CHAPTER 4

OPTIMISATION OF PLASTOHYDRODYNAMIC TUBE SINKING PARAMETERS

4.1 Introduction

The plastohydrodynamic analysis of tube sinking presented in the previous chapters provides for the determination of lubricant film thickness in the deformation zone during the sinking process. The lubricant film thickness has been found to be dependent on the sinking process parameters and the modified Sommerfeld number, which is a dimensionless quantity involving the physical constants of the lubricant and the tube material along with the speed of drawing. In order to reduce the drawing load in the tube sinking operation a full fluid film is desirable, however to achieve an acceptable surface finish of the final product by burnishing action, lubricant film should be made to break at some point in the deformation zone during the sinking operation. This break down of lubricant film occurs when the minimum film thickness falls below some critical value, which is a function of roughness of the mating surfaces (die and tube). In a previous analysis (Nauhria and Bedi, 1985) a suitable combination of sinking parameters along with a critical Sommerfeld number has been suggested, so as to provide a full fluid film lubrication in the deformation zone, together with an acceptable surface finish of the drawn product.

4.2 Breakdown Of Hydrodynamic Lubrication In Tube Sinking

Ocvirk and Dubois (1959) have studied the onset of breakdown of full fluid film lubrication in journal bearings. Their experiments verified the hypothesis that hydrodynamic lubrication breaks down when surface asperities of shaft and
bearing tend to come into contact. This concept has been successfully applied (Bedi and Hillier, 1970) for predicting the breakdown of hydrodynamic lubrication in strip rolling, assuming that the breakdown of hydrodynamic lubrication occurs when the minimum lubricant film thickness (at the exit section of rolling) is of the same magnitude as the sum of peak heights of the asperities of the two mating surfaces. On the similar lines in the present analysis it has been assumed that the breakdown of hydrodynamic lubrication in tube sinking occurs when the minimum lubricant film thickness in the deformation zone (die inlet or exit depending upon the sinking parameters) is less than the critical film thickness which is a function of the surface conditions of the two mating surfaces (die and tube).

Tarasov (1945) concluded after observation of various surface profiles that there is a 'predominant roughness' consisting of relatively small irregularities at small intervals and in addition there are 'high peaks' and 'deep valleys' at relatively large intervals. He has further shown that the peak roughness can be expressed as a multiple $T_f$ (Table 4.1) of the r.m.s profilometer roughness $\delta_s$. If subscripts $t$ and $d$ refer to tube and die respectively, the lubrication will first break in tube sinking when:

$$H_{\text{min}} \leq H_{\text{crit.}}$$

(4.1)

Where the critical film thickness ($H_{\text{crit.}}$) is defined as:

$$H_{\text{crit.}} = T_f \delta_s + T_d \delta_d$$

(4.2)

Where $T_f$ and $T_d$ are Tarasov factors for tube and die surfaces respectively and $\delta_s$ and $\delta_d$ r.m.s values of roughness for tube and die respectively.
4.3 Results And Discussion:

The results of hydrodynamic analysis presented in figures 2.19 and 2.20 shall be used for discussing the optimisation of plastohydrodynamic tube sinking parameters.

The optimisation of plastohydrodynamic tube sinking is achieved, when the thickness of the lubricant film is sufficient to ensure a full fluid film in the deformation zone and at the same time should not be so thick as to adversely affect the surface finish of the drawn product because of absence of burnishing action.

The choice of the optimal film thickness for the tube sinking process depends on the surface finish of the mating surfaces (tube and die). The critical film thickness, which is a function of the surface roughness of the mating surfaces can be obtained by using equation (4.2).

With reference to figure (2.19), if we select the critical film thickness value($H_{\text{crit.}} = 1.00 \times 10^{-3}$) as an example, the values

<table>
<thead>
<tr>
<th>Surface Condition</th>
<th>Factor $T_f$ (Tarasov, 1945)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ground</td>
<td>4.5</td>
</tr>
<tr>
<td>Hyprolapped</td>
<td>6.5</td>
</tr>
<tr>
<td>Sand papered</td>
<td>7.0</td>
</tr>
<tr>
<td>Superfinished</td>
<td>7.0</td>
</tr>
<tr>
<td>Loose abrasive lapped</td>
<td>10.0</td>
</tr>
</tbody>
</table>
of modified Sommerfeld numbers for obtaining this critical value. The film thickness shall be 28x10^{-8}, 36x10^{-8}, 55x10^{-8} and 148x10^{-8} corresponding to the tube outside diameter reductions of 20, 15, 10 and 5 percent respectively. This means that the condition of hydrodynamic lubrication (for the sinking conditions represented by figure 2.19) prevail only for values of modified Sommerfeld numbers greater than or equal to these values. Similarly with reference to figure 2.20, if we select the critical film thickness (H_{crit} again as 1.00x10^{-3}), the values of modified Sommerfeld numbers corresponding to the critical film thickness shall be 35x10^{-8}, 56x10^{-8}, 83x10^{-8}, 118x10^{-8} and 169x10^{-8} corresponding to the parameter \gamma values of 0.1, 0.2, 0.3, 0.4 and 0.5 respectively.

The optimisation procedure for plastohydrodynamic tube sinking can be summarised as follows:

1. Decide upon the critical film thickness value, depending upon the roughness of the mating surfaces (tube and die) by making use of equation (4.2).

2. Find the value of modified Sommerfeld number which will result in the critical film thickness for the given set of sinking parameters. This value of the modified Sommerfeld number is defined as the critical Sommerfeld number.

3. The optimisation of plastohydrodynamic tube sinking shall be achieved when the process is operated close to, but slightly lower than the critical modified Sommerfeld number. If the process is operated at modified Sommerfeld number, significantly less than the critical value, the lubricant film thickness shall not be sufficient to ensure full fluid film lubrication in the deformation zone. Further if the
modified Sommerfeld number is significantly high than the critical value, the surface finish of the drawn product shall be adversely affected in the absence of burnishing action as a result of formation of a thick film of lubricant in the deformation zone.

By operating close to but slightly lower than critical modified Sommerfeld number value, the lubricant film thickness will reduce to critical film thickness at some point in the deformation zone (die inlet or exit in the case of tube sinking depending on parameter as shown in figures 2.12 and 2.13), so that an optimum surface finish may be obtained because of burnishing action at the point in the deformation zone at which the minimum film thickness reduces to the critical film thickness, along with conditions of hydrodynamic lubrication at all other points in the deformation zone.

4.4 Conclusions:

The modified Sommerfeld number appears to be the deciding factor in the optimisation of plastohydrodynamic tube sinking process. The modified Sommerfeld number (defined by equation 2.64) is a function of lubricant viscosity at atmospheric pressure, the speed of drawing the yield stress of the tube material and the initial tube outside diameter. As the initial tube outside diameter and yield stress of the tube material are fixed for a given tube sinking schedule, the manipulating parameters to control the value of modified Sommerfeld number are the lubricant viscosity and drawing speed. If the speed of drawing has to be increased for higher production rates, the lubricant viscosity should be proportionately reduced to have the same value of modified Sommerfeld number.
CHAPTER 5
PLASTOHYDRODYNAMIC LUBRICATION IN PLUG DRAWING

5.1 Introduction
The plug drawing of tubes is performed to control the internal diameter of the tube, with the objective of obtaining an uniform thickness of the drawn tube. The mechanics of plug drawing has been studied by Baldwin and Sachs (1955), Blazynski and Cole (1960) and Smith and Bramley (1974) by assuming a constant friction coefficient in the deformation zone. The mechanics of plug drawing with variable die friction and the effect of lubricant viscosity and other process parameters on the lubrication film thickness has not been investigated to date. In the present work a plastohydrodynamic analysis of plug drawing with variable die friction has been presented under conditions of full fluid film lubrication. The analysis provides for the calculation of lubricant film thickness at the die-tube and tube-plug interfaces based on the principle of minimum energy dissipation rate (Hillier, 1967 and Bedi, 1968). The hydrodynamic analysis of plug drawing also identify the parameters affecting the lubricant film thickness in the deformation zone. The optimisation of plug drawing parameters has been suggested to ensure hydrodynamic lubrication at die-tube and plug-tube interfaces.

5.2 Basic Assumptions
(a) For Tube Material:
   (i) The tube material is homogeneous, isotropic and plastically incompressible.
   (ii) The material obeys the Von-Mises yield criteria and its associated flow rule.
FIG. 5.1 STRESSES ON AN ELEMENT OF TUBE MATERIAL IN PLASTOHYDRODYNAMIC PLUG DRAWING
(iii) The strain rate effects in the tube material being drawn are negligible.
(iv) The yield stress of the material remains constant and does not vary with change of temperature of the tube material as a result of plug drawing.

(b) For Lubricant:
(i) The lubricant fluid is incompressible and Newtonian.
(ii) The lubricant flow in the deformation zone is laminar.
(iii) The inertia effects are negligible.
(iv) The viscosity of the lubricant is a function of pressure and temperature.

(c) For Plug Drawing Process:
(i) There is full fluid film lubrication at the die-tube and plug-tube interfaces.
(ii) Plane sections of the tube remain plane during the deformation.
(iii) The thickness of the lubricant film is small compared to the outgoing tube thickness.
(iv) The die opening for a particular plug drawing set up constant.
(v) The ratio of tube thickness to the initial outside diameter of the tube is small (equal to 0.05 ) so that lubricant pressure at the die-tube and plug-tube interfaces may be taken as equal.

5.3 Analysis

Figure 5.1 shows the stresses acting on an element of tube material undergoing plastic deformation in a plastohydrodynamic plug drawing process. Considering the axial equilibrium of the tube element, there are five force components acting in the axial direction:
(i) Due to change in the longitudinal stress:

\[(\sigma_x + d\sigma_x)(t + dt)\mu(D + dD) - \sigma_x\etaDt\]

which is equal to,

\[\sigma_x\etaDt + \sigma_x\eta t dD + d\sigma_x\etaDt\]  \hspace{1cm} (5.1)

(By ignoring the product of small quantities, \(d\sigma_x\etaDt\) and \(d\sigma_x\eta t dD\)).

(ii) Due to fluid pressure on the circumference, die-tube interface:

\[p(\eta D\frac{dx}{\cos \alpha})\sin \alpha = p\eta D\tan \alpha dx\] \hspace{1cm} (5.2)

(iii) Due to fluid pressure on the circumference, plug-tube interface:

\[-p(\eta D\frac{dx}{\cos \beta})\sin \beta = -p\eta D\tan \beta dx\] \hspace{1cm} (5.3)

Where \(\beta\) is the semi plug angle.

(iv) Due to shear stress of the lubricant, die-tube interface:

\[(\eta D\frac{dx}{\cos \alpha})\cos \alpha = \tau\eta D dx\] \hspace{1cm} (5.4)

(v) Due to shear stress of the lubricant, plug-tube interface:

\[(\eta D\frac{dx}{\cos \beta})\cos \beta = \tau\eta D dx\] \hspace{1cm} (5.5)

Under steady state plug drawing, the forces represented by equations (5.1) through (5.5) must be in equilibrium.

So that in equilibrium:

\[\sigma_x\eta Dt + \sigma_x\eta t dD + d\sigma_x\eta Dt + \eta Dp(\tan \alpha - \tan \beta)dx + 2\tau\eta D dx = 0\] \hspace{1cm} (5.6)
From the geometry figure 5.1, we get:

\[ \frac{dx}{dt} = \frac{dD}{2} \tan \alpha \]  
(5.7)

\[ \frac{dt}{dD} = \frac{dD}{2} \left( 1 - \frac{\tan \beta}{\tan \alpha} \right) \]  
(5.8)

From equations (5.7) and (5.8), we get:

\[ \frac{dt}{dx} (\tan \alpha - \tan \beta) \]  
(5.9)

From equations (5.6) and (5.9):

\[ \sigma_x \frac{dt}{dD} + \sigma_x \frac{t}{D} + d \sigma_x \frac{t}{dD} + p \frac{dt}{dD} \]

\[ + \frac{2 \tau dt}{dD(\tan \alpha - \tan \beta)} = 0 \]

Or

\[ \sigma_x \left( \frac{dt}{dD} + \frac{t}{D} \right) + d \sigma_x \left( \frac{t}{dD} \right) \]

\[ + p \frac{dt}{dD} \left( 1 + \frac{2 \tau}{p(\tan \alpha - \tan \beta)} \right) = 0 \]  
(5.10)

Or

\[ \sigma_x \left( \frac{dt}{dD} + \frac{t}{D} \right) + d \sigma_x \left( \frac{t}{dD} \right) \]

\[ + p \frac{dt}{dD} \left( 1 + \frac{\mu_a}{K \tan \alpha} \right) = 0 \]  
(5.11)

Where

\[ K_p = \frac{1}{2} \left( 1 - \frac{\tan \beta}{\tan \alpha} \right) \]

And \[ \mu_a = \frac{\tau}{p} \]  
(5.12)

\( \mu_a \) is defined as the apparent coefficient of friction in the deformation zone.

From the Von-Mises criteria for the axi-symmetric case of plug drawing, we get:

\[ \sigma_x + p = \bar{\sigma} \]  
(5.13)

Where \( \bar{\sigma} \) is the generalised yield stress of the tube material and is given as (Swift, 1952) follows:

\[ \bar{\sigma} = A_0 \left( B + \bar{e}^n \right) \]  
(5.14)
From equations (5.10) and (5.14), we get:

\[
\sigma_x \left( \frac{dt}{dD} + \frac{t}{D} \right) + \sigma_x \left( \frac{t}{dD} \right) + (\sigma - \sigma_x) \left( 1 + \frac{\mu_a}{K_p \tan \alpha} \right) \frac{dt}{dD} = 0
\]

Or

\[
d\sigma_x = \sigma_x \left( \frac{\mu_a}{K_p \tan \alpha} \frac{dt}{t} - \frac{dD}{D} \right) - \sigma (1 + \frac{\mu_a}{K_p \tan \alpha}) \frac{dt}{t}
\]

Or

\[
d\sigma_x = \sigma_x \left( \frac{\mu_a}{K_p \tan \alpha} \frac{\bar{dt}}{\bar{t}} - \frac{\bar{dD}}{\bar{D}} \right) - \sigma (1 + \frac{\mu_a}{K_p \tan \alpha}) \frac{\bar{dt}}{\bar{t}}
\]

(5.15)

Where \( \bar{dt} = \frac{dt}{t_2}, \bar{dD} = \frac{dD}{t_2}, \bar{D} = \frac{D}{t_2} \)

and \( \bar{t} = \frac{t}{t_2} \) are all dimensionless quantities.

Hydrodynamic Lubrication:

For a fully developed Newtonian laminar flow of lubricant between the die-tube and the plug-tube interfaces, the generalised Reynolds equation (Pinkus and Sternlicht, 1961) as modified by Wilson (1978) for axi-symmetric case can be written as:

\[
\frac{1}{2} Hu \frac{H^3}{12 \eta} \frac{dp}{ds} = Q
\]

(5.16)

Where \( Q \) is the volume flow rate of lubricant, \( H \) is the lubricant film thickness at any section in the deformation zone, \( u \) is the lubricant velocity, \( \eta \) is the lubricant viscosity at pressure \( p \) and \( \frac{dp}{ds} \) is the pressure gradient in the deformation zone.

The surface shear stress on the tube material due to viscous shearing of the lubricant (Pinkus and Sternlicht, 1961)
is given by:

\[ \tau = \frac{\eta}{H} \left( \frac{u}{H} + \frac{1}{2} H \frac{dp}{ds} \right) \]  \hspace{1cm} (5.17)

If \( H \) is sufficiently small and \( \frac{dp}{ds} \) is not large,

\[ \tau = \frac{\eta u}{H} \]  \hspace{1cm} (5.18)

Where \( \eta = \eta_0 e^{-\gamma_0 t} \)  \hspace{1cm} (5.19)

From the geometry of figure 5.1, we get:

For die-tube interface, \( u = \frac{V}{\cos \alpha} \)  \hspace{1cm} (5.20)

And for plug-tube interface, \( u = \frac{V}{\cos \beta} \)  \hspace{1cm} (5.21)

From equations (5.18), (5.20) and (5.21), we get:

\[ \tau = \frac{\eta V}{H_0 \cos \alpha} = \frac{\eta V}{H_p \cos \beta} \]  \hspace{1cm} (5.22)

Where \( H_D \) and \( H_P \) are the lubricant film thicknesses at the die-tube and plug-tube interfaces, at any section in the deformation zone respectively.

From equation (5.22), we get:

\[ H_D \cos \alpha = H_P \cos \beta \]

Or

\[ H_P = H_D \frac{\cos \alpha}{\cos \beta} \]  \hspace{1cm} (5.23)

Incompressibility Of Tube Material:

From the condition of incompressibility of tube material, we get:

\[ A_1 V_1 = A V = A_2 V_2 \]

Or

\[ \frac{\pi}{4} \left[ \frac{D^2}{D_1} - \left( D_1 - 2t_1 \right)^2 \right] V_1 = \frac{\pi}{4} \left[ \frac{D^2}{D_2} - (D_2 - 2t_2)^2 \right] V_2 \]
Or \[ V_{1} t_{1} (D_{1} - t_{1}) = V_{1} t \ (D - t) = V_{2} t_{2} (D_{2} - t_{2}) \]

Or \[ V_{1} \tilde{t}_{1} (\tilde{D}_{1} - \tilde{t}_{1}) = V_{1} \tilde{t} (\tilde{D} - \tilde{t}) = V_{2} (\tilde{D}_{2} - 1) \] (5.24)

**Principle Of Minimum Energy Dissipation:**

As the lubricant film thickness increases the rate of shear work of the lubricant tends to decrease, but at the same time the outgoing tube thickness decreases, thus increasing the rate of plastic work in hydrodynamic plug drawing. The lubricant film thickness at the die-tube and plug-tube interface may be obtained by applying the principle of minimum energy dissipation rate (Hillier, 1967 and Bedi, 1968) in the form of:

\[
\frac{\partial}{\partial H_{2D}} (\dot{E}) = 0
\] (5.25)

Where \( \dot{E} = \dot{W}_{P} + \dot{W}_{S1} + \dot{W}_{S2} \) (5.26)

So that equation (5.25) can be written as:

\[
\frac{\partial}{\partial H_{2D}} (\dot{W}_{P} + \dot{W}_{S1} + \dot{W}_{S2}) = 0
\] (5.27)

Where \( H_{2D} \) is the lubricant film thickness at the die-tube interface at the die-exit, \( \dot{W}_{P} \) is the rate of plastic work and \( \dot{W}_{S1} \) and \( \dot{W}_{S2} \) are rates of shear work at die-tube and plug-tube interfaces respectively.

For the homogeneous deformation the rate of plastic work is given by:

\[
\dot{W}_{P} = A_{2} V_{2} \int_{0}^{\tilde{e}} \tilde{\sigma} \ \tilde{de}
\] (5.28)

In order to include the effect of redundant work \( \tilde{e} \) shall be replaced by \( \tilde{e}^{*} = f (\tilde{e}) \), where \( f \) is the redundant
work factor and is defined by equation (2.37).

From equation (5.28), after including the redundant work, we get:

\[
W_p = \frac{\eta}{4} \int_0^1 \bar{\sigma} \cdot \bar{e}_2^* \, d\varepsilon
\]

\[
= \Pi t_2 \left( D_2 - t_2 \right) V_2 \bar{\sigma}_2 \int_0^1 \frac{\bar{e}_2^*}{\bar{\sigma}} \, d\varepsilon
\]

(5.29)

Differentiating equation (5.29), we get:

\[
\frac{\partial W_p}{\partial H_{2D}} = \Pi t_2 \left( D_2 - t_2 \right) V_2 \bar{\sigma}_2 \int_0^1 \frac{\partial}{\partial \bar{e}_2} \frac{\bar{e}_2^*}{\bar{\sigma}} \, d\varepsilon \frac{d\bar{e}_2}{dH_{2D}}
\]

\[
= \Pi t_2 \left( D_2 - t_2 \right) V_2 \bar{\sigma}_2 \int_0^1 \frac{d\bar{e}_2}{dH_{2D}}
\]

(5.30)

The expression for the generalised plastic strain at the die exit has been given by Blazynski and Cole (1963-64), as follows:

\[
\bar{e}_2 = \frac{V_2}{2} \left[ \frac{2}{3} \left( \ln \frac{A_1}{A_2} \right)^2 + \left( \ln \frac{D_2 - t_2}{D_1 - t_1} \right)^2 \right]^{1/2}
\]

\[
= \frac{V_2}{2} \left[ \frac{2}{3} \left( \ln \frac{t_1 (D_1 - t_1)}{t_2 (D_2 - t_2)} \right)^2 + \left( \ln \frac{D_2 - t_2}{D_1 - t_1} \right)^2 \right]^{1/2}
\]

\[
= \frac{V_2}{2} \left[ \frac{2}{3} \left( \ln \frac{\bar{E}_1 (D_1 - \bar{E}_1)}{(D_2 - 1)^2} \right)^2 + \left( \ln \frac{D_2 - 1}{D_1 - \bar{E}_1} \right)^2 \right]^{1/2}
\]

(5.31)
The incremental plastic strain ($\text{d}e$) has been given by Green (1960) and has been defined by equations (2.31), through (2.35).

For a particular plug drawing set up, the die opening is constant, so that:

$$2t_2 + 2H_{2D} + 2H_{2P} = D_{D2} - D_{P2}$$  \quad (5.32)

Where $t_2$ is the tube thickness at the die outlet, $H_{2P}$ is the lubricant film thickness at the plug-tube interface at the exit section, $D_{D2}$ is the die diameter at the die exit and $D_{P2}$ is the plug diameter at the exit section.

Differentiating equation (5.32), after substituting the value of $H_{2P}$ in terms of $H_{2D}$ by making use of equation (5.23) we get:

$$2 \frac{dt_2}{dH_{2D}} + 2 + 2 \frac{\cos \alpha}{\cos \beta} = 0$$

(As $H_{2P} = H_{2D} \frac{\cos \alpha}{\cos \beta}$)

So that, $$\frac{dt_2}{dH_{2D}} = - \left(1 + \frac{\cos \alpha}{\cos \beta}\right)$$  \quad (5.33)

From equation (5.30), we get:

$$\frac{\partial W_p}{\partial H_{2D}} = \Pi t_2 (D_2 - t_2) V_2 \sigma_2^f \frac{d\bar{e}_2}{dt_2} \frac{dt_2}{dH_{2D}}$$  \quad (5.34)

Substituting equation (5.33) in equation (5.34),

$$\frac{\partial W_p}{\partial H_{2D}} = -\Pi t_2 (D_2 - t_2) V_2 \sigma_2^f \left(1 + \frac{\cos \alpha}{\cos \beta}\right) \frac{d\bar{e}_2}{dt_2}$$  \quad (5.35)

Substituting the value of $\bar{e}_2$ from equation (5.31) in equation (5.35), it can be written as follows:(Appendix 2)
\[
\frac{\partial W}{\partial H_{2D}} = \frac{\Pi V_{2} \overline{v}_{2} f \left(1 + \frac{\cos \alpha}{\cos \beta}\right) M_{2}}{\sqrt{2} \left( F_{2} \right)^{\frac{1}{2}}}
\]

Where \( F_{2} = \left[ \left( \ln \frac{(\overline{D}_{1} - \overline{\varepsilon})}{\overline{H}_{1}} \right)^{2} \right] + \left( \ln \frac{\overline{D}_{2} - 1}{\overline{D}_{1} - \overline{H}_{1}} \right)^{2}
\]

And \( M_{2} = \ln \frac{\overline{D}_{1} - \overline{H}_{1}}{\overline{D}_{2} - 1} \left[ \left( \frac{2 \overline{E}_{1} (\overline{D}_{2} - 1)}{K_{p} (\overline{D}_{1} - \overline{H}_{1}) \overline{H}_{1}} + (\overline{D}_{2} - 3) \right) t_{2} \right]
\]

\[+ \ln \frac{\overline{D}_{2} - 1}{\overline{D}_{1} - \overline{H}_{1}} \left( \frac{\overline{D}_{1} - \overline{H}_{1}}{K_{p} (\overline{D}_{1} - \overline{H}_{1}) \overline{H}_{1}} + (\overline{D}_{2} - 2) \right) + \frac{\overline{D}_{2} - 1}{\overline{H}_{1}} \]

The rate of shear work of lubricant at the die-tube interface is given as:
\[
W_{S1} = \int_{S} \Pi D \tau u \, ds
\]

From the geometry of figure 5.1, we get:
\[
ds = -\frac{1}{2} \left( \sec \alpha \, dD + \cot \alpha \, dH_{D} \right)
\]

Substituting equation (5.40) in equation (5.39), we get:
\[
W_{S1} = -\int_{D_{1}}^{D_{2}} \frac{1}{2} \Pi D \tau u \left( \sec \alpha + \cot \alpha \frac{dH_{D}}{dD} \right) dD
\]

Assuming the lubricant film thickness profile at the die-tube interface to be exponential (Bedi, 1968), we get:
\[
H_{D} = H_{2D} e^{Y \left( D-D_{2} \right)}
\]

Where \( Y \) is a constant to be determined from the end conditions:
\[
H_{D} = H_{1D} \text{ at } D = D_{1} \text{ and } H_{D} = H_{2D} \text{ at } D = D_{2}
\]
From equations (5.42) and (5.43), we get:

\[
\frac{H_{1D}}{H_{2D}} = e^{Y \left( D_1 - D_2 \right)} \\
\frac{Y \, t_2 (D_1 - D_2)}{t_1 (D_1 - D_2)} = e^{Y \left( D_1 - D_2 \right)}
\]  

(5.44)

As the lubricant film thickness, \(H_D\) is small and velocity of the lubricant \(u\) is large, from equation (5.16), we get:

\[
Q = \frac{1}{2} \cdot H_D \cdot u \cdot \Pi_D
\]  

(5.45)

As the volume flow rate, \(Q\) shall be constant in the deformation zone, from equation (5.45), we get:

\[
H_{1D} \cdot u_1 \cdot D_1 = H_D \cdot u \cdot D = H_{2D} \cdot u_2 \cdot D_2
\]  

(5.46)

From equations (5.46) and (5.20), we get:

\[
D_1 \cdot H_{1D} \cdot V_1 = D \cdot H_D \cdot V = D_2 \cdot H_{2D} \cdot V_2
\]  

Or

\[
\frac{H_{1D}}{H_{2D}} = \frac{D_2 \cdot V_2}{D_1 \cdot V_1}
\]  

(5.47)

Substituting equation (5.24) in (5.47), we get:

\[
\frac{H_{1D}}{H_{2D}} = \frac{D_2}{D_1} \cdot \frac{t_1 (D_1 - t_1)}{t_2 (D_2 - t_2)}
\]  

(5.48)

Combining equation (5.44) and (5.48), we get:

\[
Y \left( D_1 - D_2 \right) = \frac{D_2}{D_1} \cdot \frac{t_1 (D_1 - t_1)}{t_2 (D_2 - t_2)}
\]  

Or

\[
Y \left( D_1 - D_2 \right) = \ln \frac{D_2}{D_1} \cdot \frac{t_1 (D_1 - t_1)}{t_2 (D_2 - t_2)}
\]  

Or

\[
Y = \frac{1}{D_1 - D_2} \ln \frac{t_1 (D_1 - t_1)}{t_2 (D_2 - t_2)}
\]  

(5.49)
From equations (5.42) and (5.49), we get:

\[ H_D = H_{2D} e^{\frac{D}{D_1-D_2} \ln \frac{t_1(D_1-t_1)}{t_2(D_2-t_2)}} \]

Or

\[ \ln H_D = \ln H_{2D} + \frac{D-D_2}{D_1-D_2} + \ln \frac{t_1(D_1-t_1)}{t_2(D_2-t_2)} \]

Differentiating equation (5.50), we get:

\[ \frac{1}{H_D} \frac{dH_D}{dD} = \frac{1}{D_1-D_2} \]

Or

\[ \frac{dH_D}{dD} = \frac{H_D}{D_1-D_2} \] (5.51)

Substituting equation (5.51) in equation (5.41), we get:

\[ \dot{W}_{S1} = -\int_{D_1}^{D_2} \frac{1}{2} \Pi_D \frac{u}{H_D} \left( \csc \alpha + \cot \alpha \frac{H_D}{D_1-D_2} \right) dD \] (5.52)

Substituting equation (5.17) in equation (5.52), we get:

\[ \dot{W}_{S1} = -\int_{D_1}^{D_2} \frac{1}{2} \Pi_D \left( \frac{\eta}{H_D} u + \frac{1}{2} H_D p' \right) u \left( \csc \alpha + \cot \alpha \frac{H_D}{D_1-D_2} \right) dD \] (5.53)

Where \( p' = \frac{dp}{ds} \) and has been defined as (Snidle, Dowson and Parson, 1973) follows:

\[ p' = \frac{4}{D} \left[ \frac{\bar{t} \sin \xi + \bar{t} \cos \xi}{D} \right] \]

\[ = \frac{4t_2}{D} \left[ \frac{\bar{t} \sin \xi + \bar{t} \cos \xi}{D} \right] \] (5.54)

Substituting the value of \( u \) from equation (5.20) and \( H_D \) from equation (5.42) in equation (5.53), we get:

\[ \dot{W}_{S1} = -\frac{C_5I_5}{H_{2D}} - C_6I_6 H_{2D} - C_7I_7 - C_8I_8 H_{2D}^2 \] (5.55)
Where \( C_5 = \frac{\Pi v^2 t^2 (D_2-t_2)^2 \eta_0}{2 \cos^2 \alpha \sin \alpha} \)

\[ = \frac{\Pi m A_0 v^2 (\bar{D}_2-1) t^2}{2 \cos \alpha \sin \alpha} \]

(5.56)

Where \( m = \frac{\eta_0 v^2}{A_0 \bar{D}_1} \) \hspace{1cm} (5.57)

\( m \) is defined as modified Sommerfeld number

\[ C_6 = \frac{\Pi v^2 t^2 (D_2-t_2)^2}{4 \cos \alpha \sin \alpha} = \frac{\Pi v^2 t^2 (\bar{D}_2-1)}{4 \cos \alpha \sin \alpha} \hspace{1cm} (5.58) \]

\[ C_7 = \frac{\Pi v^2 t^2 (D_2-t_2)^2 \eta_0}{2 (D_1 - D_2) \cos \alpha \sin \alpha} \]

\[ = \frac{\Pi v^2 A_0 t^4}{2 \cos \alpha \sin \alpha} \frac{(\bar{D}_2-1) \bar{D}_1}{(\bar{D}_1 - \bar{D}_2) \cos \alpha \sin \alpha} \hspace{1cm} (5.59) \]

\[ C_8 = \frac{\Pi v^2 t^2 (D_2-t_2)^2}{4 (D_1-D_2) \sin \alpha} = \frac{\Pi v^2 t^2 (\bar{D}_2-1)}{4 (\bar{D}_1-\bar{D}_2) \sin \alpha} \hspace{1cm} (5.60) \]

And \( I_5 = \int_{D_1}^{D_2} \frac{D e^\gamma_0 p}{Y(D-D_2) t^2 (D-t)^2} dD \)

\[ = \frac{\bar{D}_2 \bar{D} e^\gamma_0 p}{\bar{D}_1 e t^2 (\bar{D}-\bar{D}_2) t^2 (\bar{D}-\bar{D}_1) t^2} \hspace{1cm} (5.61) \]

\[ I_6 = \int_{D_1}^{D_2} \frac{Y(D-D_2)}{t (D-t)} dD \]

\[ = \int_{\bar{D}_1}^{\bar{D}_2} \frac{Y t_2 (\bar{D}-\bar{D}_2)}{t (\bar{D}-\bar{D}_1)} dD \hspace{1cm} (5.62) \]

\[ I_7 = \int_{D_1}^{D_2} \frac{D e^\gamma_0 p}{t^2 (D-t)^2} dD \]
Differentiating equation (5.55), we get:
\[
\frac{\partial W_{S1}}{\partial H_{2D}} = \frac{C_5}{H_{2D}^2} I_5 - C_6 I_6 - 2C_8 I_8 H_{2D}
\] (5.65)

The rate of shear work of lubricant at the plug-tube interface is given as:
\[
\dot{W}_{S2} = \int_S \tau D' \dot{\tau} u \, ds
\] (5.66)

Where \( D' \) is the internal diameter of the tube at any section and is given as:
\[
D' = D - 2t
\] (5.67)

From the geometry of figure 5.1, we get:
\[
ds = \left( -\frac{1}{2} \cosec \beta \, dD' + \cot \beta \, dH_p \right)
\] (5.68)

Where \( \beta \) is the semi plug angle and \( H_p \) is the lubricant film thickness at plug-tube interface.

Substituting equation (5.68) in equation (5.64), we get:
\[
\dot{W}_{S2} = \int_{D_1}^{D_2} \int_{D_1}^{D_2} \tau D' \dot{\tau} \left( -\frac{1}{2} \cosec \beta + \cot \beta \frac{dH_p}{dD'} \right) dD'
\] (5.69)
Substituting equation (5.17) in equation (5.69), we get:

\[ W_{S2} = - \int_{D_1'}^{D_2'} \Pi_1 D' u \left( \frac{\gamma}{H_p} + \frac{1}{2} \frac{\partial H_p}{\partial s} \right) \left( \frac{1}{2} \csc \beta + \cot \beta \frac{\partial H_p}{\partial s} \right) dD' \]

(5.70)

Assuming exponential variation of lubricant film thickness in the plug-tube interface (Bedi, 1968), we get:

\[ H_p = H_{2p} e^{Y'(D'-D')_2} \]

(5.71)

Where \( Y' \) is a constant to be determined from the end conditions:

\[ H_p = H_{1p} \text{ at } D' = D_1' \text{ and } H_p = H_{2p} \text{ at } D' = D_2' \]

(5.72)

From equations (5.71) and (5.72), we get:

\[ \frac{H_{1p}}{H_{2p}} = e^{Y'(D'_1 - D'_2)} \]

(5.73)

As the volume flow rate in the deformation zone remains constant:

\[ Q = \frac{1}{2} H_p \Pi_1 D' \]

(5.74)

Substituting the value of \( u \) from equation (5.21) in equation (5.74), we get:

\[ \frac{1}{2} H_{1p} \frac{V_1}{\cos \beta} \Pi_1 D'_1 = \frac{1}{2} H_p \frac{V}{\cos \beta} \Pi_1 D' = \frac{1}{2} H_{2p} \frac{V_2}{\cos \beta} \Pi_1 D'_2 \]

Or

\[ H_{1p} \frac{V_1 D'_1}{V_2 D'_2} = H_p V D' = H_{2p} \frac{V_2 D'_2}{V_1 D'_1} \]

Or

\[ \frac{H_{1p}}{H_{2p}} = \frac{V_2 D'_2}{V_1 D'_1} \]

(5.75)
Substituting the value of \( \frac{V_2}{V_1} \) from equation (5.24) in equation (5.75), we get:

\[
\frac{H'_1}{H_2} = \frac{(D_1-t_1)t_1}{(D_2-t_2)t_2} \cdot \frac{D'_2}{D'_1}
\]  
(5.76)

Combining equations (5.76) and (5.73), we get:

\[
\begin{align*}
Y'(D'_1 - D'_2) &= \frac{(D_1-t_1)t_1}{(D_2-t_2)t_2} \cdot \frac{D'_2}{D'_1} \\
\text{Or} \quad Y'(D'_1 - D'_2) &= \ln \left( \frac{(D_1-t_1)t_1}{(D_2-t_2)t_2} \right) \cdot \frac{D'_2}{D'_1} \\
\text{Or} \quad Y' &= \frac{1}{D'_1 - D'_2} \ln \left( \frac{(D_1-t_1)t_1}{(D_2-t_2)t_2} \right) \cdot \frac{D'_2}{D'_1} 
\end{align*}
\]  
(5.77)

Substituting equation (5.77) in equation (5.71), we get:

\[
H'_p = H'_2 e^{D'_1 - D'_2} \ln \left( \frac{(D_1-t_1)t_1}{(D_2-t_2)t_2} \right) \cdot \frac{D'_2}{D'_1}
\]

\[
\text{Or} \quad \ln H'_p = \ln H'_2 + \frac{D'_1 - D'_2}{D'_1 - D'_2} \ln \left( \frac{(D_1-t_1)t_1}{(D_2-t_2)t_2} \right) \cdot \frac{D'_2}{D'_1}
\]  
(5.78)

Differentiating equation (5.78), we get:

\[
\frac{1}{H'_p} \frac{dH'_p}{dD'} = \frac{1}{D'_1 - D'_2}
\]  
(5.79)

Substituting equation (5.79) in equation (5.70), we get:

\[
W_{S2} = \int_{D'_1}^{D'_2} \frac{V}{\cos \beta} \left( \frac{V}{\cos \beta} H'_p + \frac{1}{2} H'_p p' \right) \left( \frac{1}{2} \cosec \beta + \cot \beta \frac{H'_p}{D'_1 - D'_2} \right) dD'
\]  
(5.80)
Since \( u = \frac{V}{\cos \beta} \), equation (5.21)

Where \( P' = \frac{4}{D'} (\bar{g} \sin \beta + \bar{t} \cos \beta) \)

\[
= \frac{4t_2}{D'} (\bar{g} \sin \beta + \bar{t} \cos \beta)
\]

(5.81)

Substituting the value of \( V \) from equation (5.24) and \( H_p \) from equation (5.71) in equation (5.80), we get:

\[
W = - \frac{C_9}{H_2D} I_9 - C_{10} I_{10} H_2D - C_{11} I_{11}
\]

\[
- \frac{C_{12}}{H_2D} I_{12}
\]

Where \( C_9 = \frac{\Pi V_2 t_2 (D_2-t_2) \cos \alpha}{2 \cos \beta \sin \beta \cos \alpha} \)

\( C_{10} = \frac{\Pi V_2 t_2 (D_2-t_2) \cos \alpha}{4 \cos^2 \beta \sin \beta} \)

\( C_{11} = \frac{\Pi V_2 \lambda_0 m t_2 (\bar{D}_2-1) \bar{D}_1}{\cos \beta \sin \beta (\bar{D}_1' - \bar{D}_2')} \)

\( C_{12} = \frac{\Pi V_2 t_2 (D_2-t_2) \cos \alpha}{2 \cos^2 \beta \sin \beta (D_1'-D_2')} \)

(5.83)

Where \( (m = \frac{\gamma_0 V_2}{\lambda_0 \bar{D}_1}, \text{is defined as modified Sommerfeld number}) \)

\[
I_9 = \int_{D_1}^{D_2} \frac{dD'}{D'} e \sqrt{Y'(D'-D_2')} \ dD'
\]

\[
= \int_{D_1}^{D_2} \frac{\bar{D}_1' \ e \sqrt{\gamma_0 \bar{p}}}{\bar{D}_1' \ t_2 \ t_2 (\bar{D}_1'-\bar{D}_2')} \ e \sqrt{Y' \ t_2 (\bar{D}_1'-\bar{D}_2')}
\]

(5.87)

(As \( D' = D-2t \), so \( D-t = D'+t \))
Similarly, \( I_{10} = \int_{\bar{D}_1}^{\bar{D}_2} \frac{p' \bar{D}' e^{y' t_2 (\bar{D}' - \bar{D}_1)}}{\bar{\epsilon} (\bar{D}' + \bar{\epsilon})} \, d\bar{D}' \) \hspace{1cm} (5.88)

\[ I_{11} = \int_{\bar{D}_1}^{\bar{D}_2} \frac{\bar{D}' e^{y_0 p}}{t_2^2 \bar{\epsilon}^2 (\bar{D}' + \bar{\epsilon})} \, d\bar{D}' \] \hspace{1cm} (5.89)

\[ I_{12} = \int_{\bar{D}_1}^{\bar{D}_2} \frac{p' \bar{D}'}{\bar{\epsilon} (\bar{D}' + \bar{\epsilon}) e^{2Y' t_2 (\bar{D}' - \bar{D}_2)}} \, d\bar{D}' \] \hspace{1cm} (5.90)

Differentiating equation (5.82), we get:

\[ \frac{\partial S_{22}}{\partial H_{2D}} = \frac{C_9}{H_{2D}^2} I_9 - C_{10} I_{10} - 2C_{12} H_{2D} I_{12} \] \hspace{1cm} (5.91)

Substituting equation (5.36), equation (5.65) and equation (5.91) in equation (5.27), we get:

\[ \frac{\pi V_2 \bar{\sigma}_2 \ell (1 + \frac{\cos \alpha}{\cos \beta}) M_2}{\sqrt{2} (F_2)} + \frac{C_5}{H_{2D}^2} I_5 - C_6 I_6 \]

\[ - 2C_8 I_8 H_{2D} + \frac{C_9}{H_{2D}^2} I_9 - C_{10} I_{10} - 2C_{12} H_{2D} I_{12} = 0 \] \hspace{1cm} (5.92)

Or

\[ X_2 + \frac{1}{H_{2D}^2} (C_5 I_5 + C_9 I_9) - 2H_{2D} (C_8 I_8 + C_{12} I_{12}) \]

\[ - (C_6 I_6 + C_{10} I_{10}) = 0 \] \hspace{1cm} (5.93)

Where

\[ X_2 = \frac{\pi V_2 \bar{\sigma}_2 \ell (1 + \frac{\cos \alpha}{\cos \beta}) M_2}{\sqrt{2} (F_2)^{1/2}} \] \hspace{1cm} (5.94)

Equation (5.93 can also be written as:

\[ X_2 + \frac{X_3}{H_{2D}^2} - 2X_4 H_{2D} - X_5 = 0 \]

Or

\[ 2X_4 H_{2D}^2 - (X_2 - X_5) H_{2D}^2 - X_3 = 0 \] \hspace{1cm} (5.95)
\( T \)here \( X = C_1 + C_2 \) \( (5.96) \)

\( X_4 = C_8 I_8 + C_{12} I_{12} \) \( (5.97) \)

\( X_5 = C_6 I_6 + C_{10} I_{10} \) \( (5.98) \)

5.4 **Temperature Effect:**

To include the temperature effect in the hydrodynamic analysis of plug drawing, an approach similar to that of tube sinking analysis with temperature effect discussed in chapter 3, has been used. The lubricant flow in the deformation zone has been modified by replacing lubricant velocity \( u \) by \( u(W) \), [where \( W \) is defined by equation (3.34)] in the hydrodynamic analysis of plug drawing given in section 5.3 for including the temperature effect as has been done in the case of tube sinking.

5.5 **Solution Procedure:**

The equations of plastohydrodynamic analysis of plug drawing have been solved numerically by using DEC-20 computer. The procedure to be adopted is similar to the procedure discussed in section 2.4 in case of tube sinking. The drawing stress and the plastic strain at the die exit has been calculated by using an incremental approach. The incremental drawing stress \( d\sigma_X \) and incremental strain \( d\varepsilon \) for plug drawing has been calculated by using equations (5.15) and (2.31) respectively. The drawing stress and plastic strain at the subsequent sections in the deformation zone has been obtained by adding the incremental drawing stress and incremental plastic strain values to the initial values of the drawing stress and plastic strain. The drawing stress and plastic strain values at the die inlet has been taken as zero and as 0.002 corresponding to the concept of proof stress.
The calculation of total energy dissipation in plug drawing has been obtained by calculating the plastic work \( \dot{W}_p \) and lubricant shear work at the die-tube interface \( \dot{W}_{sl} \) and at the plug tube interface \( \dot{W}_{s2} \) by making use of equations (5.29), (5.55) and (5.82) respectively and employing the equation (5.26) to calculate the total energy dissipation rate \( \dot{E} \).

The lubricant film thickness at the die-tube and plug-tube interfaces in plug drawing has been obtained by using an iterative approach. The final value of lubricant film thickness at die tube interface, at the die exit \( H_{2D} \) has been obtained by making use of the approximate value of \( H_{2D} \) and refined value of \( H_{2D} \) to be calculated in a manner similar to that discussed in section 2.4 for calculating the lubricant film thickness at the die exit \( H_2 \) in case of tube sinking. After obtaining the final value of \( H_{2D} \), the lubricant film thickness at the plug tube interface has been obtained by making use of equation (5.23). To obtain the lubricant film thickness at the die tube interface and plug tube interface within the deformation zone, the equations (5.42) and (5.71) have been used.

The apparent friction coefficient in the deformation zone has been obtained by using an approach exactly similar to that of tube sinking discussed in section 2.4.

5.6 Numerical Data:

The following numerical data has been used to illustrate the application of plastohydrodynamic analysis of plug drawing:

(i) Tube Material: (Johnson and Mellor, 1962)
Annealed Aluminium
\[
\sigma_0 = 1.5296 \times 10^8 \text{ N/m}^2, B=0, n=0.25
\]

(ii) Lubricant (Reid and Schey, 1978)
Bright Stock,
\[
\eta_0 = 0.973 \text{ Ns/m}^2, \gamma_0 = 2.6 \times 10^{-8} \text{ m}^2/\text{N},
\]
\[
\theta = 0.0467 \text{ K}^{-1}
\]
Semi die angle (Radians) $\alpha$, 0.15

Semi plug angle (Radians) $\beta$, 0.01, 0.02, 0.03, 0.04, 0.05, 0.06, 0.07, 0.08, 0.09, 0.10, 0.11, 0.12, 0.13 and 0.14.

Initial tube outside diameter ($D_1$): 0.0250 m.

Initial tube thickness ($t_1$): 0.00125 m.

Modified Sommerfeld number ($m \times 10^{-8}$): 0.25, 1.0, 1.21, 1.44, 2.56, 5.76 and 9.0.

Reduction in tube thickness (percent): 10, 12, 14, 15, 16, 18, 20.

5.7 Results And Discussion:

The results of the hydrodynamic analysis of plug drawing by using the numerical data given in section 5.6 are shown in figure 5.2 through 5.11.

Figures 5.2 and 5.3 show the effect of semi plug angle on the energy consumption rate and dimensionless drawing stress respectively. As is clear from figure 5.2 and figure 5.3 the energy consumption rate and the dimensionless drawing stress increases exponentially with increase of semi plug angle in the hydrodynamic plug drawing. A small semi plug angle is therefore desirable. This observation is in confirmation with the industrial practice of using a cylindrical plug ($\beta = 0$) for plug drawing (Blazynski and Cole, 1960).

Figures 5.4 and 5.5 depict the effect of draft (reduction of tube thickness) on the sink (reduction of tube outside diameter) during the plug drawing process. It can be seen from these figures that the reduction of tube thickness in plug drawing causes a very small change in the tube outside diameter.
The effect of semi plug angle on the maximum strain at the die exit in case of plug drawing has been shown in figure 5.6. The figure also indicate the critical strain ($\varepsilon_{\text{crit}}$) for the tube material. The maximum strain during the plug drawing should not exceed this critical value for the given set of parameters such as magnitude of draft. The critical dimensionless drawing stress corresponding to the critical strain for the given tube material (annealed aluminium) has been calculated as 0.70, and this value has not been exceeded. This figure also suggests that by providing a small value of semi plug angle during plug drawing increased draft can be achieved without exceeding the critical strain value.

Figure 5.7 shows the variation of apparent friction coefficient in the case of plug drawing. It is clear from this figure that the coefficient of friction varies considerably within the deformation zone during plug drawing. This observation is in confirmation with the view that the consideration of variable die friction in the analysis of plug drawing is more appropriate.

Figure 5.8 shows the variation of lubricant film thickness at the die tube and plug tube interfaces during the plug drawing process. The figure reveals that the lubricant film thickness is minimum at the die inlet in case of plug drawing. The magnitude of lubricant film thickness at the plug tube interface is smaller than the lubricant film thickness at the die-tube interface in case of plug drawing as is shown in the figure.

Figure 5.9 shows the effect of semi plug angle on the minimum lubricant film thickness in hydrodynamic plug drawing. A study of this figure reveals that the increase of semi plug angle
causes an increase of lubricant film thickness at the die tube and plug tube interfaces. This observation suggests that providing of a small plug angle may be helpful for the cause of hydrodynamic lubrication.

The effect of modified Sommerfeld number on the dimensionless film thickness at the die-tube and plug-tube interfaces has been shown in figure 5.10. It can be seen from the figure that the increase of modified Sommerfeld number causes an increase of lubricant film thickness. The minimum lubricant film thickness at the die-tube and plug-tube interfaces decide the breakdown of hydrodynamic lubrication in plug drawing. The choice of a proper modified Sommerfeld number is helpful in achieving full fluid film lubrication in plug drawing, together with an acceptable surface finish of the drawn product. A more detailed discussion on this aspect shall be presented later.

Figure 5.11 shows the variation of tube temperature in the deformation zone during the plug drawing. As can be seen from the figure, the tube temperature varies exponentially in the deformation zone during the plastohydrodynamic plug drawing.

5.8 Optimisation Of Plastohydrodynamic Plug Drawing:

The magnitude of lubricant film thickness at the die-tube and plug-tube interfaces during hydrodynamic plug drawing is of importance in achieving the optimisation of plastohydrodynamic plug drawing. As has been discussed above the lubricant film thickness in plug drawing varies linearly with the modified Sommerfeld number. The optimisation of hydrodynamic plug drawing is achieved when the lubricant film thickness at the die-tube and plug-tube interfaces is sufficient to ensure full fluid
film lubrication at these interfaces, but at the same time not so thick so as to result in a poor surface finish of the drawn product. The hydrodynamic lubrication in plug drawing will first break when the lubricant film thickness at the die-tube and/or plug-tube interfaces falls below a specified film thickness which is defined as the critical film thickness. This critical film thickness is a function of the surface roughness of the meeting surfaces (tube and die for the die-tube interface) and (tube and plug for the plug-tube interface) and can be expressed as follows:

(i) For die-tube interface:

$$H_{crit} = \delta_t T_{ft} + \delta_d T_{fd}$$  \hspace{1cm} (5.99)

Where $\delta_t$ and $\delta_d$ are the r.m.s. values of surface roughness for die and tube surface respectively and $T_{fd}$ and $T_{ft}$ are the Tarasov factor for the die and tube surfaces (table 4.1).

(ii) For plug-tube interface:

$$H_{crit} = \delta_p T_{fp} + \delta_t T_{ft}$$  \hspace{1cm} (5.100)

Where $\delta_p$ and $T_{fp}$ are the r.m.s. surface roughness value and Tarasov factor for the plug surface.

As shown in figure 5.10, the lubricant film thickness at the die tube and plug tube interfaces are dependent on the modified Sommerfeld number for a given plug drawing set up.

The choice of the critical film thickness depending on the surface roughness of the meeting surfaces as discussed above will decide the value of modified Sommerfeld number which will produce the film thickness of magnitude equal to the critical film thickness. This value of modified Sommerfeld number is defined as the critical modified Sommerfeld number. With reference to figure 5.10, if we select the critical film thickness ($H_{crit} = 0.130 \times 10^{-3}$)
ANNEXED ALUMINIUM
LUBRICANT BRIGHT STOCK
\( \alpha = 0.15 \text{ RADIANS} \)
\( D_1 = 0.0250 \text{ m.} \)
\( t_1 = 0.00125 \text{ m.} \)
\( m = 1.0 \times 10^{-6} \)
\( S_f = 1 - \frac{t_2}{t_1} = 0.10 \)

**FIG. 5.2** EFFECT OF SEMI PLUG ANGLE ON TOTAL ENERGY CONSUMPTION IN PLASTOHYDRODYNAMIC PLUG DRAWING
**Annealed Aluminium**

**Lubricant Bright Stock**

\[ \alpha = 0.15 \text{ radian} \]
\[ D_1 = 0.25 \text{ m.} \]
\[ t_1 = 0.00125 \text{ m.} \]
\[ m = 1.0 \times 10^{-8} \]
\[ s_1 = 1 - \frac{t_2}{t_1} = 0.15 \]

**Fig. 5.3.** Effect of semi plug angle on dimensionless drawing stress in plug drawing.
Fig. 5.4. EFFECT OF REDUCTION IN TUBE THICKNESS ON REDUCTION IN TUBE OUTSIDE DIAMETER IN PLASTOHYDRODYNAMIC PLUG DRAWING.
Fig. 5.5. Variation of dimensionless tube outside diameter in the deformation zone in plastohydrodynamic plug drawing.
FIG. 5.6. EFFECT OF SEMI PLUG ANGLE ON MAXIMUM STRAIN AT DIE EXIT IN PLUG DRAWING.
ANNÉELED ALUMINIUM

LUBRICANT BRIGHT STOCK

$\alpha = 0.15 \text{ Radian}$

$\beta = 0.01 \text{ Radian}$

$d_1 = 0.0250 \text{ m.}$

$t_1 = 0.00125 \text{ m.}$

$m = 1.0 \times 10^{-8}$

$s_1 = 1 - \frac{t_2}{t_1} = 0.15$

**FIG. 5.7. VARIATION OF APPARENT FRICTION COEFFICIENT IN THE DEFORMATION ZONE IN PLUG DRAWING.**
ANNEXED ALUMINIUM
LUBRICANT BRIGHT STOCK
\[ \alpha = 0.15 \text{ RADIANS} \]
\[ \beta = 0.01 \text{ RADIANS} \]
\[ D_1 = 0.250 \text{ m.} \]
\[ t_1 = 0.0125 \text{ m.} \]
\[ m = 1.0 \times 10^{-8} \]
\[ S_1 = 1 - \frac{t_2}{t_1} = 0.15 \]

\[ H_D = \text{LUBRICANT FILM THICKNESS AT DIE-TUBE INTERFACE} \]
\[ H_P = \text{LUBRICANT FILM THICKNESS AT PLUG-TUBE INTERFACE} \]

FIG. 5.8. VARIATION OF DIMENSIONLESS LUBRICANT FILM THICKNESS IN THE DEFORMATION ZONE IN PLASTOHYDRODYNAMIC PLUG DRAWING.
ANNEALED ALUMINIUM
LUBRICANT BRIGHT STOCK

$\alpha = 0.15$ Radian

$D = 0.0250 \text{ m}$

$t_1 = 0.00125 \text{ m}$

$m = 1.0 \times 10^{-8}$

$S_1 = 1 - \frac{t_2}{t_1} = 0.15$

\[ \overline{H}_D \quad \overline{H}_P \]

$\overline{H} = \text{MINIMUM FILM THICKNESS AT DIE-TUBE INTERFACE.}$

$\overline{H}_P = \text{MINIMUM FILM THICKNESS AT PLUG-TUBE INTERFACE.}$

$H_{\text{CRIT.}}$

FIG. 5.9. EFFECT OF SEMI PLUG ANGLE ON DIMENSIONLESS MINIMUM FILM THICKNESS IN PLUG DRAWING.
FIG. 5.10. EFFECT OF MODIFIED SOMMERFELD NUMBER ON DIMENSIONLESS MINIMUM FILM THICKNESS IN PLUG DRAWING.
ANNEXED ALUMINIUM
LUBRICANT BRIGHT STOCK

$\alpha = 0.15 \text{ Radian}$
$\beta = 0.01 \text{ Radian}$

$D_1 = 0.0250 \text{ m.}$
$t_1 = 0.00125 \text{ m.}$

$s_1 = 1 - \frac{t_2}{t_1} = 0.15$

$m = 1.0 \times 10^{-8}$

**FIG. 5.11. VARIATION OF TUBE TEMPERATURE IN THE DEFORMATION ZONE IN PLUG DRAWING.**
as an example, we find that the modified Sommerfeld number values for die-tube interface is $1.00 \times 10^{-8}$ and plug-tube interface is $1.21 \times 10^{-8}$, corresponding to the critical film thickness. It means that the plug drawing process for the conditions shown in figure 5.10 should be operated at modified Sommerfeld number value close to $1.21 \times 10^{-8}$ for achieving optimum results.

5.9 Conclusions:
The conclusions of plastohydrodynamic analysis of plug drawing may be summarised as follows:

1. The energy consumption rate and the dimensionless drawing stress in plug drawing increases exponentially with increase of semi plug angle.

2. The effect of draft (reduction in tube thickness) on the sink (reduction in tube outside diameter) is small.

3. The apparent friction coefficient varies considerably in the deformation zone during plastohydrodynamic plug drawing.

4. The minimum lubricant film thickness in the deformation zone occurs at the die inlet in case of plug drawing.

5. The magnitude of lubricant film thickness at the plug interface is small compared with the magnitude of lubricant film thickness at the die-tube interface for the same plug drawing set up.

6. The lubricant film thickness increases exponentially with increase of semi plug angle.

7. The lubricant film thickness varies linearly with the modified Sommerfeld number.

8. The tube temperature increases exponentially in the deformation zone in plug drawing.