Literature Survey on Load flow methods and Sensitivity Analysis
Chapter 2

Literature Survey on Load Flow Methods and Sensitivity Analysis.

2.1 Introduction:

In this chapter, various load flow methods developed for the distribution networks with main feeder and main feeder with laterals, are discussed. The survey is initiated with the “Modified Fast Decoupled Load Flow” (FDLF) methods and then explained the need for the development of exclusive load flow methods for the distribution networks.

Later, the survey is continued on the sensitivity analysis of power systems networks, and studied about the utility of the “Adjoint Network Method” to the power systems transmission networks.

2.2 Survey on Load Flow methods:

It is known that, the “Fast Decoupled Load Flow” (FDLF) method is a suitable load flow method for the transmission networks, because it reliable and the fastest method in obtaining the convergence. Generally, the transmission networks are loop configured and have branches of low \((r/x)\) ratio. The assumption that, “the series branch conductance \((G_p)\) should be made maintained smaller than the series branch susceptance \((B_p)\)” is the decoupling assumption used for the FDLF load flow solution.

The researchers attempted to obtain the load flow solution of a distribution network using the FDLF method. As the distribution networks are generally radial in configuration with branches of high \((r/x)\) ratios, it was observed that the “Fast Decoupled Load Flow” (FDLF) method could not solve the problems with regard to non-convergence and long execution time. The non-convergence and long execution time are
lowered to reasonable extent with the modifications proposed in the network parameters and the load flow method used is called as "Modified FDLF method".

2.2.1 Modified FDLF method \cite{5,6,15,29,38}:

In the suggested modification certain compensating devices are added which modifies the network parameter values. The external compensating devices varies the conductance of the branch $G_{pq}$ maintaining the decoupling assumption that the conductance of the branch should exceed the susceptance of the branch. Due to the modification, the ratio $(r/x)$ of the branch is lowered and led to a load flow solution with better convergence. It is reported that, many compensation methods\cite{13} are proposed namely series, parallel and or series-parallel compensation methods. The compensation values are obtained from the empirical equations or from the experimental methods. It was observed that the load flow solution was not a fully converged one, even with the compensated values.

Hence various methods of load flow solution were developed for the distribution networks to obtain a greater computational speed, low storage and more reliability and better convergence. As no one method possesses all the desirable features of the others, the load flow solution of radial distribution networks has become a significant field of interest to the researcher.

2.2.2 Exclusive Load Flow methods developed for Distribution networks:

The load flow solution of the distribution networks is attempted by changing the view of the network as a simple RL circuit in the form of ladder network and the power flow equations are developed to determine the node voltage magnitudes.

Ladder network method \cite{8,11}:

As the distribution network is treated like a simple RL circuit (ladder circuit), the basic Kirchoff's laws are utilized to develop the power flow equations. The load flow
solution is found iteratively. The power flow equations are the real and reactive powers flowing in the branches and the magnitude of the terminal node voltage which are developed using the power conservation principle. The principle is, that is the power fed to the node is equal to the sum of the powers consumed by the branch and the power fed to the load at the node.

The model distribution network is represented as a ladder network shown in the Figs 2.1, 2.2 and 2.3.

From the ladder network it can be observed that, the \( V_1 \) is the source node voltage and the branch between the nodes is represented with impedances \( Z_1, Z_2, ..., Z_n \) and the loads connected at the nodes are represented with shunt admittance \( Y_L, Y_L, ..., Y_L \).

**Linear Ladder load flow method:**

The method begins with the assumption of the end node voltage \( V_n \).

The current \( I_{Un} \) drawn by the load is calculated with the eqn.\([2.1]\)

\[
I_{Un} = Y_L V_n
\]  \([2.1]\)

With the \( I_{L(n-1)} \) known, the series branch current is computed as \( I_{n-1} = I_{L(n-1)} \).

The potential drop across the \((n-1)\)th series branch is computed using the eqn.\([2.2]\).

\[
V_{n-1} = I_{L(n-1)} Z_{n-1}
\]  \([2.2]\)

Knowing the node voltage \( V_{n-1} \), the load current \( I_{Un-1} \) can be computed using the eqn.\([2.3]\).

\[
I_{Un-1} = Y_{L(n-1)} V_{n-1}
\]  \([2.3]\)

This process is continued till the source node voltage magnitude \( V_1 \) is calculated.

The calculated node voltage magnitude \( V_1 \) is compared with the known input voltage \( E_1 \) and a Correction Factor (CF) is computed.

\[
CF = \frac{E_1}{V_1}
\]  \([2.4]\)

This correction factor is multiplied to the voltages and currents computed to obtain the updated voltages and currents. The updated voltages and currents are used to compute the branch power losses. The process is continued till the convergence is achieved. If the

*Sri Venkateswara University, TIRUPATI.*
Load flow study and Sensitivity Analysis of Distribution networks using Tellegen theorem

distribution network is a non-linear network, the loads are represented as constant complex power elements and non-linear load flow method is used.

Non-Linear Ladder load flow method:

The load flow solution is obtained by using either of the methods “Backward update method” or “Forward update method”. The two methods differ in the solution methodology and initial assumptions made.

Backward update method:

This load flow method is initiated with the assumption of the end node voltage $V_n$.

With the known end node voltage and on single phase basis, the load current $I_{Ln}$ is computed using the eqn.[2.5]

$$I_{Ln} = \left( \frac{S_{Ln}}{V_n} \right)^*$$  \[2.5\]

Knowing the current through the $(n-1)^{th}$ branch, the voltage drop in that branch is computed and the $(n-1)^{th}$ node voltage magnitude is computed. Likewise the remaining node voltages are computed in the backward direction, till the source node voltage $V_1$ is computed. This is called as “Estimated source node voltage”. It is compared with the “Initial source node voltage” $E_0$. Due to the non-linearity of the network, a simple correction factor cannot be determined and cannot be used to compute all other calculations of currents and voltages. Hence the change in voltage value $\Delta V$ is computed.

$$\Delta V = E_0 - V_1$$  \[2.6\]

This calculated $\Delta V$ is applied to the assumed voltage $V_n$

$$V_{n\ new} = V_{n\ old} + \Delta V$$  \[2.7\]

The backward computation process is continued and the source node voltage magnitude is computed and the $\Delta V$ is maintained within a specified tolerance value. Since the computation process is carried in a backward direction, this computation process is called “Backward update method”.

Forward update method:

The load flow method is initiated with the branch power losses to zero. With this assumption, the total power injected into the network is made equal the total load

*Sri Venkateswara University, Tirupati.*
connected to the distribution network. The source node voltage $E_s$ is known and the
terminal node voltage of each series branch is computed using the branch power flow
eqn. [2.8] and [2.9].

\[ P_{i+1} = P_i - L_{pj} P_{Li} \]  \hspace{1cm} [2.8]  \\
\[ Q_{i+1} = Q_i - L_{qj} Q_{Li} \]  \hspace{1cm} [2.9]

Where
- $P_i, Q_i$ are the real and reactive powers sending end of the branch between the “i” and
  “i+1”.
- $P_{i+1}, Q_{i+1}$ are the real and reactive powers terminal node of the branch between the “i”
  and “i+1”.
- $P_{Li}, Q_{Li}$ are the real and reactive powers flowing through the loads connected at “i + 1”.
- $L_{pj}$ and $L_{qj}$ are the real and reactive branch power losses of “j”th branch.

The terminal node voltages and branch power losses are computed using the
power flow equations. The iteration process is continued till the end node of the network.
The branch power losses are subjected to tolerance check and iteration process is
continued until convergence is met.

The “Backward update method” and “Forward update methods” are called
exclusive load flow methods of Distribution networks.

S.K. Goswamy, et al. proposed a loop based load flow solution method for a
general distribution network as a method as a direct solution method.
Using basic Kirchoff’s voltage law, the following loop equations are written.

\[ E_s = Z_{11} I_1 + Z_{12} I_2 + ... + Z_{1n} I_n \]
\[ E_s = Z_{21} I_1 + Z_{22} I_2 + ... + Z_{2n} I_n \]
\[ ... \]
\[ E_s = Z_{n1} I_1 + Z_{n2} I_2 + ... + Z_{nn} I_n \]  \hspace{1cm} [2.10]
Load flow study and Sensitivity Analysis of Distribution networks using Tellegen theorem

As the standard form of distribution network assumes that, no node is a junction of more than three branches namely one incoming and two outgoing branches, the network having any other configuration is transformed in to the standard form. This is possible by inserting a dummy node and a dummy line of zero impedances in the appropriate positions. The method restructuring the network to obtain the standard form is called as “preprocessing the original network”.

The loop impedance matrix is formulated with the diagonal term as the summation of all impedance elements through which the current of the \( i^{th} \) load node flows. The off diagonal term is the impedance of the branch between the \( i^{th} \) and \( j^{th} \) node. The impedance matrix formed is solved using the LU decomposition.

Baran and Wu \cite{20} proposed a load flow solution for the radial distribution network with main feeder as well as for the network with main feeder and laterals. The branch power flow equations are developed are called “DistFlow branch equations”. The equations represent the real, reactive power flowing through the node and voltage at the node.

The \( P_{i} \), \( Q_{i} \) and \( V_{i} \) represent the quantities at the sending end of the branch and equations are written for the receiving end quantities \( P_{i+1} \), \( Q_{i+1} \) and \( V_{i+1} \).

The DistFlow power equations are

\[
\begin{align*}
    P_{i+1} &= P_{i} - r_{i} \left( P_{i}^2 + Q_{i}^2 \right) / V_{i}^2 - P_{L_{i+1}} \\
    Q_{i+1} &= Q_{i} - x_{i} \left( P_{i}^2 + Q_{i}^2 \right) / V_{i}^2 - Q_{L_{i+1}} \\
    V_{i+1}^2 &= V_{i}^2 - 2 \left( r_{i} P_{i} + x_{i} Q_{i} \right) + \left( r_{i}^2 + x_{i}^2 \right) \left( P_{i}^2 + Q_{i}^2 \right) / V_{i}^2
\end{align*}
\]

[2.11]

Where

\[
\begin{align*}
    \text{Real power loss} &= r_{i} \left( P_{i}^2 + Q_{i}^2 \right) / V_{i}^2 \\
    \text{Reactive power loss} &= x_{i} \left( P_{i}^2 + Q_{i}^2 \right) / V_{i}^2
\end{align*}
\]

[2.12]

The above set of eqns. [2.11] can also be written for the sending end node quantities from the receiving end node quantities. The \( P_{i} \), \( Q_{i} \) and \( V_{i} \) represent the

Sri Venkateswara University, TIRUPATI.
quantities at the receiving end quantities of the branch and equations are written for the sending end quantities $P_{i+1}$, $Q_{i+1}$ and $V_{i+1}$.

\[ P_{i+1} = P_i + r_i \frac{(P'^2 + Q'^2)}{V_i^2} + P_i \]

\[ Q_{i+1} = Q_i + x_i \frac{(P'^2 + Q'^2)}{V_i^2} - Q_i \]

\[ V_{i+1} = V_i + 2 \left( r_i \frac{P_i + Q_i}{V_i} + x_i \right) \frac{(P'^2 + Q'^2)}{V_i^2} \]

[2.13]

Where

Real power loss \[ = \frac{r_i \frac{(P'^2 + Q'^2)}{V_i^2}}{V_i} \]

Reactive power loss \[ = \frac{x_i \frac{(P'^2 + Q'^2)}{V_i^2}}{V_i} \]

where

\[ P' = P_i + P_{i+1} \]

\[ Q' = Q_i + Q_{i+1} \]

The load flow method is initiated with known or estimated quantities $P_o$, $Q_o$, $V_o$ at the first node of the distribution network. The same quantities at the other nodes are calculated by applying the above branch equations.

The load flow solution is obtained using forward update methods. The expression for branch power loss is considered as an objective function. The branch power losses are initialized to zero and the total power injected in to the first node is the total load on the network. From this information and the known source node voltage, the voltages of the remaining nodes are determined using the “Forward update method”. Then the branch power losses are updated and convergence check is applied.

D.Das, et al [30,31,32] proposed a load flow method called as “Forward sweeping method” for radial distribution network with only main feeder as well as main feeder with laterals. The power conservation principle is used to determine the power flowing through terminal node of the branch and the corresponding node voltage is computed using the 4th order equation developed. The branch power losses are updated with the updated voltages and a convergence check is made on these updated branch power losses.

Sri Venkateswara University, TIRUPATI.
Though convergence is guaranteed, the computation time is very high due to the $4^{th}$ order expression developed for the terminal node voltage.

Jasmorn and Lee [29,27,28] have proposed a solution method called “DistFlow method” for radial Distribution networks. The power flow equations, representing the real, reactive powers flowing through the branches and the terminal node voltage derived by Baran and Wu [30], are used for the solution determination. The network is reduced into a single equivalent network having a branch with equivalent resistance and reactance values. An iterative procedure is used to compute the total power required by the distribution network. The repeated voltage computations are avoided, thus saving a large computation time. But the expressions developed are of higher order expressions and iterative process is complicated. Due to this, the load problem is restricted to main feeder case only and no attempt has been made for the distribution network with laterals.

M. AfSari et al. [31] presented separate algorithms to estimate the terminal node voltage to start the iteration process. The iterative process is initiated in the backward direction from the terminal node to the source node and is considered to be having a built-in corrective feature. The voltage is considered as a complex variable having magnitude and phase angle. The phase angle is not included in the power flow equations. A good estimate of the terminal node voltage is very much needed to reduce the number of iterations otherwise, the number of iteration increases with bad start of the terminal voltage.

It is observed that, the exclusive load flow methods use the power conservation principle extensively and iteration process is heavily dependent on the initial assumptions made. Hence, it is thought if the same power conservation principle at the network level, is applied to the distribution network, a new load flow method can be developed. As the power conservation principle is a reality, the load flow solution can also be near to the real and practical situation.
2.3 Survey on Sensitivity Analysis:

The Automated transmission planning system requires a proper sensitivity analysis of a network so as to plan the strategies for the feasible network expansion. The system planner uses many analytical methods and the results of the automated methods are used as guide lines. The automated transmission planning requires the knowledge of generation, the location of the load centers and their capacities, and available transmission corridors. The real problem of the present day transmission system is, to determine the new capacity of the lines to be added or to reinforce the existing lines so that no overloading is taking place under normal steady state conditions.

Several approaches have been proposed to solve this problem. Initially DC power flow methods are developed. The voltage magnitude is set as important variable and the phase angle is ignored. Later AC power flow flow methods are developed using both magnitude and phase angle. The methods used the Tellegen theorem and its extensions. The procedure involved is the determination of solution of two networks the original network and other is called an Adjoint network. The "Adjoint Network" is a fictitious network defined mathematically.

SW Director and RA Rohere [2] have first developed the theory to simulate an “Adjoint Network” of a given original network and sensitivity computation of network is carried out using the perturbation analysis.

The corollary theorem of Tellegen Theorem called “Quasi power theorem” or “Difference form of Tellegen theorem” is used to develop the “Adjoint Network”.

It is reported that “Adjoint Network” of a resistive network is another resistive network with identical element values. This formed as a basis for the “Adjoint Network” simulation for a practical power system network. It is also proved that that “Adjoint Network” matrix is the transpose of the bus admittance matrix of the power system network. The sensitivity expression developed is a simple linear expression in terms of
voltages of the original and "Adjoint Network" and admittance values of the perturbed elements. The sensitivity of the node voltage is computed non-recursively.

John W Randler, Rudolph F Seviora\(^{[3]}\) applied this "Adjoint Network" approach to computer-aided microwave theory. Earlier direct search methods were used to optimization of the microwave network. Since the "Adjoint Network" model is dependent on the network topology, the computation time is very much decreased. The sensitivity computation involved, the actual network voltages and "Adjoint Network" parameters. Hence the "Adjoint Network" based methods have replaced the direct search methods.

Louis AFM Ferreira\(^{[14,16,23]}\) attempted the Tellegen theorem and "Adjoint Network Method" on the power system network to compute the sensitivity of node voltage to the changes in the load or network parameters.

The types of perturbations caused are load power perturbations and network parameter perturbations. For a given power system network, the "Adjoint Network" is modeled and a linear first order sensitivity expression for node voltage magnitude is written.

The "Adjoint Network" concept was first applied for the purpose of computing the network sensitivities based on DC load flow models and later AC load flow models are developed. The computation of first order sensitivities was developed and later on some higher order terms have been added in the problem formulation and analysis is carried out in the form of layered solutions. The layer 0 contains basic core results without any non-linearity and layer2 contains integrated results of all the linear as well as non-linear systems. The exact analysis of the power system response due to perturbations could not be obtained based on the methods of Tellegen theorem and many researchers continued to depend on the conventional load flow equations. The Jacobian matrix inverse computation was a big constraint in the analysis. The Inverse Matrix Modification Lemma (IMML) was the most important computational tool used in the perturbation. The IMML is a special type of matrix, used for each network perturbation.
thus avoiding reconstructing and refactorizing the un-perturbed network matrices. The power flow solution under network perturbations required a solution of Adjoint system which is perturbation invariant.

The methods based on the “Adjoint Network” are applied to the sensitivity evaluation of a power system. The analysis is split in to PQ and PQ-PV analysis. In the PQ analysis the power variation is assumed to be zero, that is, all the buses are considered as constant power buses. The variation of the node voltage due to branch perturbation is computed. In PQ-PV analysis, the reactive power at specified buses is varied and voltage variation is evaluated.

The “Adjoint Network” method is used to compute the sensitivity of the node voltages of the radial Distribution network due to the load perturbation both real and reactive power at one or many nodes. As no work has been reported on the sensitivity analysis of radial distribution network, there is no data for the comparison of the efficacy or and any other performance criterion.