this chapter we have in fact proved error estimates for the difference between a strong solution of the continuous system and a weak solution of the fully implicit time discretization of this system under the additional assumption that $p=$constant.

### 5.2 Use of the study:

This study gives a model of the ERF under the influence of an electric field which basically informs us about the stress and velocity of the
fluid as functions of applied electric field. This is what basically required to tune the viscosity of the ERF with the help of an electric field. Two theorems proved in the thesis are the major work of the research done by the researcher which can be used even for other fluids which follow Maxwell's equations and other conditions cited in the theorems. The entire analysis is logically formulated with practical approximations.

5.3 Future scope:

The study of electrorheological fluids is a relatively new field in mathematics compared to study done on it experimentally. In this study the ERF was modelled considering only the electric field. However, some fluids under go changes under electric and magnetic fields and there is no model for such fluids. Development of a model of such ERF under electric and magnetic field is a future scope in the field.