Chapter 5

Micropolar ferromagnetic liquid flow due to stretching of an elastic sheet


5.1 Introduction

Fluid elements such as liquid crystals, polymeric fluids and animal blood exhibit microscopic effects due to local structure and micromotions. Eringen [50, 51] provided the mathematical model to study the behavior of such fluids and the theory of a micropolar fluids was formulated. Since then the dynamics of micropolar fluid flow has been a popular area of research. An interesting fluid mechanical application is found in polymer extrusion processes, where an object on passing between two closely placed solid blocks is stretched into a liquid region. The stretching imparts a unidirectional orientation to the extrudate, thereby improving its mechanical properties [54]. The liquid is basically meant to cool the stretching sheet whose property as a final product depends greatly on the rate at which it is cooled.

The flow of a viscous fluid past a linearly stretching surface in otherwise quiescent surroundings was first considered by Crane [40] for a Newtonian fluid and subsequently extended to fluids obeying non-Newtonian constitutive equations.

Kelson and Farrell [72] worked on micropolar flow over a porous stretching sheet with strong suction or injection. Kelson and Desseaux [73] considered the effects of surface conditions on the flow of micropolar fluid driven by a porous stretching sheet. Mostafa and Mahmoud [92] considered the effects of thermal radiation on the flow of a micropolar fluid over a stretching surface with variable thermal conductivity. Agarwal and Dhanapal [13] examined the boundary layer flow of a micropolar fluid past a flat plate subject to suction and heat sources. Effects of radiation and magnetic field on mixed convection flow over a vertical stretching in a porous medium was investigated by Hayat et al. [64]. Bachok and

All the above investigations deal with the flows over a linear stretching sheet. Work on micropolar fluid over a non-linear stretching surface is considered in Vajravelu, Roslinda et al. and Abel et al. [119, 94, 11]. Abel et al. [4, 8] studied the effects of variable thermal conductivity in viscoelastic fluid and power law fluids, respectively, over a stretching sheet in the presence of a non-uniform heat source and an external magnetic field. Abel et al. [10] investigated the effects of thermal buoyancy and variable conductivity on the MHD flow over a vertical stretching sheet in the presence of a non-uniform heat source.

Ferrofluids are artificially synthesized and consist of highly concentrated colloidal suspensions of fine magnetic particles in a non-conducting carrier fluid. The resulting fluid behaves like a normal fluid except that it experiences a force due to the magnetization. A particular attractive feature of the ferrofluids is the dependence of the magnetization upon the temperature, and this thermo magnetic coupling makes ferrofluids useful in various practical applications, see for example, Neuringer [95] and Andersson [16]. The two classical problems in fluid mechanics, namely the Blasius boundary layer flow along a flat plate and the stagnation point flow were extended for a saturated ferrofluid under the combined influence of thermal and magnetic field gradients by Neuringer [95]. Andersson and Valnes [16] extended Crâne’s problem by studying the influence of the magnetic field,
due to a magnetic dipole, on a shear driven motion (flow over a stretching sheet) of a viscous non-conducting ferrofluid.

The ferro-micropolar fluid is representative of the liquid surrounding the stretching sheet that serves the purpose of a controlled cooling system in the presence of a magnetic field. In view of this Hayat et al. [61] studied the mixed convection flow of a micropolar fluid over a non-linear stretching sheet. Abraham [125] worked on Rayleigh-Benard convection in a micropolar ferromagnetic fluid. Siddheshwar and Pranesh [112] investigated the stability of magnetoconvection in a micropolar liquid. Rahman et al. [100] studied the heat transfer in micropolar fluid along an inclined permeable plate with variable fluid properties. The magnetohydrodynamic heat and mass transfer in the micropolar fluid flow over a stretching sheet was considered by Bhargava et al. [28]. Recently, Abel et al. [11] carried out a numerical study of the hydromagnetic flow due to a horizontal and vertical stretching sheets of a non-uniform property micropolar liquid and presented a scientific approach in arriving at initial values required to implement a shooting technique.

In this study the influence of the magnetic field due to magnetic dipole on the shear driven motion of a viscous and non-conducting micropolar ferromagnetic fluid due to stretching is explored. The focus of attention is on the magneto-thermomechanical interaction and heat transfer at the stretching sheet. The fluid dynamics over a stretching surface is important in many practical applications such as extrusion of plastic sheets, paper production, glass blowing and drawing plastic films, to name a few. The quality of the final product depends on the rate of heat transfer at the stretching surface.
5.2 Mathematical formulation

Consider a steady two-dimensional flow of an incompressible, viscous and electrically non-conducting micropolar ferrofluid driven by an impermeable sheet in the horizontal direction. By applying two equal and opposite forces along the horizontal direction which is taken as the $x$ - axis, with the $y$ - axis in a direction normal to the flow, the sheet is stretched with a velocity $u_w(x)$ which is proportional to the distance from the origin. A magnetic dipole is located with its centre on the $y$ - axis at a distance ‘$a$’ from the sheet. The magnetic field due to the dipole points in the positive $x$ - direction giving rise to a magnetic field of sufficient strength to saturate the ferrofluid. The sheet temperature $T_w$ is kept fixed below the Curie temperature $T_c$, while the fluid elements far away from the sheet are assumed to be at a temperature $T_\infty = T_c$ and hence incapable of being magnetized until they begin to cool upon entering the thermal boundary layer adjacent to the sheet.
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Figure 5.1: Schematic representation of flow configuration (broken lines represent the magnetic field)

The equations governing the motion and heat transfer are given by (see Andersson 1998):

\[
\begin{align*}
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} &= 0, \\
\frac{\partial u}{\partial x} + \frac{v}{\partial y} &= \nu \frac{\partial^2 u}{\partial y^2} + \frac{\mu_0}{\rho} M \frac{\partial H}{\partial x} + k_1 \frac{\partial \omega}{\partial y}, \\
G \frac{\partial^2 \omega}{\partial y^2} - 2\omega - \frac{\partial u}{\partial y} &= 0, \\
\rho c_p \left( \frac{u}{\partial x} + \frac{v}{\partial y} \right) + \mu_0 \left( \frac{\partial M}{\partial T} \left( \frac{u}{\partial x} + \frac{v}{\partial y} \right) \right) &= k \frac{\partial^2 T}{\partial y^2} + \frac{\delta}{\rho c_v} (\nabla \times \vec{\omega}) \cdot \nabla T,
\end{align*}
\]

where \(u\) and \(v\) are the velocity components along \(x\) and \(y\) directions respec-
tively, $\rho$ is the fluid density, $\mu$ is the dynamic viscosity, $\nu = \mu/\rho$ is the kinematic viscosity, $c_p$ is the specific heat at constant pressure, $k$ is the thermal conductivity, $k_1$ is the microrotation diffusivity parameter, $G$ is the microrotation coupling parameter, $\mu_0$ is the magnetic permeability, $M$ is the magnetization, $H$ is the magnetic field, $T$ is the temperature of the fluid, $\delta$ the micropolar heat conduction parameter and $c_v$ is the specific heat at constant volume. $\vec{\omega} = (0, 0, \omega)$ is the spin along the $z$ direction.

Hence $(\nabla \times \vec{\omega}) \cdot \nabla T = \frac{\partial \omega}{\partial y} \frac{\partial T}{\partial x} - \frac{\partial \omega}{\partial x} \frac{\partial T}{\partial y}$.

The assumed boundary conditions are:

\[
\begin{align*}
\begin{cases}
  u(x, 0) = cx, \\
  v(x, 0) = 0, \\
  \omega(x, 0) = s \left[ \omega(x, 0) + \frac{1}{2} \frac{\partial u}{\partial y} \right], \\
  T(x, 0) = T_w = T_c - A \left( \frac{x}{L} \right) \quad \text{in PST}, \\
  -k \frac{\partial T}{\partial y}(x, 0) = q_w = D \left( \frac{x}{L} \right) \quad \text{in PHF},
\end{cases}
\end{align*}
\]

at $y = 0$, \hspace{1cm} (5.5)

\[
\begin{align*}
\begin{cases}
  u(x, \infty) \to 0, \\
  \omega(x, \infty) \to 0 \\
  \text{and} \quad T(x, \infty) \to T_c
\end{cases}
\end{align*}
\]

as $y \to \infty$, \hspace{1cm} (5.6)

where $k$ is the thermal conductivity of the fluid, $q_w$ is the wall heat flux, $A$ and $D$ are positive constants, $L = \sqrt{\nu/c}$ is the characteristic length and $s$ is the parameter relating to microrotation to the asymmetric part of the stress with $0 \leq s < \infty$. The condition on $\omega(x, 0)$ in (5.5) reduces to no relative spin and no
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asymmetric part of the stress on the boundary in the limits $s \to 0$ and $s \to \infty$ respectively. The flow of a micropolar ferrofluid is affected by the magnetic field due to the magnetic dipole whose magnetic scalar potential is given by

$$\phi = \frac{\gamma}{2\pi} \left\{ \frac{x}{x^2 + (y + a)^2} \right\},$$

(5.7)

where $\gamma$ is the dipole moment per unit length. The magnetic field $H$ has components

$$H_x = -\frac{\partial \phi}{\partial x} = \frac{\gamma}{2\pi} \left\{ \frac{x^2 - (y + a)^2}{[x^2 + (y + a)^2]^2} \right\},$$

(5.8)

$$H_y = -\frac{\partial \phi}{\partial y} = \frac{\gamma}{2\pi} \left\{ \frac{2x(y + a)}{[x^2 + (y + a)^2]^2} \right\}$$

(5.9)

Here we note that magnetic body force is proportional to the gradient of the magnitude of $H$ and using

$$H = \left[ \left( \frac{\partial \phi}{\partial x} \right)^2 + \left( \frac{\partial \phi}{\partial y} \right)^2 \right]^{\frac{1}{2}},$$

(5.10)

we obtain

$$\frac{\partial H}{\partial x} = -\frac{\gamma}{2\pi} \left\{ \frac{2x}{(y + a)^4} \right\},$$

$$\frac{\partial H}{\partial y} = \frac{\gamma}{2\pi} \left\{ \frac{-2}{(y + a)^3} + \frac{4x^2}{(y + a)^5} \right\}.$$ 

(5.11)

The variation of magnetization $M$ with temperature $T$ is approximated by a linear equation

$$M = K(T_c - T),$$

(5.12)
where $K$ is the pyromagnetic coefficient.

### 5.3 Solution Procedure

We introduce the non-dimensional variables as assumed by Andersson (1998):

\begin{align}
(\xi, \eta) &= \sqrt{\nu} (x, y), \\
(U, V) &= \left(\frac{u,v}{\nu \sigma}\right), \\
N &= \frac{\omega}{c}, \\
\Theta &= \frac{T_c - T_w}{T_c - T_w},
\end{align}

where $T_c - T_w = A \left(\frac{x}{L}\right)$ in PST case and $T_c - T_w = \frac{DL}{k} \left(\frac{x}{L}\right)$ in PHF case. The boundary layer equations (5.1) - (5.4), in case of PST, on using (5.11) - (5.13) take the following form:

\begin{align}
\frac{\partial U}{\partial \xi} + \frac{\partial V}{\partial \eta} &= 0, \\
U \frac{\partial U}{\partial \xi} + V \frac{\partial U}{\partial \eta} &= \frac{\partial^2 U}{\partial \eta^2} + k^* \frac{\partial N}{\partial \eta} - \beta \frac{2 \xi \Theta}{(\eta + \alpha)^4}, \\
G^* \frac{\partial^2 N}{\partial \eta^2} - 2N - \frac{\partial U}{\partial \eta} &= 0,
\end{align}
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\[
Pr \left[ U \left( \frac{\partial \Theta}{\partial \xi} + \frac{\Theta}{\xi} \right) + V \frac{\partial \Theta}{\partial \eta} \right] + \\
\beta \lambda (\epsilon - \Theta) \left[ -\frac{2\xi U}{(\eta + \alpha)^2} - \frac{2V}{(\eta + \alpha)^3} + \frac{4\xi^2 V}{(\eta + \alpha)^5} \right] \\
= \frac{\partial^2 \Theta}{\partial \eta^2} + \delta^* Pr \left[ \frac{\partial N}{\partial \eta} \left( \frac{\partial \Theta}{\partial \xi} + \frac{\Theta}{\xi} \right) - \frac{\partial N \partial \Theta}{\partial \xi \partial \eta} \right]. \tag{5.17}
\]

The eight dimensionless parameters, which appear explicitly in equations (5.14) - (5.17), are the Prandtl number \( Pr \), the viscous dissipation parameter \( \lambda \), the dimensionless Curie temperature ratio \( \epsilon \), the ferrohydrodynamic interaction parameter \( \beta \), the scaled microrotation coupling parameter \( G^* \), the scaled microrotation diffusivity parameter \( k^* \), the scaled micropolar heat conduction parameter \( \delta^* \) and the dimensionless distance \( \alpha \) from the origin to the center of the magnetic pole, defined respectively as

\[
Pr = \frac{\mu c_p}{k} ;
\]

\[
\lambda = \frac{\alpha \mu^2}{\rho k (T_c - T_w)} ;
\]

\[
\epsilon = \frac{T_c}{T_c - T_w} ;
\]

\[
\beta = \frac{\gamma \rho}{2\pi \mu^2 \mu_0 K (T_c - T_w)} ,
\]

\[
G^* = \frac{c_1 c_p}{\nu} ,
\]

\[
k^* = \frac{k}{\nu} ,
\]

\[
\delta^* = \frac{\delta}{\rho c_v L^2} ,
\]

and \( \alpha = \frac{\alpha}{L} \).
The boundary conditions given in (5.5) and (5.6) now take the form

\[
\begin{align*}
U(\xi, 0) &= \xi, \\
V(\xi, 0) &= 0, \\
N(\xi, 0) &= s \left[ N(\xi, 0) + \frac{1}{2} \frac{\partial U}{\partial \eta}(\xi, 0) \right], \\
\Theta(\xi, 0) &= 1 \quad \text{in PST}, \\
\frac{\partial \Theta}{\partial \eta}(\xi, 0) &= -1 \quad \text{in PHF}, \\
U(\xi, \infty) &\to 0 \\
N(\xi, \infty) &\to 0 \\
\text{and} \quad \Theta(\xi, \infty) &\to 0.
\end{align*}
\]

(5.18)

We now introduce the stream function \(\psi(\xi, \eta)\) such that

\[
\begin{align*}
U &= \frac{\partial \psi}{\partial \eta}, \\
V &= -\frac{\partial \psi}{\partial \xi}.
\end{align*}
\]

(5.19)

Equation (5.19) automatically satisfies the continuity equation (5.14) and equations (5.15) - (5.18) take the form:

\[
\frac{\partial^3 \psi}{\partial \eta^3} + \frac{\partial}{\partial (\xi, \eta)} \left( \psi, \frac{\partial \psi}{\partial \eta} \right) + k^* \frac{\partial N}{\partial \eta} - \beta \frac{2\xi \Theta}{(\eta + \alpha)^4} = 0, 
\]

(5.20)

\[
G^* \frac{\partial^2 N}{\partial \eta^2} - 2N - \frac{\partial^2 \psi}{\partial \eta^2} = 0, 
\]

(5.21)
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\[
\frac{\partial^2 \Theta}{\partial \eta^2} + Pr \frac{\partial (\Theta, \delta^*N - \psi)}{\partial (\xi, \eta)} + Pr \frac{\partial (\delta^*N - \psi)}{\partial \eta} \left[ \frac{-2\xi \frac{\partial \psi}{\partial \eta}}{(\eta + \alpha)^4} + \frac{2 \frac{\partial \psi}{\partial \xi}}{(\eta + \alpha)^3} - \frac{4\xi^2 \frac{\partial \psi}{\partial \xi}}{(\eta + \alpha)^5} \right] = 0, \tag{5.22}
\]

\[
\begin{align*}
\frac{\partial \psi}{\partial \eta} (\xi, 0) &= \xi, \\
\frac{\partial \psi}{\partial \xi} (\xi, 0) &= 0, \\
N(\xi, 0) &= -S \frac{\partial^2 \psi}{\partial \eta^2} (\xi, 0), \\
\Theta(\xi, 0) &= 1 \quad \text{in PST}, \\
\frac{\partial \Theta}{\partial \eta} (\xi, 0) &= -1 \quad \text{in PHF}, \\
\frac{\partial \psi}{\partial \eta} (\xi, \infty) &\to 0 \\
N(\xi, \infty) &\to 0 \\
\text{and} \quad \Theta(\xi, \infty) &\to 0.
\end{align*}
\tag{5.23}
\]

where \( S = \frac{s}{2(s - 1)} \), \( 0 \leq S \leq \frac{1}{2} \). The parameter \( S \) relates the microrotation to the asymmetric part of the stress. The value \( S = 0 \) indicates the concentrated particle flows at the wall in which the microelements close to the wall surface are unable to rotate. This case is also known as the strong concentration of microelements. The other extreme value \( S = \frac{1}{2} \) indicates the vanishing of the antisymmetric part of the stress tensor and denotes weak concentration of microelements. Further, \( S = 1 \) is used for the modeling of turbulent boundary layer flows.

In order to convert the partial differential equations (5.20) - (5.22) into ordi-
nary differential equations the following similarity transformations are adopted.

\[ \psi(\xi, \eta) = \xi f(\eta), \]
\[ N(\xi, \eta) = \xi g(\eta), \]
\[ \Theta(\xi, \eta) = \begin{cases} 
\theta_1(\eta) + \xi^2 \theta_2(\eta) & \text{in PST} \\
\phi_1(\eta) + \xi^2 \phi_2(\eta) & \text{in PHF} 
\end{cases} \]

On using (5.24) in (5.20) - (5.23) we obtain the following boundary value problems:

(i) **PST**

\[ f''' + f f'' - f'^2 + k^* g' - \frac{2\beta \theta_1}{(\eta + \alpha)^4} = 0, \]
\[ (5.25) \]

\[ G^* g'' - 2g - f' = 0, \quad \frac{2\beta \theta_1}{(\eta + \alpha)^4} = 0, \]
\[ (5.26) \]

\[ \theta_1'' + Pr f \theta_1' + \frac{2\beta \lambda f (\theta_1 - \epsilon)}{(\eta + \alpha)^3} = 0, \]
\[ (5.27) \]

\[ \theta_2'' - Pr [3f' \theta_2 - f \theta'_2 - \delta^* (3g' \theta_2 - g \theta'_2)] + \frac{2\lambda \beta f \theta_2}{(\eta + \alpha)^3} - \lambda \beta (\theta_1 - \epsilon) \left[ \frac{2f'}{(\eta + \alpha)^4} + \frac{4f}{(\eta + \alpha)^5} \right] = 0, \]
\[ (5.28) \]
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\[
\begin{align*}
  f(0) &= 0, \\
  f'(0) &= 1, \quad g(0) = -Sf''(0), \\
  \theta_1(0) &= 1, \\
  \theta_2(0) &= 0, \\
  f'(&\infty) \to 0, \\
  g(&\infty) \to 0, \\
  \theta_1(&\infty) \to 0 \\
  \text{and} \quad \theta_2(&\infty) \to 0.
\end{align*}
\]

(ii) PHF

\[
\begin{align*}
  f''' + ff'' - f'^2 + k^*g' - \frac{2\beta\phi_1}{(\eta + \alpha)^4} &= 0, \\
  G^*g'' - 2g - f' &= 0, \\
  \phi''_1 + Prf\phi'_1 + \frac{2\beta\lambda f(\phi_1 - \epsilon)}{(\eta + \alpha)^3} &= 0, \\
  \phi''_2 - Pr[3f'\phi_2 - f\phi'_2 - \delta^*(3g'\phi_2 - g\phi'_2)] + \frac{2\lambda\beta f\phi_2}{(\eta + \alpha)^3} \\
  -\lambda^2(\phi_1 - \epsilon) \left[ \frac{2f'}{(\eta + \alpha)^4} + \frac{4f}{(\eta + \alpha)^5} \right] &= 0.
\end{align*}
\]
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\[
\begin{align*}
  f(0) &= 0, \\
  f'(0) &= 1, \\
  g(0) &= -Sf''(0), \\
  \phi_1'(0) &= -1, \\
  \phi_2'(0) &= 0, \\
  f'(\infty) &\to 0, \\
  g(\infty) &\to 0, \\
  \phi_1(\infty) &\to 0 \\
  \text{and} \quad \phi_2(\infty) &\to 0.
\end{align*}
\]

(5.34)

Here the primes denote differentiation with respect to \( \eta \). The local skin friction coefficient \( C_f \), which is a dimensionless form of the shear stress at the sheet is given by

\[
C_f = -\frac{2\tau_{xy}}{\rho u_w^2} = -2Re^{-1/2} f''(0). \quad (5.35)
\]

In the PST case where we are fixing the surface temperature, we compute the heat transfer using,

\[
Nu_x = Re^{1/2} [\theta_1'(0) + \xi^2 \theta_2'(0)]. \quad (5.36)
\]

In the PHF case we fix the surface heat flux and hence compute the surface temperature as

\[
T(\xi, 0) = T_c - \frac{DL}{k} [\xi \phi_1(0) + \xi^3 \phi_2(0)]. \quad (5.37)
\]

Equations (5.25) - (5.29) and (5.30) - (5.34) constitute two sets of nonlinear, two-
point boundary value problems, which are solved by means of a standard shooting technique. The higher order ordinary differential equations are decomposed into a set of nine first order equations and integrated as an initial value problem using the adaptive stepping Runge-Kutta-Fehlberg (RKF45) method. Trial values of \( f''(0), g'(0), \theta'_1(0), \theta'_2(0) \) and \( \phi_1(0), \phi_2(0) \) were adjusted iteratively by Newton-Raphson’s method to assure a quadratic convergence of the iterative trial values required in order to fulfill the outer boundary conditions.

### 5.4 Results and Discussion

In this section the influence of physical parameters on the velocity, the micro-rotation velocity and the temperature fields are presented.

The magnetic parameter \( \beta \) highlights the effect of external magnetic field induced by the magnetic dipole on the dynamics of the fluid. The presence of magnetic field acts as the retarding force on the velocity field and therefore as \( \beta \) increases so does the retarding force and hence the axial velocity decreases. It is observed that when the fluid is under the influence of magnetic field, the velocity component \( f'(\eta) \) is lesser than the corresponding one in hydrodynamic case (\( \beta = 0 \)) as shown in figure 5.2 (a). This is because of the interaction between the motion of the fluid and the action of the magnetic field. This interaction decreases the velocity thereby increasing frictional heating between the fluid layers which is responsible for increasing the thermal boundary layer thickness as shown in figure 5.2 (c). Increasing \( \beta \) increases angular velocity profile as shown figure 5.2 (b). It is also observed from figure 5.2 that in the presence of magnetic dipole the linear velocity in case of PST is more than that of PHF and the angular
velocity and the temperature in case of PST are less than those of PHF.

The viscous dissipation parameter $\lambda$ signifies the effect of frictional heating due to the viscosity and the applied magnetic field on the temperature distribution. Since, the equations of conservation of linear, angular momentum and energy are coupled, $\lambda$ affects linear and angular velocity profiles as well. Figure 5.3 captures the effect of $\lambda$ on linear, angular velocity and temperature profiles in PST and PHF cases. The effect of increasing values of $\lambda$ is to decrease the thermal boundary layer thickness in both PST and PHF cases as can be seen from figure 5.3 (c). This is contrary to the hydrodynamic case where increasing values of viscous dissipation parameter results in enhancing the temperature in the boundary layer region. It is also observed from figure 5.3 (c) that PHF boundary heating produces more heat as compared to PST type heating. Increasing values of $\lambda$ results in enhancement in the linear velocity and reduction in the angular velocity as seen from figure 5.2 (a) and (b). For smaller values of $\lambda$ it is found that the linear velocity in the case of PST is more than the PHF one while the opposite holds at larger values of $\lambda$. The angular velocity has exactly the opposite trend as compared to the linear velocity.

The effects of micropolar fluid parameters $G^*$, $k^*$ and $\delta^*$ on the angular velocity are depicted in figures 5.4 (a) - (c) respectively. The effects of microrotation coupling parameter $G^*$ and microrotation diffusivity parameter $k^*$ are to increase the angular velocity profiles in both PST and PHF cases. The micropolar heat conduction parameter $\delta^*$ quantifies the amount of heat diffused via microrotation of the suspended particles into the fluid. It also represents the coupling between the thermal and the micropolar effects. Increasing values of $\delta^*$ results in increasing the angular velocity profile in both PST and PHF cases. With respect to all
these parameters the angular velocity in case of PHF exceeds that in PST.

The skin friction coefficient is shown as a function of viscous dissipation parameter $\lambda$ for different values of ferrohydrodynamic interaction parameter $\beta$ in figures 5.5 (a) and (b) in the cases of PST and PHF respectively. Clearly, in the hydrodynamic case the skin friction coefficient remains a constant with respect to $\beta$ and in the presence of magnetic dipole it is an increasing function of $\lambda$ in both PST and PHF cases. The skin friction coefficient is a decreasing function of $\beta$, but at higher values of $\beta$ the $f''(0)$-$\lambda$ curve increases abruptly.

The local Nusselt number $Nu_x\Re_x^{-1/2}$ is shown as a function of streamwise location $\xi$ in figures 5.6 - 5.8 for different values of $\beta$, $\lambda$ and $Pr$ in the PST case. It is evident from these figures that the local Nusselt number is an increasing function of streamwise location. From figure 5.6 we see that in the hydrodynamic case ($\beta = 0$) the Nusselt number behaves as a constant function of $\xi$ and in the presence of magnetic field generated by the dipole it increases with $\xi$. It is also seen that increasing the strength of magnetic dipole leads to enhancement of the local Nusselt number. From figure 5.7 we see that the local Nusselt number behaves as decreasing function of $\lambda$ up to a certain location in the streamwise direction and follows opposite trend there after. From figure 5.8 it is clear that the local Nusselt number is a decreasing function of $Pr$.

The variation of wall temperature $[T_e - T(\xi, 0)]k/\Delta L$ with the streamwise location $\xi$ is depicted in figures 5.9 - 5.11 for different values of $\beta$, $\lambda$ and $Pr$ in the PHF case. It can be readily seen from these figures that the wall temperature is an increasing function of streamwise location. From figure 5.9 we see that in the hydrodynamic case ($\beta = 0$) the wall temperature is a linear function of $\xi$ and in the presence of applied magnetic field it is a cubic function of $\xi$. It is also seen
that increasing the values of $\beta$ results in increasing the wall temperature. From figure 5.10 we see that the wall temperature in the PHF case is a decreasing function of $\lambda$ up to a certain location in the streamwise direction and follows opposite trend thereafter. From figure 5.11 it is clear that the wall temperature in PHF case is a decreasing function of $Pr$.

5.5 Conclusions

The effect of magnetic field due to a dipole was studied on the flow of a micropolar fluid induced by the stretching of an elastic sheet. It was found that the presence of a dipole magnetic field reduces the axial velocity profile and increases the angular velocity and temperature profiles in both types of boundary heating. Further, the local Nusselt number in case of PST and the wall temperature in case of PHF were found to increase in the presence of a dipole magnetic field.
Figure 5.2: Effect of ferrohydrodynamic interaction parameter \( \beta \) on linear, angular velocity and temperature profiles.
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Figure 5.3: Effect of viscous dissipation parameter $\lambda$ on linear, angular velocity and temperature profiles.
Figure 5.4: Effect of micropolar fluid parameters $G^*$, $k^*$ and $\delta^*$ on the angular velocity profiles.
Figure 5.5: Variation of $f''(0)$ with $\lambda$ for different values of $\beta$. 

(a) PST
- $\beta = 0$
- $\beta = 1$
- $\beta = 2$

(b) PHF
- $\beta = 0$
- $\beta = 1$
- $\beta = 2$
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Figure 5.6: Variation of $\frac{Nu_x Re_x^{1/2}}{\beta}$ with streamwise location $\xi$ for different values of $\beta$. 

---

Figure 5.6: Variation of $Nu_x Re_x^{-1/2}$ with streamwise location $\xi$ for different values of $\beta$. 

- **PST**
  - $\beta = 0$
  - $\beta = 1$
  - $\beta = 2$
Figure 5.7: Variation of $Nu_x Re_x^{-1/2}$ with streamwise location $\xi$ for different values of $\lambda$
Figure 5.8: Variation of $Nu_x Re_x^{-1/2}$ with streamwise location $\xi$ for different values of $Pr$. 

$\text{PST}$
- $Pr = 1$
- $Pr = 3$
- $Pr = 5$
- $Pr = 7$
Figure 5.9: Variation of \((T_c - T)k/\Delta L\) with streamwise location \(\xi\) for different values of \(\beta\).
Figure 5.10: Variation of \((T_c - T)k/DL\) with streamwise location \(\xi\) for different values of \(\lambda\).
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Figure 5.11: Variation of \((T_c - T)k/DL\) with streamwise location \(\xi\) for different values of \(Pr\).