Reliability deals with different phases of life time of a component or system. During this time period reliability engineering can have greatest effect for enhancing the safety and quality of the system. This chapter introduces basic concepts and their relationships.

1.0 INTRODUCTION

Products fail due to mistakes in design, variation in features such as dimensions, parameters, strength, etc., or due to the environment in which it would have to endure. The wear and degradation imposed by use and time also are to be considered. On production side, the effects of process variations on quality, yield, and cost should be minimum. On maintenance side, the product should be serviceable. Causes of failures include bad engineering design, faulty construction or manufacturing process, human error, poor maintenance, inadequate testing and inspection, improper use and lack of protection against excessive environmental stress.

Reliability in service can be improved if necessary precautions are taken during development and production. Greater care in design, more effort on training, use of better materials and processes along with more effective testing will enhance the Reliability of a product. Greater care and skill along with more effective inspection will give better quality to the product. However there is an optimum level of effort that should be expended on quality and reliability, beyond which further effort would be counter-productive. Reduced failures of products will limit the cost both to the customer and manufacturer. Reliability and maintainability are not only an important part of the engineering design process but also necessary in life-cycle costing, cost benefit analysis, operational capability studies, repair and facility resourcing, inventory and spare part requirement determination, replacement decisions, and the establishment of preventive maintenance programs.

The history of formal reliability studies goes back to the days of the second world war when it was felt necessary to develop methods for estimating the success rate of complex weapons. After the war, these methods were applied to electronic devices and in space technology. Most of the applications concerned with non-repairable devices and systems where the first failure would terminate the useful life.
of the device or system. The primary reason for reliability and maintainability engineering is to improve the reliability and availability of the product or system being developed and thereby add to its value.

Essentially, reliability studies provide predictions. They predict the future behavior of a device or system, based on past information and experience. Since predictions cannot be made with certainty, they are inherently probabilistic. Obtaining reliability figure/index of complex systems requires collection of field data, life testing experiments etc., Reliability will be evaluated based on the function the product /component which is supposed to perform with respect to environment and time.

At a given point in time, a component or system is either functioning or it has failed. A working component or system will eventually fail. The failed state will continue forever, if the component or system is non-repairable. A repairable component or system will remain in the failed state for a period of time, while it is being repaired and then transcends back to the functioning state when the repair is completed. The change from a functioning to a failed state is failure while the change from a failure to a functioning state is referred to as repair. It is also assumed that repairs bring the component or system back to an "as good as new" condition. This cycle continues with the repair-to-failure and the failure-to-repair process; and then, repeats over and over for a repairable system.

Engineering products can fail in service due to many reasons [1] These include:

- Variation of parameters and dimensions, leading to weakening, component mismatch, incorrect fits, vibration etc.,
- When applied stress exceeds the strength of a component (overstress) such as mechanical overstress leading to a fracture or bending of a beam or electrical overstress leading to local melting of an integrated circuit transistor or breakdown of the dielectric of a capacitor
- Wear-out due to time dependent mechanisms such as material fatigue, wear, corrosion, insulation deterioration, etc., which progressively reduce the strength of the component so that it can no longer withstand the stress applied.
There are many other causes of failures, such as electromagnetic interference in electronic systems, backlash in mechanical drives, friction leading to incorrect operation of mechanisms, leaks and excessive vibration etc. Knowledge, skill and effort can make a product reliable. It therefore follows that measurement of Reliability is only a statement of history. Statistical methods have been developed for measuring, analyzing and predicting Reliability.

Reliability is the ability of an item to perform a required function, under given environmental and operational conditions and for a stated period of time.

- The term “item” is used here to denote any component, subsystem or system that can be considered as an entity.

- A “required function” may be a single function or a combination of functions that is necessary to provide a specified service.

- All technical items (components, subsystems, systems) are designed to perform one or more (required) functions. Some of these functions are active and some functions are passive. For assessing the reliability, the required functions under consideration need to be specified.

- For hardware item to be reliable, it must exceed the initial factory performance or quality specification i.e., it must operate satisfactorily for a specific period of time in the actual application for which it is intended.

1.1 PROBABILITY FUNCTIONS

For computing reliabilities, four related probability functions [2] are required: the reliability function, the cumulative distribution function, the probability density function and the hazard rate function. Specifying any one of these functions will uniquely and completely characterize the failure process.
1.11 The Reliability Function

The Reliability [2], \( R(t) \), of a component or system is defined as the probability that a component or system will function over some time period \( t \). To express this relationship mathematically, the continuous random variable \( T \), the time to failure of the system or component is considered. Then Reliability can be expressed as

\[
R(t) = \Pr\{T \geq t\} 
\]

Where \( R(t) \geq 0 \), \( R(0) = 1 \), and \( \lim_{t \to \infty} R(t) = 0 \). For a given value of \( t \), \( R(t) \) is the probability that the time to failure is greater than or equal to \( t \).

1.12 Cumulative Distribution Function

The Cumulative Distribution Function [CDF], also known as Unreliability, \( F(t) \), of a component or system is defined as the probability that the component or system experiences the first failure or has failed one or more times during the time interval zero to time \( t \), given that it was operating or repaired to a like new condition at time zero.

\[
R(t) + F(t) = 1 \ \text{or Unreliability } F(t) = 1 - R(t) 
\]

If \( F(t) \) is defined as

\[
F(t) = 1 - R(t) = \Pr\{T < t\} 
\]

Where \( F(0) = 0 \)

and \( \lim_{t \to \infty} F(t) = 1 \)

then \( F(t) \) is the probability that a failure occurs before time \( t \).

1.13 Probability Density Function

This function, \( f(t) \) describes the shape of the failure distribution.

\[
f(t) = \frac{dF(t)}{dt} = \frac{dR(t)}{dt} 
\]

\[
... 1.3
\]
The PDF, \( f(t) \) has the following properties:

\[
f(t) \geq 0 \text{ and } \int_{0}^{\infty} f(t) \, dt = 1
\]

Given the PDF, \( f(t) \), then

\[
F(t) = \int_{0}^{t} f(t') \, dt' \quad \cdots 1.4
\]

\[
R(t) = \int_{t}^{\infty} f(t') \, dt' \quad \cdots 1.5
\]

In other words, both the Reliability function and CDF represent areas under the curve defined by \( f(t) \). Therefore, since the area beneath the entire curve is equal to one, both reliability and the failure probability will be defined so that

\[
0 \leq R(t) \leq 1 \quad 0 \leq F(t) \leq 1
\]

The function \( R(t) \) is normally used when reliabilities are being computed, and the function \( F(t) \) is normally used when failure probabilities are being computed. Graphing the PDF, \( f(t) \), provides a visual representation of the failure distribution. The three functions are illustrated in Fig. 1.1.

### 1.14 Hazard Rate Function

In addition to the probability functions defined earlier, another function, called the failure rate or hazard rate function, is often used in reliability.

\( \lambda(t) \) is known as the instantaneous hazard rate function or failure rate function. Conditional failure rate or failure intensity, \( \lambda(t) \), can be defined as the anticipated number of times an item will fail in a specified time period, given that it was as good as new at time zero and is functioning at time \( t \).

\[
\lambda(t) = \frac{f(t)}{R(t)} \quad \cdots 1.6
\]

The failure rate function \( \lambda(t) \) provides an alternative way of describing a failure distribution. Failure rates in some cases may be characterized as increasing (IFR), decreasing (DFR), or constant (CFR) when \( \lambda(t) \) is an increasing, decreasing or constant function.
(a) The reliability function
(b) The cumulative distribution function
(c) The probability density function

Fig. 1.1: Probability functions
Equation (1.7) can be used to derive the Reliability function from a known hazard rate function.

1.2 AVAILABILITY AND UNAVAILABILITY

In Reliability engineering and Reliability studies, it is the general convention to deal with unreliability and unavailability values rather than Reliability and availability [3]. The numerical value of both availability and unavailability are also expressed as a probability from 0 to 1 with no units.

The Availability, A(t), of a component or system is defined as the probability that the component or system is operating at time t, given that it was operating at time zero.

The Unavailability, Q(t), of a component or system is defined as the probability that the component or system is not operating at time t, given that is was operating at time zero.

Therefore, the following relationship holds good since a component or system must be either operating or not operating at any time:

\[ A(t) + Q(t) = 1 \] ...1.8

1.3 FAILURE CATEGORIES

The following three-level list of failure categories [4] is provided as an example:

- Type I - Failure: Severe operational incidents that would definitely result in a service call, such as part failures, unrecoverable equipment hangs, consumables that fail/deplete before their specified life, onset of noise, and other critical problems.
These constitute “hard-core” failure modes that would require the services of a trained repair technician to recover.

- **Type II** – Intervention: Any unplanned occurrence or failure of product mission that requires the user to manually adjust or otherwise intervene with the product or its output. These tend to be “nuisance failures” that can be recovered by the customer, or with the aid of phone support. Depending on the nature of the failure mode, groups of the Type II failures could be upgraded to Type I if they exceed a predefined frequency of occurrence.

- **Type III** – Event: Events will include all other occurrences that do not fall into either of the categories above. This might include events that cannot directly be classified as failures, but whose frequency is of engineering interest and would be appropriate for statistical analysis. Examples include failures caused by test equipment malfunction or operator error.

1.4 THE ROLE OF RELIABILITY PREDICTION (FAILURE RATES)

Reliability Prediction has many roles in the Reliability engineering process. The impact of proposed design changes on Reliability is determined by comparing the Reliability predictions of the existing and proposed designs.

A Reliability prediction can also assist in evaluating the significance of reported failures. Ultimately, the results obtained by performing a Reliability prediction analysis can be useful when conducting further analyses such as a FMECA (Failure Modes, Effects and Criticality Analysis), RBD (Reliability Block Diagram) or a FTA (Fault Tree Analysis). The Reliability predictions are used to evaluate the probabilities of failure events.

Failures are inevitable and although unavoidable, the life of a product, a component or a system can be extended with the help of various improved methods, techniques, and technical advancements. Complexity of the systems is increasing day...
by day in an effort to meet the ever growing demands of the customer, which imposes unavoidable cost factor for Reliability improvement.

Reliability predictions are carried out based on failure rates. Failure rate calculations are based on complex models which include factors using specific component data such as temperature, environment, and stress. In the prediction model, assembled components are structured serially. Thus, calculated failure rates for assemblies are a sum of the individual failure rates for components within the assembly.

There are three common basic categories of failure rates [3] which are given below:

- Mean Time Between Failures (MTBF)
- Mean Time To Failure (MTTF)
- Mean Time To Repair (MTTR)

### 1.41 Mean Time Between Failures (MTBF)

Mean time between failures (MTBF) is a basic measure of reliability for repairable items. MTBF can be described as the time passed before a component, assembly, or system fails, under the condition of a constant failure rate. Another way of stating MTBF is the expected value of time between two consecutive failures, for repairable systems. It is a commonly used variable in Reliability and maintainability analyses.

MTBF can be calculated as the inverse of the failure rate, $\lambda$, for constant failure rate systems. For example, for a component with a failure rate of 2 failures per million hours, the MTBF would be the inverse of that failure rate, $\lambda$, or:

$$MTBF = \frac{1}{\lambda} = \frac{1}{2 \text{ failures} \times 10^6 \text{ hours}} = \frac{1}{500,000 \text{ hours} \text{ / failure}} \approx 1.9$$
1.42 Mean Time To Failure (MTTF)

Mean time to failure (MTTF) is a basic measure of Reliability for non-repairable systems. It is the mean time expected until the first failure of a piece of equipment. MTTF is a statistical value and is intended to be the mean over a long period of time and with a large number of units. For constant failure rate systems, MTTF is the inverse of the failure rate, \( \lambda \). If failure rate, \( \lambda \), is in failures/million hours, \( \text{MTTF} = 1,000,000 / \text{Failure Rate, } \lambda \), for components with exponential distributions.

\[
\text{MTTF} = \frac{1}{\lambda \text{ failures/10⁶ hours}} \quad \ldots 1.10
\]

For repairable systems, MTTF is the expected span of time from repair to the first or next failure.

The mean time to failure (MTTF) is defined by

\[
\text{MTTF} = E(t) = \int_{0}^{\infty} tF(t) \, dt \quad \ldots 1.11
\]

(or)

\[
\text{MTTF} = \int_{0}^{\infty} R(t) \, dt \quad \ldots 1.12
\]

1.43 Mean Time To Repair (MTTR)

Mean time to repair (MTTR) is defined as the total amount of time spent performing all corrective or preventive maintenance repairs divided by the total number of those repairs. It is the expected span of time from a failure (or shut down) to the repair or maintenance completion. This term is typically only used with repairable systems.

1.5 COMPARISON OF THE MEASURES OF CENTRAL TENDENCY

The mean of the failure distribution is only one of several measures of central tendency of the failure distribution. Another is the median time to failure, defined by

\[
R(t_{\text{med}}) = 0.5 = Pr \{T \geq t_{\text{med}}\} \quad \ldots 1.13
\]
The median divides the distribution into two halves, with 50 percent of the failures occurring before the median time to failure and 50 percent occurring after the median. The median may be preferred to the mean when the distribution is highly skewed.

A third frequently used average is mode, or most likely observed failure time, defined by
\[
\text{mode} = \max_{t \in \text{domain}} f(t)
\] ... 1.14

Fig. 1.2 shows the locations of the mean, the median, and the mode for a distribution skewed to the right.

![Fig. 1.2: Measures of central tendency](image)

Obviously, the MTTF alone will not uniquely characterize a failure distribution. Other measures are necessary. One measure that is often used to further describe a failure distribution is its variance \( \sigma^2 \), defined by
\[
\sigma^2 = \int_{0}^{\infty} (t - \text{MTTF})^2 f(t) \, dt \] ... 1.15

The variance can also be written as
\[
\sigma^2 = \int t^2 f(t) \, dt - (\text{MTTF})^2
\] ... 1.16
1.6 FAILURE FREQUENCIES

There are four failure frequencies [3], which are commonly used in Reliability analyses.

- **Failure Density \( f(t) \):** The failure density of a component or system, \( f(t) \), is defined as the probability per unit time that the component or system experiences its first failure at time \( t \), given that the component or system was operating at time zero.

- **Failure Rate \( r(t) \):** The failure rate of a component or system, \( r(t) \) or \( \mu(t) \), is defined as the probability per unit time that the component or system experiences a failure at time \( t \), given that the component or system was operating at time zero and has survived to time \( t \).

- **Conditional Failure Intensity (or Conditional Failure Rate) \( \lambda(t) \):** The conditional failure intensity of a component or system, \( \lambda(t) \), is defined as the probability per unit time that the component or system experiences a failure at time \( t \), given that the component or system was operating, or was repaired to be as good as new, at time zero and is operating at time \( t \).

- **Unconditional Failure Intensity or Failure Frequency \( \omega(t) \):** The unconditional failure intensity of a component or system, \( \omega(t) \), is defined as the probability per unit time that the component or system experiences a failure at time \( t \), given that the component or system was operating at time zero.

1.7 REPAIRABLE AND NON-REPAIRABLE ITEMS

It is important to distinguish between repairable and non-repairable items [3] when predicting or measuring reliability.

1.7.1 Non-repairable Items

Non-repairable items are components or systems such as a light bulb, transistor, rocket motor, etc. Their reliability is the survival probability over the items expected life or over a specific period of time during its life, when only one failure can occur. During the component or systems life, the instantaneous probability of the first and only failure is called the hazard rate or failure rate, \( r(t) \). Life values such as MTTF described above are used to define non-repairable items.
1.72 Repairable Items

For repairable items, Reliability is the probability that failure will not occur in the time period of interest; or when more than one failure can occur, Reliability can be expressed as the failure rate, \( \lambda \), or the rate of occurrence of failures (ROCOF). In the case of repairable items, Reliability can be characterized by MTBF, but only under the condition of constant failure rate.

There is also the concern for availability, \( A(t) \), of repairable items since repair takes time. Availability, \( A(t) \), is affected by the rate of occurrence of failures (failure rate, \( \lambda \)) or MTBF plus maintenance time; where maintenance can be corrective (repair) or preventive (to reduce the likelihood of failure). Availability, \( A(t) \), is the probability that an item is in an operable state at any time.

\[
\text{Availability, } A(t) = \frac{\text{MTBF}}{\text{MTBF + MTTR}}
\]

Some systems are considered both repairable and non-repairable, such as a missile. It is repairable while under test on the ground; but becomes a non-repairable system when fired.

1.8 Failure Patterns

The bathtub curve occupies a place of considerable importance in reliability practice. The curve represents the idea that the operation of a population of devices can be viewed as comprised of three distinct periods viz., burn-in, useful life and wear-out periods. Bathtub curve is usually adopted to represent general trend of hazard rate function.

1.8.1 Non-repairable Items

There are three patterns [3] of failures for non-repairable items, which can change with time. The failure rate (hazard rate) may be decreasing, increasing or constant.

- Decreasing Failure Rate

A decreasing failure rate (DFR) can be caused by an item, which becomes less likely to fail as the survival time increases. This is demonstrated by electronic
equipment during their early life or the burn-in period. This is demonstrated by the first half of the traditional bathtub curve (Fig. 1.3) for electronic components or equipment where failure rate is decreasing during the early life period.

- **Constant Failure Rate**
  
  A constant failure rate (CFR) can be caused by the application of loads at a constant average rate in excess of the design specifications or strength. These are typically externally induced failures.

- **Increasing Failure Rate**
  
  An increasing failure rate (IFR) can be caused by material fatigue or by strength deterioration due to cyclic loading. Its failure mode does not accrue for a finite time, and then exhibits an increasing probability of occurrence.

This failure pattern is also demonstrated by electronic equipment that has aged beyond its useful life and the failure rate is rapidly increasing with time.

![Bathtub Curve Diagram](image)

**Fig. 1.3: The bathtub curve**

### 1.82 Repairable Items

There are three patterns of failures, for repairable items, which can change with time. The failure rate (hazard rate) may be decreasing, increasing or constant.
• **Decreasing Failure Rate**
  An item whose reliability is improved by progressive repair and/or burn-in can cause a decreasing failure rate (DFR) pattern.

• **Constant Failure Rate**
  A constant failure rate (CFR) is indicative of externally induced failures as in the constant failure rate of non-repairable items. This is typical of complex systems subject to repair and overhaul.

• **Increasing Failure Rate**
  This increasing failure rate (IFR) pattern is demonstrated by repairable equipment when wear out modes begin to predominate or electronic equipment that has aged beyond its useful life and the failure rate is increasing with time.

1.9 **REDUNDANCY**

Redundancy [3] is defined as the existence of two or more means, not necessarily identical, for accomplishing a given single function. There are different types of redundancy.

**Active Redundancy**: It has all items operating simultaneously in parallel. All items are working and in use at the same time, even though only one item is required for the function. There is no change in the failure rate of the surviving item after the failure of a companion item.

**Pure Parallel**: It has no change in the failure rate of surviving items after failure of a companion item.

**Shared Parallel**: It has failure rate of remaining items which change after failure of a companion item.

**Standby Redundancy**: It has alternate items activated upon failure of the first item. Only one item is operating at a time to accomplish the function. One item’s failure rate affects the failure characteristics of others as they are now more susceptible to failure because they are now under load.

15
Hot Standby: Same as Active Standby or Active Redundancy.

Cold Standby (Passive): Normally not operating. Do not fail when they are on cold standby. Failure of an item forces standby item to start operating.

Warm Standby: Normally active or operational, but not under load. Failure rate will be less due to lower stress.

r-out-of-n Systems: Redundant system consisting of n items in which r of the n items must function for the system to function (voting decision).

1.10 PROBABILITY MODELS

The following two probability functions [2] are utilized in the present study.

- The time-invariant Exponential Reliability function
- The time-dependant Weibull Reliability function

1.101 The Exponential Reliability Model

A failure distribution that has a constant failure rate is called an exponential probability distribution. The exponential distribution [2] is one of the most important Reliability distributions. Many systems exhibit constant failure rates and exponential distribution in many respects the simplest Reliability distribution to analyze.

Failures due to completely random or chance events will follow this distribution. It should dominate during the useful life of a system or component. It is also one of the easiest distributions to analyze statistically.

Assuming that \( \lambda(t) = \lambda, \ t \geq 0, \ \lambda > 0 \) then

\[
R(t) = e^{-\lambda t}, \quad t \geq 0
\]

And

\[
F(t) = 1 - e^{-\lambda t}
\]

Then

\[
f(t) = \lambda e^{-\lambda t}
\]

\[
\text{MTTF} = \frac{1}{\lambda}, \quad \ldots \ 1.19
\]

\[
\sigma^2 = \frac{1}{\lambda^2}, \quad \ldots \ 1.20
\]
For this model, the mean time to failure is the reciprocal of the failure rate. Another observation concerning this distribution is that \( R \) (MTTF) = 0.368, i.e. a component having a CFR has a slightly better than one-third chance of surviving to its mean time to failure.

The design life of a component having an exponentially distributed failure time may be obtained from the inverse of the reliability function. That is for a given reliability \( R \),

\[
R (t_R) = e^{-\lambda t} = R
\]

Then \( t_R = \frac{1}{\lambda} \ln R \) \hspace{1cm} \ldots \text{1.21}

\[
t_{\text{mod}} = \frac{0.69315}{\lambda} \hspace{1cm} \ldots \text{1.22}
\]

A well known characteristic of the CFR model, one not shared by other failure distributions is its lack of memory (memorylessness). That is, the time to failure of a component is not dependent on how long the component has been operating. There is no aging or wear out effect. The probability that the component will operate for the next 1000 hr is the same regardless of whether the component is brand new, has been operating for several years, or has been operating for several thousand years. This process is consistent with the completely random and independent nature of the failure process.

1.102 The Weibull Model

The hazard rate function of this model is not constant over time but is time dependent. The Weibull distribution [2] may be used to model both increasing and decreasing failure rates. It is characterized by a hazard rate function of the form

\[
\lambda (t) = at^b \hspace{1cm} \ldots \text{1.23}
\]

which is a power function. The function \( \lambda (t) \) is increasing for \( a > 0, b > 0 \) and is decreasing for \( a > 0, b < 0 \). For mathematical convenience \( \lambda (t) \) is expressed as
\[ \lambda(t) = \beta \left( \frac{t}{\theta} \right)^{\beta-1} t > 0, \beta > 0, t \geq 0 \] ...

\[ R(t) = \exp \left[ -\int_0^t \left( \frac{t}{\theta} \right)^{\beta-1} dt \right] = e^{-\left( \frac{t}{\theta} \right)^{\beta}} \] ...

\[ f(t) = \frac{-dR(t)}{dt} = \beta \left( \frac{t}{\theta} \right)^{\beta-1} \cdot e^{-\left( \frac{t}{\theta} \right)^{\beta}} \] ...

Beta (\beta) is referred to as the shape parameter. For \( \beta < 1 \), the PDF is similar in shape to the exponential, and for large values of \( \beta \) (e.g., \( \beta \geq 3 \)), the PDF is somewhat symmetrical, like the normal distribution. For \( 1 < \beta < 3 \), the density function is skewed. When \( \beta = 1 \), \( \lambda(t) \) is a constant and the distribution is identical to the exponential with \( \lambda = 1/\theta \). In this distribution 63.2 percent of all Weibull failures will occur by time \( t = \theta \) regardless of the value of the shape parameter. The hazard rate function can be increasing or decreasing depending on the value of \( \beta \).

Theta (\( \theta \)) is a scale parameter that influences both the mean and the spread, or dispersion, of the distribution. As \( \theta \) increases, the reliability increases at a given point of time and the slope of the hazard rate decreases. The parameter \( \theta \) is also called the characteristic life and it has units identical to that of the failure time, T.

1.11 RELIABILITY OF SYSTEMS

Many systems [5] can be considered to be series or parallel systems, or a combination of both. A series system is one in which all components are so interrelated that the entire system will fail if any one of its components fails; a parallel system is one that will fail only if all of its components fail.

1.1.1 Series Configuration

Consider a system of \( n \) components connected in series, and assume that the components are independent, namely the performance of any one part does not affect the Reliability of the others. Under these conditions, the probability that the system will function is given by the special rule of multiplication for probabilities:

\[ R_s = R_1 \cdot R_2 \cdot \ldots \cdot R_n \] ...

18
Where $R_i$ is the reliability of the $i^{th}$ component and $R_s$ is the reliability of the series system. This simple product law of probabilities, applicable to series system of independent components, vividly demonstrates the effect of increased complexity on Reliability.

The Reliability of the five-component series system having individual component Reliability of 0.970 is $(0.970)^5 (= 0.859)$. Increasing system complexity to 10 components will decrease the system Reliability to $(0.970)^{10} (= 0.737)$.

1.1.12 Parallel Configuration

One way to increase the Reliability of a system is to replace certain components by several similar components connected in parallel. If a system consists of $n$ independent components connected in parallel, it will fail to function only if all $n$ components fail. Thus, if $F_i = 1 - R_i$ is the unreliability of the $i^{th}$ component, the unreliability of the parallel system ($F_p$) can be written as

$$F_p = \prod_{i=1}^{n} R_i$$ ...

and $R_p = 1 - F_p$ is the reliability of the parallel system. Thus for parallel systems a product law of unreliabilities is expressed as

$$R_p = 1 - \prod_{i=1}^{n} (1 - R_i)$$ ...

for the Reliability of a parallel system.

1.1.13 Series versus Parallel Configuration

Suppose that a system consists of $n$ components connected in series, and that these components have the respective failure rates $\lambda_1, \lambda_2, ..., \lambda_n$. The product law of Reliabilities for the exponential model can be written as

$$R_s(t) = \prod_{i=1}^{n} e^{-\lambda_i t} = \exp \left( - \sum_{i=1}^{n} \lambda_i t \right) = \exp(-\lambda t)$$ ...

where $\lambda_i = \sum \lambda_i$.
And it can be seen that the reliability function of the series system also satisfies the exponential assumptions. The failure rate of the entire series system is readily identified as

\[ \sum_{i=1}^{n} \lambda_i \]

the sum of the failure rates of its components.

For parallel systems the results are not quite so simple. If a system consists of \( n \) components in parallel, having the respective failure rates \( \lambda_1, \lambda_2 \ldots \lambda_n \), the system unreliability to time \( t \) is given by

\[
F_p(t) = \prod_{i=1}^{n} \left(1 - e^{-\lambda t}\right)
\]

Thus the failure distribution of a parallel system is not exponential even when each of its components satisfies the exponential assumption.

### 1.12 Application Areas of Reliability

The main objective of a Reliability study is to provide information as a basis for taking decisions. Reliability technology has a potentially wide range of application areas.

- **Safety/risk analyses** Reliability techniques are often applied in risk analyses to evaluate availability and applicability of safety systems ranging from single component to complex systems.

- **Environmental protection** Reliability studies may be used to improve the design and operational regularity of antipollution systems like gas/water cleaning systems.

- **Quality** The concepts of quality and Reliability are closely related. Reliability is considered to be a quality characteristic.

- **Optimization of operation and maintenance** The important connection between maintenance and Reliability is fully realized by many industries such as nuclear power, aviation, defense, and shipping industry etc.
**Engineering design** Reliability assurance is one of the most important quality characteristic during the engineering design process. Many industries have an integrated Reliability program in the design process.

**Verification of quality/reliability** Reliability analyses and testing are the necessary tools in the verification process.