CHAPTER 3

REFINEMENT OF PLASTOHYDRODYNAMIC ANALYSIS FOR TUBE SINKING

3.1 Introduction

The plastohydrodynamic analysis of tube sinking presented in chapter 2, assumed a linear variation of tube thickness in the deformation zone during the sinking process. It further assumed the lubricant flow to be isothermal and did not incorporate the effect of temperature change on the lubricant viscosity and film thickness in the deformation zone during the sinking process.

The present analysis examines the different tube thickness profiles in the deformation zone during the sinking process, with the objective of establishing the most appropriate tube thickness profile. The best tube thickness profile has been determined by employing the principle of minimum energy dissipation (Hillier, 1967 and Bedi, 1968). To include the effect of temperature on the lubricant viscosity and film thickness in the deformation zone during the sinking process, Wilson's (1973) approach has been used to modify the lubricant flow because of temperature effect.

3.2 Study Of Tube Thickness Profiles

The following tube thickness profiles have been examined for the tube sinking process in the present study:

\[ t = t_1 - K_1 (D-D_1) \]  \( \text{Or} \quad \ddot{\xi} = \ddot{\xi}_1 - K_1 (\ddot{D}-1) \)  \( (3.1) \)

\[ t = t_1 + K_2 (D-D_1) \]  \( \text{Or} \quad \ddot{\xi} = \ddot{\xi}_1 + K_2 (\ddot{D}-1) \)  \( (3.2) \)

\[ t = t_1 e^{K_3} (D-D_1) \]  \( \text{Or} \quad \ddot{\xi} = \ddot{\xi}_1 e^{K_3 D_1 (\ddot{D}-1)} \)  \( (3.3) \)

\[ t = t_1 e^{-K_4(D-D_1)} \]  \( \text{Or} \quad \ddot{\xi} = \ddot{\xi}_1 e^{-K_4 D_1 (\ddot{D}-1)} \)  \( (3.4) \)

Equations (3.1) and (3.2) represent linear tube thickness
profiles with decreasing and increasing tube thickness respectively.

Equations (3.3) and (3.4) represent exponential tube thickness profiles with increasing and decreasing tube thickness in the deformation zone.

The constants $K_1, K_2, K_3$ and $K_4$ used in equations (3.1) through (3.4) are to be determined from the following end conditions:

$$t = t_1 \text{ at } D = D_1 \text{ and } t = t_2 \text{ at } D = D_2$$  (3.5)

The values of $K_1, K_2, K_3$ and $K_4$ can be obtained (Appendix 1) through use of equations (3.1) to (3.4) along with equation (3.5) as follows:

$$K_1 = \frac{1}{(D_2-1)} \left( \frac{M_1}{M_1 - t_1} \right)$$  (3.6)

$$K_2 = \frac{1}{(D_2-1)} \left( \frac{M_1 - t_1}{M_1} \right)$$  (3.7)

$$K_3 = \frac{1}{D_1(D_2-1)} \ln \left( \frac{M_1}{M_1 - t_1} \right)$$  (3.8)

$$K_4 = \frac{1}{D_4(1-D_2)} \ln \left( \frac{M_1}{M_1 - t_1} \right)$$  (3.9)

Where $M_1$ is defined by equation (2.20).

For using the tube thickness profiles represented by equations (3.1) through (3.4) in the analysis presented in chapter 2, equation (2.44) may be written in the general form as follows:

$$\frac{\partial \hat{W}_P}{\partial H_2} = \frac{\sqrt{2 \pi f \sigma_2} K G}{(P_1)^{\frac{1}{2}}}$$  (3.10)

In equation (3.10) the value of $K$ is to be selected.
from equations (3.6) through (3.9) depending on the choice of tube thickness profiles represented by equations (3.1) to (3.4). $F_1$ is defined by equation (2.46) for all the tube thickness profiles. The value of $G$ in equation (3.10), corresponding to tube thickness profiles represented by equations (3.1) through (3.4) may be calculated as follows: (Appendix 1)

$$
G_1 = \left[ 2 \left( \frac{1}{K_1} + 1 \right) \bar{\varepsilon}_2 - \left( \bar{D}_2 - \bar{\varepsilon}_2 \right) \right] D_1 \ln \frac{1-\bar{\varepsilon}_1}{\bar{D}_2 - \bar{\varepsilon}_2} + \left[ \left( \frac{1}{K_1} + 1 \right) \bar{\varepsilon}_2 - 2 \left( \bar{D}_2 - \bar{\varepsilon}_2 \right) \right] D_1 \ln \frac{\bar{\varepsilon}_1}{\bar{\varepsilon}_2} \quad (3.11)
$$

$$
G_2 = \left[ \left( \bar{D}_2 - \bar{\varepsilon}_2 \right) + 2(-\frac{1}{K_2} - 1) \bar{\varepsilon}_2 \right] D_1 \ln \frac{1-\bar{\varepsilon}_1}{\bar{D}_2 - \bar{\varepsilon}_2} + \left[ 2 \left( \bar{D}_2 - \bar{\varepsilon}_2 \right) + (-\frac{1}{K_2} - 1) \bar{\varepsilon}_2 \right] D_1 \ln \frac{\bar{\varepsilon}_1}{\bar{\varepsilon}_2} \quad (3.12)
$$

$$
G_3 = \left[ \left( \bar{D}_2 - \bar{\varepsilon}_2 \right) \bar{\varepsilon}_2 D_1^2 + 2 \bar{\varepsilon}_2 D_1 \left( \frac{1}{K_3} - \bar{\varepsilon}_2 D_1 \right) \right] \ln \frac{1-\bar{\varepsilon}_1}{\bar{D}_2 - \bar{\varepsilon}_2} + \left[ 2 \left( \bar{D}_2 - \bar{\varepsilon}_2 \right) \bar{\varepsilon}_2 D_1^2 + \left( \frac{1}{K_3} - \bar{\varepsilon}_2 D_1 \right) \right] \ln \frac{\bar{\varepsilon}_1}{\bar{\varepsilon}_2} \quad (3.13)
$$

$$
G_4 = \left[ 2 \left( \frac{1}{K_4} + \bar{\varepsilon}_2 D_1 \right) \bar{\varepsilon}_2 D_1^2 - \left( \bar{D}_2 - \bar{\varepsilon}_2 \right) \bar{\varepsilon}_2 D_1^2 \right] \ln \frac{1-\bar{\varepsilon}_1}{\bar{D}_2 - \bar{\varepsilon}_2} + \left[ \left( \frac{1}{K_4} + \bar{\varepsilon}_2 D_1 \right) \bar{\varepsilon}_2 D_1^2 - 2 \left( \bar{D}_2 - \bar{\varepsilon}_2 \right) \bar{\varepsilon}_2 D_1^2 \right] \ln \frac{\bar{\varepsilon}_1}{\bar{\varepsilon}_2} \quad (3.14)
$$

Where $G_1$, $G_2$, $G_3$ and $G_4$ correspond to tube thickness profiles represented by equations (3.1) through (3.4).

The best tube thickness profile in the deformation zone shall be determined by comparing the total energy dissipation rate ($\dot{E}$) defined by equation (2.28), for all the profiles. The tube
thickness profile providing the minimum energy shall be the most appropriate profile.

3.3 Numerical Data

The following numerical data has been used to compare the total energy dissipation rate for different tube thickness profiles:

(i) Tube Material: (Johnson and Mellor, 1962)
Annealed Aluminium \( A_0 = 1.5296 \times 10^8 \text{ N/m}^2, B = 0.25 \)

(ii) Lubricant: (Reid and Schey, 1978)
Bright Stock \( \eta_0 = 0.973 \text{ Ns/m}^2, \gamma_0 = 2.6 \times 10^{-8} \text{ m}^2/\text{N} \)

(iii) Parameter \( \bar{U} \): 0.1, 0.2, 0.3, 0.4, 0.5

(iv) Die Geometry \( \phi \): 0.15 radian

(v) Initial Tube Outside Diameter: 0.0125 m.

(vi) Modified Sommerfeld Number: \( 121 \times 10^{-8} \)

3.4 Results And Discussion

The results of the present study have been tabulated in table 3.1. It is evident from the observation of the table that tube thickness profiles represented by equations (3.1) and (3.2) provide the minimum total energy. The tube thickness profile represented by these equations is a linear profile, which appears to be the most appropriate profile for the tube sinking process as compared to exponential profile represented by equations (3.3) and (3.4). It can be seen from table 3.1 that the linear tube thickness profiles with decreasing and increasing tube thickness represented by equations (3.1) and (3.2) result in the same total energy which is less than the total energy provided by exponential profiles with increasing and decreasing tube thickness represented by equations (3.3) and (3.4).
<table>
<thead>
<tr>
<th>Reduction of tube outside diameter %</th>
<th>Parameter</th>
<th>Tube Thickness Profiles</th>
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</thead>
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<tr>
<td></td>
<td>(3.1)</td>
<td>(3.2)</td>
</tr>
<tr>
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<td>240.99</td>
</tr>
<tr>
<td></td>
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<td></td>
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<tr>
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<td>0.3</td>
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</tr>
<tr>
<td></td>
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<td></td>
<td>0.5</td>
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</tr>
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</tr>
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<tr>
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</tr>
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</tr>
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</tr>
<tr>
<td></td>
<td>0.3</td>
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</tr>
<tr>
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<td>0.4</td>
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</tr>
<tr>
<td></td>
<td>0.5</td>
<td>5189.31</td>
</tr>
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</table>
FIG. 3.1 TEMPERATURE EFFECTS ON THE LUBRICANT FLOW IN DEFORMATION ZONE IN TUBE SINKING PROCESS (WILSON 1973)
3.5 **Plastohydrodynamic Analysis Of Tube Sinking With Temperature Effect:**

To include the temperature effect in the hydrodynamic analysis of tube sinking by using Wilsons'(1973) approach, the following assumptions have been made:

(i) The lubricant viscosity $\eta_T$ at any temperature $T$ is given by:

$$\eta_T = \eta e^{-\theta (T - T_d)} \quad (3.15)$$

Where $\eta$ is the viscosity of the lubricant at die temperature ($T_d$) and pressure $p$, $\theta$ is the temperature coefficient of lubricant viscosity.

(ii) All the plastic work is converted into heat during the sinking process. The temperature of the tube material increases as a result of plastic work converted into heat. As has been shown later, the rise of tube temperature because of this heat is not significant (less than 50°C), the flow stress of the tube material has been assumed to be constant (Johnson and Mellor, 1973).

(iii) The die temperature ($T_d$) has been assumed to be constant at room temperature.

(iv) The heat dissipated in the lubricant is neglected.

With reference to figure 3.1, the temperature of the lubricant ($T$) at distance $z$ from the die surface is given by:

$$T = T_d + \frac{(T_t - T_d)z}{H} \quad (3.16)$$

Where $H$ is the lubricant film thickness, $T_t$ is the tube temperature and $z$ is the distance coordinate perpendicular to the tube surface in the deformation zone.
From equations (3.15) and (3.16), the lubricant viscosity at any point in the deformation zone can be expressed as:

$$\eta_T = \eta e^{-\theta (T_t - T_d)z \frac{H}{n}}$$  \hspace{1cm} (3.17)

The shear stress of the lubricant is given as:

$$\tau = \eta \frac{du}{dz}$$  \hspace{1cm} (3.18)

Substituting for \(\eta\) from equation (3.17) in (3.18), we get:

$$\frac{du}{dz} = -\theta (T_t - T_d)z \frac{T_h e^{-\theta (T_t - T_d)z \frac{H}{n}}}{n}$$

Or

$$\frac{du}{dz} = \frac{\tau}{\eta} e^{-\theta (T_t - T_d)z \frac{H}{n}}$$  \hspace{1cm} (3.19)

Integrating equation (3.19), we get:

$$u = \frac{\tau H}{\eta} e^{-\theta (T_t - T_d)z \frac{H}{n}} + Y_1$$  \hspace{1cm} (3.20)

Where \(Y_1\) is a constant of integration and can be found from the boundary condition:

$$u = 0 \text{ at } z = 0$$  \hspace{1cm} (3.21)

From equations (3.20) and (3.21), we get:

$$Y_1 = \frac{-\tau H}{\eta} \frac{1}{(T_t - T_d) \theta}$$  \hspace{1cm} (3.22)

From equations (3.20) and (3.22), we get:

$$u = \frac{\tau H}{\eta} \frac{1}{(T_t - T_d) \theta} \left[ e^{-\theta (T_t - T_d)z \frac{H}{n}} - 1 \right]$$  \hspace{1cm} (3.23)
From figure 3.1, we get:

\[ u = \frac{V}{\cos \alpha} \text{ at } z = H \quad (3.24) \]

So that from equations (3.23) and (3.24), we get:

\[ \frac{V}{\cos \alpha} = \frac{H}{\eta} \cdot \frac{1}{(T_t - T_d) \theta} \left[ e^{\theta (T_t - T_d)} - 1 \right] \quad (3.25) \]

From equations (3.23) and (3.25), we get:

\[ \frac{u}{V} \cos \alpha = \frac{\frac{\theta(T_t - T_d)z}{H}}{e^{\frac{\theta(T_t - T_d)}{Lz} - 1}} \quad (3.26) \]

Or

\[ u = \frac{V}{\cos \alpha} \left[ \frac{Lz}{e^H - 1} \right] \quad (3.27) \]

Where \( L = \theta (T_t - T_d) \) \quad (3.28)

The volume flow rate of lubricant in the deformation zone is given as:

\[ Q = \int_{0}^{H} u (\pi D) dz \quad (3.29) \]

Substituting equation (3.27) in equation (3.29), we get:

\[ Q = \int_{0}^{H} \frac{V}{\cos \alpha} \left( \frac{Lz}{e^H - 1} \right) (\pi D) dz \]

Or

\[ Q = \frac{V \pi D}{\cos \alpha L} \left( \frac{e^{Lz}}{e^H - 1} - 1 \right) dz \]

Or

\[ Q = \frac{V \pi DH}{\cos \alpha L} \left[ \frac{e^{Lz}}{e^L - 1} \right] \quad (3.30) \]
For the isothermal case \((L = 0)\), equation (3.30) reduces to the following form:

\[
Q_1 = \frac{V(\Pi D)H}{2 \cos \alpha}
\]  

(3.31)

Where \(Q_1\) is the volume flow rate of the lubricant under isothermal conditions.

Comparing equations (3.30) and (3.31), we get:

\[
Q = \left[ \frac{2(e^L - L - 1)}{L(e^L - 1)} \right] Q_1
\]  

(3.32)

Or

\[
Q = W Q_1
\]  

(3.33)

Where \(W = \left[ \frac{2(e^L - L - 1)}{L(e^L - 1)} \right] \)  

(3.34)

\((W\) is defined as dimensionless flow of lubricant\)

The isothermal analysis of plastohydrodynamic tube sinking can be modified to include the temperature effect on the lubricant film thickness by replacing the lubricant velocity \(u\) at any point in the deformation zone by \(u(W)\) as suggested by Mahdavian and Wilson (1976).

To determine the value of \(L\) (defined as dimensionless temperature difference), it is assumed that the tube material is initially at die temperature \(T_d\) and all the plastic work during the deformation of the tube material is converted into heat. So that:

\[
\frac{W_p}{J} = M S (T_t - T_d)
\]  

(3.35)

Where \(J\) is the mechanical equivalent of heat, \(S\) is the specific heat of tube material, and \(M\) is the mass flow rate of lubricant in the deformation zone.
Equation (3.35) can also be written as:
\[ \dot{W}_p \frac{J}{J} = A \dot{V} \rho S (T_t - T_d) \]

Where \( \rho \) is the density of the tube material.

Or
\[ \dot{W}_p \frac{J}{J} = A_2 \dot{V} \rho S (T_t - T_d) \]

(Since \( A \dot{V} = A_2 \dot{V} \), from the condition of incompressibility of tube material)

Or
\[ \dot{W}_p \frac{J}{J} = A_2 \dot{V} \rho S \frac{(T_t - T_d)}{\theta} \quad (3.36) \]

Substituting equation (3.28) in (3.36), we get:
\[ \dot{W}_p \frac{J}{J} = \frac{A_2 \dot{V} \rho S L}{\theta} \]

Or
\[ L = \dot{W}_p \frac{\theta}{A_2 \dot{V} \rho S J} \quad (3.37) \]

Equation (3.37) can be used to determine the value of \( L \) for calculating the value of dimensionless flow from equation (3.34) for modifying the lubricant velocity.

3.6 Numerical Data

Following numerical data has been used for studying the temperature effect on the hydrodynamic tube sinking process:

(i) Modified Sommerfeld Number: \( m \),
\[ 4 \times 10^{-8}, 16 \times 10^{-8}, 25 \times 10^{-8}, 64 \times 10^{-8}, 121 \times 10^{-8}, 225 \times 10^{-8} \text{ and } 400 \times 10^{-8} \]

(ii) Reduction of tube outside diameter(%):
5, 10, 15, 20

The rest of the numerical data as given in section 3.3 has been used.
3.7 Results And Discussion

The results of plastohydrodynamic tube sinking analysis with temperature effect are given in figures 3.2 through 3.5.

Figure 3.2 shows the variation of tube temperature in the deformation zone during the sinking process. As can be seen from this figure, the tube temperature increases exponentially as it undergoes plastic deformation in the deformation zone.

The effect of temperature on the lubricant film thickness in the deformation zone during the tube sinking process is shown in figure 3.3. The figure indicates a reduction of lubricant film thickness because of thermal effects. Similar analytical results of reduced film thickness have been reported by Dow, Kannel and Bupara (1975) and Wilson and Mahdavian (1974) for hydrodynamic cold rolling and hydrostatic extrusion processes, when they incorporated thermal effects in the isothermal analyses of these metal forming processes.

Figure 3.4 shows the temperature effect on the variation of lubricant film thickness with modified Sommerfeld number for different sinking conditions. It also indicates a reduction of lubricant film thickness because of temperature effect.

The maximum tube temperature at the die exit because of plastic deformation in the die zone has been shown in figure 3.5 for the different sinking conditions.

3.8 Conclusions

The conclusions of refined hydrodynamic analysis of tube sinking can be summarised as follows:

1. The linear tube thickness profile in the deformation zone appears to be the most appropriate profile as it provides minimum total energy.
ANNIELED ALUMINIUM
LUBRICANT BRIGHT STOCK

$\alpha = 0.15 \text{ radian}$

$\gamma = 0.40$

$m = 6.4 \times 10^{-8}$

$R_1 = 1 - \frac{D_1}{D_2} = 0.10$

$D_1 = 0.0125 \text{ m.}$

**Fig. 3.2. Variation of Tube Temperature in the Deformation Zone for Tube Sinking.**
Fig. 3.3. Effect of temperature on dimensionless film thickness in deformation zone.

Annealed aluminium lubricant bright stock

\( \alpha = 0.15 \) radian

\( \tau = 0.40 \)

\( m = 64 \times 10^{-8} \)

\( R_1 = 1 - \frac{D_2}{D_1} = 0.10 \)
ANNAPLED ALUMINIUM LUBRICATION BRIGHT STOCK
\( \alpha = 0.15 \text{ Radian} \)
\( R_1 = 0.10 \)
\( D_1 = 0.0125 \text{ m} \)

\[ H_{mn} \times 10^{-3} \]

\[ \sqrt{m} \times 10^{-4} \]

**Fig. 3.4. Effect of Modified Sommerfeld Number on Dimensionless Minimum Film Thickness in Tube Sinking With Temperature Effect.**
ANNEALED ALUMINIUM
LUBRICANT BRIGHT STOCK
\( \alpha = 0.15 \) RADIUS
\( m = 64 \times 10^{-8} \)
\( d_L = 0.0125 \) m.

**Fig. 3.5.** Effect of Reduction in Tube Outside Diameter on Maximum Tube Temperature at the Die Exit in Plasto-Hydrodynamic Tube Sinking.
2. The temperature effect tends to decrease the lubricant film thickness in the deformation zone during the tube sinking process.

3. The variation of tube temperature within the deformation zone is exponential.