Load flow solution formulation using exclusive methods and Tellegen theorem method
Chapter 3

Load Flow Solution formulation Using Exclusive methods and Tellegen Theorem method

3.1 Introduction:

The topological properties of the distribution networks are exploited to devise dedicated exclusive power flow methods and at each stage the power conservation principle at each node is extensively used. The chapter presents the description of the radial distribution network and the importance of the bus and branch indexing in the load flow solution process is explained. Later the load flow problem formulation of a radial distribution network using the typical exclusive load flow methods “Forward sweeping method” and “DistFlow method” and a new load flow method “Tellegen theorem method” with necessary detailed derivations is presented.

3.2 Description of the radial Distribution network:

In general, the structure of the distribution network has to be properly explained, so that load flow problem formulation becomes easier. The notation of the node and branches used and the various electrical variables used for the power flow equations formulation are given in Appendix. Table A.11.

The distribution network is considered in two configurations. They are the distribution network with main feeder and other distribution network with main feeder and laterals.

The distribution network with the above configurations is as shown in the fig.3.1 and fig.3.2.
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The network has \( N_b \) nodes and \( N_{b-1} \) branches. The node and branch of the main feeder or laterals are indexed such that data storing and retrieval are easy. The node is identified with the number in the square brackets "[ ]" and the branch is identified with the number in normal brackets "( )". It can be observed that the sending end node and the corresponding branch connected to that node are identified with the same number. The branch \( "j" \) is represented with resistance and reactance \( r_j, x_j \). The load at node \( "i" \) is represented with \( P_{Li}, Q_{Li} \).

The power flow equations are represented as a set of equations and unknowns, by which a solution methodology can be developed. These equations and unknowns are organized through a particular node ordering. The number of equations and number of unknowns at each node are reduced with a proper bus and branch-indexing scheme. As the distribution network is radial nature, the indexing of bus and branch should be proper such that reading and retrieval of the branch and node data is easy. Hence a proper indexing scheme is demanded by the load flow solution method.

The main feeder nodes and branches are indexed first and then lateral nodes and branches are indexed. The main feeder is treated as a "lateral 1 (L=1)" and the actual laterals are numbered from 2 to \( N_{Lat} (L=2,3,..., N_{Lat}) \). The first node of the main feeder or any lateral is defined as a "source node SN (L)". The second node is called as the "Node just after the source node, LB (L)" and the last node of the main feeder or lateral is called as "End node, EB (L)".
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The following table 3.1 gives the details of the SN(L), LB(L) and EB(L) of the distribution network with main feeder and laterals as shown in the fig.3.2

Table-3.1 Indexing details of Distribution network with main feeder and laterals.

<table>
<thead>
<tr>
<th>Lateral (L)</th>
<th>Source node SN(L)</th>
<th>Node just after the source node LB(L)</th>
<th>End node EB(L)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>2</td>
<td>4</td>
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<td>2</td>
<td>2</td>
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</tr>
<tr>
<td>6</td>
<td>4</td>
<td>16</td>
<td>18</td>
</tr>
<tr>
<td>7</td>
<td>5</td>
<td>(N_{b+1})</td>
<td>(N_b)</td>
</tr>
</tbody>
</table>

3.3 Distribution networks problem formulation using Exclusive load flow methods:

The unique distinguishing feature of the radial distribution network that, a single path exists from any node of the network to the source node, is exploited in the exclusive load flow methods. These exclusive load flow methods differ in the process of updating the node voltages and branch power flows. The updating process may be carried either in forward or backward power flow directions. The assumptions used for the initiation of the iteration process have its own importance and hence the load flow methods are categorized into Loop-based methods and branch-based methods. It is reported that branch based methods are faster than loop based methods. And further, the branch-based methods are classified as “Forward update method” and “Backward update method”, on basis of the initiation of the computation process. The “Forward Update method” needs the line losses, whereas the “Backward update method” requires the information of the terminal node voltage for the load flow solution to start.
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In this section, the load flow problem is formulated using a method called “Forward sweeping method”, another method called “DistFlow method” and a new method called “Tellegen theorem method”.

3.3.1 Load flow problem formulation using “Forward Sweeping method”\[30,31,32\] :

In this section, the load flow problem is formulated using “Forward Sweeping method” on a distribution network with main feeder and main feeder with laterals. The power summation principle at a node level and basic network theory concepts are used to develop the power flow equations.

3.3.1.1 Distribution network with main feeder case:

The distribution network is as shown in the fig.3.1.

The substation node voltage $V_1$ is known and the remaining node voltages and the branch power losses are computed in the solution process. The power summation principle is used at the start of the problem formulation.

The principle is put in the form of equation as shown below.

$$P = \sum P_{Li+1} + \sum L_{Pj} \quad \quad [3.1]$$

$$Q = \sum Q_{Li+1} + \sum L_{Qj} \quad \quad [3.2]$$

The above eqns [3.1] give the initial power injected into the network.

The load on the network is assumed to be lumped at the source node of the network.

The node voltage is represented as $V \angle \delta$ where $V$ is the magnitude and $\delta$, phase angle.

The current flowing in the branch “j” in terms of terminal node voltage magnitudes and line parameters is obtained from the fig.3.3

$$I_j = (V_i \angle \delta_i - V_{i+1} \angle \delta_{i+1}) / r_j + j x_j \quad \quad [3.3]$$

The same current flowing through the branch “j” is defined in terms of node powers flowing through it.

$$I_j = (P_{i+1} - j Q_{i+1}) / V^*_{i+1} \quad \quad [3.4]$$
The eqns.[3.3] and [3.4] are one and the same. Hence
\[ (V_i \Delta \delta_i - V_{i+1} \Delta \delta_{i+1}) / r_j + j x_j = (P_{i+1} - j Q_{i+1}) / V_{i+1} \]  
[3.5]

On simplification and rearranging the terms
\[ V_{i+1}^4 + 2.0 \{ P_{i+1} r_j + Q_{i+1} x_j - 0.50 V_i^2 \} V_{i+1}^2 + (P_{i+1}^2 + Q_{i+1}^2)(r_j^2 + x_j^2) = 0 \]  
[3.6]
The above eqn.[3.6] is a 4th order equation for terminal node voltage magnitude \( V_{i+1} \).
The root of the above eqn.[3.6] is
\[ V_{i+1} = \left( (P_{i+1} r_j + Q_{i+1} x_j - 0.50 V_i^2)^2 - (P_{i+1}^2 + Q_{i+1}^2)(r_j^2 + x_j^2) \right)^{1/2} \]  
[3.7]
The eqn.[3.7] is the expression used to compute the terminal node voltage magnitude of the branch.
The exact power fed to the terminal node a branch 'j' connected between the node "i" and "i+1" is given as
\[ P_{i+1} = P_i - L_{Pj} - P_{Li} \]  
[3.8]
\[ Q_{i+1} = Q_i - L_{Qj} - Q_{Li} \]  
[3.9]
where
\( P_{Li}, Q_{Li} \) are the real and reactive loads at powers at the i" node
\( L_{Pj}, L_{Qj} \) are the real and reactive power losses flowing through the j" branch.
The eqns.[3.8] and [3.9] are the power flow equations used to compute the power injected into the terminal node "i+1".
The branch power losses \( L_{Pj} \) and \( L_{Qj} \) of the branch "j" between the nodes "i" and "i+1"
\[ L_{Pj} = (r_j \ Star \ (P_{i+1}^2 + Q_{i+1}^2)) / V_{i+1}^2 \]  
[3.10]
\[ L_{Qj} = (x_j \ Star \ (P_{i+1}^2 + Q_{i+1}^2)) / V_{i+1}^2 \]  
[3.11]
The eqns [3.10], [3.11] are the power flow equations used to compute the branch power losses.

The above power flow equations are used to compute the updated voltage magnitudes of the nodes and new branch power losses iteratively and once convergence check is achieved, the new branch power losses and new voltage magnitudes are published.

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The distribution network with main feeder and laterals is as shown in the fig.3.2. The network has one main feeder and 'N_{lat}' laterals. The main feeder is also designated as lateral 1 and remaining actual laterals are numbered from 2 to 'N_{lat}'.

The power flow equations for the main feeder with laterals use the power flow equations for the main feeder case. Since the laterals are present at the main feeder nodes, the exact power feeding through the terminal nodes is computed in a different way.

Using the power summation principle at any node, the power flowing through the terminal node of the branch is equal to the sum of the load powers connected and power losses in all the branches. It is represented in the expression form as shown below.

\[ P = \sum P_{L_{i+1}} + \sum L_{P_{j}} \]  \[ Q = \sum Q_{L_{i+1}} + \sum L_{Q_{j}} \]

where

\( P_{L_{i+1}}, Q_{L_{i+1}} \) are the real and reactive loads at powers at the \( i+1 \)th node

\( L_{P_{j}}, L_{Q_{j}} \) are the real and reactive power losses flowing through the \( j \)th branch.

Since the laterals are branching from the main feeder nodes, the exact power flowing from the main feeder nodes is computed in a different way.

From the fig 3.4

\[ P_{i+1} = T_F - P_s - S_{PL} \]  \[ Q_{i+1} = T_Q - Q_s - S_{QL} \]

where

\( P_{i+1}, Q_{i+1} \) are the real and reactive powers fed through the terminal node "\( i+1 \)".

\( T_F \) and \( T_Q \) are the total real and reactive powers fed through the source node "\( i \)".

\( P_s \) and \( Q_s \) are sum of the total real and reactive powers of the laterals connected to the source node "\( i \)".
SPL, SRL are sum of the real and reactive branch power loss of the branch between “i” and “i+1”.

The eqns.[3.13] and [3.14] are the power flow equations used to compute the power injected into the terminal node “i+1”.

The voltage magnitude of the terminal node is computed after knowing the exact power flowing through the terminal node is computed. The eqn.[3.7] is used for the voltage magnitude computation of a terminal node of a branch “j” between “i” and “i+1”.

\[ V_{i+1} = \left( \frac{((P_{i+1}r_j + Q_{i+1}x_j - 0.50 V_{i+1}^2)^2 - (P_{i+1}^2 + Q_{i+1}^2)(r_j^2 + x_j^2))^\frac{1}{2}}{(P_{i+1}r_j + Q_{i+1}x_j - 0.50 V_{i+1}^2)} \right)^2 \]

The eqn.[3.15] is the power flow equation used to compute the voltage magnitude of the “i+1” node.

The branch power losses \( L_{\sigma j} \) and \( L_{Q j} \) of the branch “j” between the nodes “i” and “i+1”

\[ L_{\sigma j} = \frac{(r_j * (P_{i+1}^2 + Q_{i+1}^2))}{V_{i+1}^2} \]

\[ L_{Q j} = \frac{(x_j * (P_{i+1}^2 + Q_{i+1}^2))}{V_{i+1}^2} \]

The eqns [3.16], [3.17] are the power flow equations used to compute the branch power losses of branch “j”.

3.3.2 Load flow Problem formulation using DistFlow method:

In this section, the load flow problem is formulated for distribution network using the DistFlow method. The problem formulation is carried out only for the main feeder case only and the distribution network main feeder with laterals is not attempted.

3.3.2.1 Distribution network with main feeder:

The distribution network as shown in the fig.3.3 is considered for the problem formulation. The distribution network has only main feeder with \( N_b \) nodes \( N_{b-1} \) branches. Each branch between the nodes has series resistance and reactance \((r, x)\) and there is a
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Load of $P_L$ and $Q_L$ that are constant in value. In the first stage of the problem formulation the distribution network is reduced to a single line equivalent network.

Using the power summation principle at any node, the power flowing through the terminal node of the branch is equal to the sum of the load powers connected and power losses in all the branches.

$$P = \sum P_{i+1} + \sum L_{pj} \quad [3.18]$$
$$Q = \sum Q_{i+1} + \sum L_{qj} \quad [3.19]$$

where

$P$ and $Q$ are the total real and reactive powers fed through the source node

$P_{i+1}$, $Q_{i+1}$ are the real and reactive loads at powers at the terminal node

$L_{pj}$, $L_{qj}$ are the real and reactive power losses flowing through the $j^{th}$ branch.

The eqns. [3.18] and [3.19] are the power flow equations used to compute the power injected into the source node.

The single line equivalent network is a network with a single branch having a source node and terminal node. The source node is the node through which the power required both real and reactive $(P, Q)$ by the network is fed. The source node voltage $V$ is known. The total load power both real and reactive is lumped at the terminal node of the single line equivalent network. The branch connecting between the source node and terminal node is represented with an equivalent resistance and reactance $r_{eq}$, $x_{eq}$.

The power flow equation at a node “$i+1$ “can be written as

$$P_{i+1} = P_i - L_{pj} - P_{i+1} \quad [3.20]$$
$$Q_{i+1} = Q_i - L_{qj} - Q_{i+1} \quad [3.21]$$

$L_{pj}$ is the real power loss and $L_{qj}$ is the reactive power loss in the branch “$j$”

$$L_{pj} = \frac{(r_j (P_i^2 + Q_i^2))}{V_i^2} \quad [3.22]$$
$$L_{qj} = \frac{(x_j (P_i^2 + Q_i^2))}{V_i^2} \quad [3.23]$$

Similarly the branch “$j+1$ “real and reactive power loss are given as

$$L_{p{j+1}} = \frac{(r_{j+1} (P_i^2 + Q_i^2))}{V_{i+1}^2} \quad [3.24]$$
$$L_{q{j+1}} = \frac{(x_{j+1} (P_i^2 + Q_i^2))}{V_{i+1}^2} \quad [3.25]$$

The ratio of real power loss and reactive power loss of branch “$j+1$” and “$j$” is written as

$L_{p{j+1}}/L_{pj}$
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\[ L_{Q+j'i} / L_{Qj} = (r_{j+1} (P_{r+i}^2 + Q_{r+i}^2)) / (P_i^2 + Q_i^2) \] 

\[ L_{P+j'i} / L_{Pj} = (x_{j+1} (P_{r+i}^2 + Q_{r+i}^2)) / (P_i^2 + Q_i^2) \]  

The eqn.[3.26] and [3.27] are rewritten as

\[ L_{P+j'i} / L_{Pj} = (r_{j+1} (P_{r+i}^2 + Q_{r+i}^2)) / (r_i (P_i^2 + Q_i^2)) \] 

\[ L_{Q+j'i} / L_{Qj} = (x_{j+1} (P_{r+i}^2 + Q_{r+i}^2)) / (x_i (P_i^2 + Q_i^2)) \] 

On further simplification

\[ L_{P+j'i} / L_{Pj} = (r_{j+1} / r_i) (P_{r+i}^2 + Q_{r+i}^2) / (P_i^2 + Q_i^2) \] 

\[ L_{Q+j'i} / L_{Qj} = (x_{j+1} / x_i) (P_{r+i}^2 + Q_{r+i}^2) / (P_i^2 + Q_i^2) \] 

The ratio \( V_{i+1}^2 / V_{i+1}^2 \) is eliminated from the expression as follows.

Let the current \( I_j \) flowing in the branch “j” can be written as

\[ I_j^2 = (P_i^2 + Q_i^2) / (V_i^2) \] 

\[ I_j^2 = ((P_{Li+i} + P_{ri+i})^2 + (Q_{Li+i} + Q_{ri+i})^2) / (V_{i+1}^2) \] 

From the eqns [3.30] and [3.31] and on simplification

\[ (V_i^2 / V_{i+1}^2) = ((P_{Li+i} + P_{ri+i})^2 + (Q_{Li+i} + Q_{ri+i})^2) / (P_i^2 + Q_i^2) \] 

The eqn.[3.32] is the power flow equation used to compute the voltage magnitudes of the “i+1”.

The eqn.[3.32] is substituted in the eqns.[3.28] and [3.29] and on simplification

\[ L_{P+j'i} / L_{Pj} = (r_{j+1} / r_i) ((P_{r+i}^2 + Q_{r+i}^2) / ((P_{Li+i} + P_{ri+i})^2 + (Q_{Li+i} + Q_{ri+i})^2) \] 

\[ L_{Q+j'i} / L_{Qj} = (x_{j+1} / x_i) ((P_{r+i}^2 + Q_{r+i}^2) / ((P_{Li+i} + P_{ri+i})^2 + (Q_{Li+i} + Q_{ri+i})^2) \] 

The eqns.[3.33] and [3.34] are used to compute the branch power losses.

The sum of the real power loss be \( W_R \) and reactive power loss be \( W_X \), after knowing the individual branch power losses

\[ W_R = \sum L_{Pj} \] 

\[ W_X = \sum L_{Qj} \]
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Let the single line equivalent network is present with \( r_{eq} \) and \( x_{eq} \) values and power fed to the line be \( P \) and \( Q \) with the source voltage \( V \).

The real power loss be and reactive power loss are written as

\[
W_R = r_{eq} \left( P^2 + Q^2 \right) / V^2 \quad [3.37]
\]

\[
W_X = x_{eq} \left( P^2 + Q^2 \right) / V^2 \quad [3.38]
\]

It can be observed that line power losses \( W_R \) and \( W_X \) from the eqns [3.37] and [3.38] are computed from the eqns [3.35] and [3.36]

\[
r_{eq} = (W_R) V^2 / (P^2 + Q^2) \quad [3.39]
\]

\[
x_{eq} = (W_X) V^2 / (P^2 + Q^2) \quad [3.40]
\]

The distribution network is reduced to a single branch network with an equivalent resistance and reactance \( (r_{eq}, x_{eq}) \) as shown in eqns [3.39] and [3.40].

The single line diagram is as shown in the fig. 3.5.

The single line equivalent network derived from the distribution network is used to determine the total real and reactive powers required in to the network.

From the fig. 3.5

\( P, Q \) are the total power injected in to the single line equivalent network and \( P_L, Q_L \) are the total real and reactive load powers lumped at the terminal node and \( V \) is the voltage magnitude of the source node.

The real power and reactive power fed in to the source node is given by the equation.

\[
P = r_{eq} \left( P^2 + Q^2 \right) / V^2 + P_L \quad [3.41]
\]

\[
Q = x_{eq} \left( P^2 + Q^2 \right) / V^2 + Q_L \quad [3.42]
\]

By rearranging the eqns [3.41] and [3.42] and eliminating the \( (P^2 + Q^2) / V^2 \), a quadratic equation in terms of \( P \) is obtained.

\[
(r_{eq}^2 + x_{eq}^2) P^2 + (2 x_{eq}^2 P_L - 2 r_{eq} x_{eq} Q_L + r_{eq}) P + (x_{eq}^2 P_L^2 + r_{eq} Q_L^2 - 2 r_{eq} x_{eq} P_L Q_L + r_{eq} V^2 P_L) = 0 \quad [3.43]
\]

This is of the form

\[
AP^2 + BP + C = 0 \quad [3.44]
\]

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Where

\[
A = (r_{eq}^2 + x_{eq}^2)
\]

\[
B = (2 x_{eq}^2 P_L - 2 r_{eq} x_{eq} Q_L + r_{eq})
\]

\[
C = (x_{eq}^2 P_L^2 + r_{eq}^2 Q_L^2 - 2 r_{eq} x_{eq} P_L Q_L + r_{eq} V^2 P_L)
\]

The roots of this quadratic equation \[3.43\] give the new real power injections in the system.

\[
P = -B \pm \sqrt{(B^2 - 4AC)} / 2A.
\] \[3.45\]

Similarly by rearranging the equation \[3.41\] and \[3.42\] and eliminating the \((P^2 + Q^2) / V^2\) a quadratic equation in terms of \(Q\) is obtained.

\[
(r_{eq}^2 + x_{eq}^2)Q^2 + (2r_{eq}^2 Q_L - 2r_{eq} x_{eq} P_L + x_{eq}) Q + (x_{eq}^2 P_L^2 + r_{eq}^2 Q_L^2 - 2 r_{eq} x_{eq} P_L Q_L + x_{eq} V^2 Q_L) = 0
\] \[3.46\]

This is of the form

\[
AA Q^2 + BB Q + CC = 0
\] \[3.47\]

Where

\[
AA = (r_{eq}^2 + x_{eq}^2)
\]

\[
BB = (2 r_{eq}^2 Q_L - 2 r_{eq} x_{eq} P_L + x_{eq})
\]

\[
CC = (x_{eq}^2 P_L^2 + r_{eq}^2 Q_L^2 - 2 r_{eq} x_{eq} P_L Q_L + r_{eq} V^2 Q_L)
\]

The roots of this quadratic equation give the new reactive power injections in the system.

\[
Q = (-BB \pm \sqrt{(BB^2 - 4AC)}) / 2AA.
\] \[3.48\]

The eqns.\[3.45\], \[3.48\] are power flow equations for the real and reactive power fed in to the network.

3.3.3 Load flow Problem formulation using Tellegen theorem method:

A new load flow solution technique for radial distribution network using the Tellegen theorem is presented. The distribution network considered has two configurations, one is the main feeder only and the other is the main feeder with laterals. The basic Tellegen theorem principle is effectively utilized to develop the core power flow equation for the load flow method. The power summation principle at a node is also used to determine the power flowing through the terminal node. Since the node voltage angles are omitted, a simple linear equation for the node voltage magnitudes is derived.

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3.3.3.1 Distribution network with main feeder:

The power flow equations are developed using the basic Tellegen theorem and the distribution network is treated like a ladder network.

The distribution network with main feeder is as shown in the fig.3.1.

It is converted into a ladder network for obtaining the power flow equations. The distribution network with main feeder is in the ladder circuit form as shown in the fig.3.6.

The ladder network elements are like sources, and series branches with \((r, x)\) and loads with \((P_L, Q_L)\).

The ladder network, as shown in the fig.3.6, has single port through which the power is fed to the network and the series branch elements and load elements dissipate this power.

The expression for the complex power in terms of the node voltage magnitudes and the node currents is written using the Tellegen theorem and the eqn.[1.52]

\[
V_1 I_1^* = (V_1 - V_2) I_1^* + V_2 I_2^* + (V_2 - V_1) I_2^* + V_3 I_3^* + \ldots + (V_{nh-1} - V_{nh}) I_{nh}^* + V_{nh} I_{nb}^* 
\]

[3.49]

The left-hand side term \(V_1 I_1^*\) is the power injected to the distribution network through the substation node. The complex power injected in the network is resolved into two components: real power and reactive power.

\[
V_1 I_1^* = P_p + j Q_p \quad [3.50]
\]

Where

\[
P_p = \text{Real Power injected into the network} \\
Q_p = \text{Reactive power injected into network}
\]

The term \((V_1 - V_2) I_1^*\) is the power consumed in the series branch element between the nodes 1 and 2. This is called as the series branch power loss.

\[
(V_1 - V_2) I_1^* + (V_2 - V_3) I_2^* + \ldots + (V_{nb-1} - V_{nb}) I_{nb}^* \quad \text{is the summation of all the branch power losses.} 
\]

\[
(V_1 - V_2) I_1^* + (V_2 - V_3) I_2^* + \ldots + (V_{nb-1} - V_{nb}) I_{nb}^* = \sum (V_i - V_{i+1}) I_i^* \quad [3.51]
\]
where the term \((V_i - V_{i+1}) I_j^*\) is the general series branch ‘j’ between the nodes the nodes “i” and “i+1”

The power loss of the branch ‘j’ between the nodes the nodes “i” and “i+1” is given as

\[
L_{p_j} = \text{Re} (V_i - V_{i+1}) I_j^* \quad [3.52]
\]

\[
L_{q_j} = \text{Im} (V_i - V_{i+1}) I_j^* \quad [3.53]
\]

The impedance of the branch “j” between the nodes “i” and “i+1” is \(Z_j = r_j + j x_j\)

The current flowing through the branch “j” having potential drop of \((V_i - V_{i+1})\) is

\[
I_j = \frac{(V_i - V_{i+1})}{Z_j} \quad [3.54]
\]

The complex conjugate of the current vector \(I_j\) is

\[
I_j^* = \frac{(V_i - V_{i+1})}{Z_j^*} \quad [3.55]
\]

On simplification

\[
I_j^* = \frac{(V_i - V_{i+1}) (r_j + j x_j)}{(r_j^2 + x_j^2)} \quad [3.56]
\]

The eqn.[3.56] is substituted in the eqns.[3.52] and [3.53]

\[
L_{p_j} = \text{Real part of } \frac{((V_i - V_{i+1}) (V_i - V_{i+1})) (r_j + j x_j)}{(r_j^2 + x_j^2)} \quad [3.57]
\]

\[
L_{q_j} = \text{Imaginary part of } \frac{((V_i - V_{i+1}) (V_i - V_{i+1})) (r_j + j x_j)}{(r_j^2 + x_j^2)} \quad [3.58]
\]

The term \(V_2 I_{2,1}^*\) is the power consumed by the load connected at the node 2.

The term \(V_2 I_{2,1}^* + V_3 I_{3,1}^* + \ldots\) is the sum of the powers consumed by the loads connected at all the nodes. Hence

\[
V_2 I_{2,1}^* + V_3 I_{3,1}^* + \ldots V_{ab} I_{ab}^* = \Sigma V_i I_i^* \quad [3.59]
\]

The load power at a node ‘i’ is resolved in to two components real power \(P_{Li}\) and reactive power \(Q_{Li}\)

\[
\Sigma P_{Li} + j Q_{Li} = \Sigma V_i I_i^* \quad [3.59]
\]

The phase angles are omitted in the problem formulation. The reason for this omission is that node voltage phase angles do not vary much for the distribution networks and the power flowing through the series branch are effectively dependent on the voltage magnitudes only.
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The eqns [3.50], [3.57], [3.58], and [3.59] are substituted in the equation [3.49] and on simplification

\[
P_p = \sum ((V_i - V_m)^2 (r_j/ (r_j^2 + x_j^2)) + \sum P_{Li} \tag{3.60}
\]

\[
Q_p = \sum ((V_i - V_m)^2 (x_j/ (r_j^2 + x_j^2)) + \sum Q_{Li} \tag{3.61}
\]

The eqns [3.60] and [3.61] are the linear algebraic equations for the real and reactive powers injected in to the network ,in terms of line parameters and voltage magnitudes. It is the objective function for the load flow solution using the Tellegen theorem method.

The eqns [3.60] and [3.61] are the power flow equations used to determine the total power injected in to the network.

The power balance equation at the node ‘i+1’ is written as

\[
P_{i+1} = P_i - L_{pj} - P_{Li} \tag{3.62}
\]

\[
Q_{i+1} = Q_i - L_{Qj} - Q_{Li} \tag{3.63}
\]

Where

\[
L_{pj} = ((V_i - V_{i+1})^2 (r_j/ (r_j^2 + x_j^2))
\]

\[
L_{Qj} = ((V_i - V_{i+1})^2 (x_j/ (r_j^2 + x_j^2))
\]

The eqns [3.62],[3.63], [3.57] and [3.58] are the power flow equations used to determine the exact power feeding through the terminal node and the branch power losses.

The current flowing in the branch “j” between the nodes “i” and “i+1”

\[
I_j^2 = (V_i - V_{i+1})^2 (r_j^2 + x_j^2) \tag{3.64}
\]

The current flowing in the line ‘j’ in terms of node injected powers and voltage magnitude

\[
I_j^2 = (P_i^2 + Q_i^2) / V_i^2 \tag{3.65}
\]

From the equations [3.64] and [3.65]

\[
V_{i+1} = (V_i^2 - K3) / V_i \tag{3.66}
\]

Where

\[
K3 = \sqrt{(P_i^2 + Q_i^2)(r_j^2 + x_j^2)} \tag{3.67}
\]

The eqns [3.66] and [3.67] are the power flow equations used to determine the node

Voltage magnitude of the terminal node.

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3.3.3.2 Distribution network with main feeder and laterals:

In this section, load flow problem is formulated for the distribution network with main and laterals using Tellegen theorem method.

A practical distribution network is a network that contains both main feeder and laterals. Each branch between the nodes is represented with branch impedance \((r + j x)\) and each node has constant power load of \(P_L + j Q_L\). Since network has both main feeder and laterals, a proper indexing scheme is to be applied otherwise the data retrieval and storing becomes a tough job. The distribution network considered here is as shown in the fig.3.4. The indexing scheme followed is as explained in the Table-3.1.

The distribution network has one main feeder and the laterals emanate from the main feeder nodes. The main feeder is treated as ‘Lateral 1’ and the all other laterals are numbered from 2, 3… \(N_{lat}\). The main feeder (Lateral 1) nodes are numbered first and the actual lateral (Lateral 2, 3… \(N_{lat}\)) nodes are numbered next. The nodes are marked in [ ] brackets. The node at which the main feeder or lateral leaves the first node is designated as start node \(SN( )\) and the node immediate to start node is designated as \(LB( )\) and the end node of the lateral is designated a \(EB( )\). The main feeder (Lateral 1) branches are numbered first and then the actual lateral (Lateral 2, 3…) branches are numbered next.

The load flow solution is obtained by treating the each lateral as a main feeder and the solution method for the main feeder case is repeatedly applied. The power flow equations for the distribution network with main feeder and laterals are same as the equations for the distribution network with main feeder only which is explained in the section “3.3.3.1 Distribution network with main feeder case”.

The Tellegen theorem is applied to the distribution network and a linear equation is developed.

\[
P_p = \sum ((V_i - V_{i+1})^2 (t_j^2 / (t_j^2 + x_i^2)) + \sum P_{li} \quad [3.68]
\]

\[
Q_p = \sum ((V_i - V_{i+1})^2 (x_j^2 / (t_j^2 + x_i^2)) + \sum Q_{li} \quad [3.69]
\]
The eqns. [3.68] and [3.69] are the linear algebraic equations for the real and reactive powers injected into the network in terms of line parameters and voltage magnitudes. It is the objective function for the load flow solution using the Tellegen theorem method.

The eqns. [3.68] and [3.69] are the power flow equations used to determine the total power injected into the network.

The power balance equation at the node 'i+1' is written as

\[ P_{i+1} = P_i - L_{Qj} - P_L \]  \hspace{2cm} [3.70]
\[ Q_{i+1} = Q_i - L_{Qj} - Q_L \]  \hspace{2cm} [3.71]

Where

\[ L_{Qj} = ((V_i - V_{i+1})^2 (r_i / (r_i^2 + x_i^2)) \]  \hspace{2cm} [3.72]
\[ L_{Qj} = ((V_i - V_{i+1})^2 (x_i / (r_i^2 + x_i^2)) \]  \hspace{2cm} [3.73]

The eqns. [3.70], [3.71], [3.72] and [3.73] are the power flow equations used to determine the exact power feeding through the terminal node and the branch power losses.

\[ V_{i+1} = (V_i^2 - K3) / V_i \]  \hspace{2cm} [3.74]

Where

\[ K3 = \sqrt{ (P_i^2 + Q_i^2)(r_i^2 + x_i^2) } \]  \hspace{2cm} [3.75]

The eqns. [3.74] and [3.75] are the power flow equations used to determine the voltage magnitude of the terminal node.