CHAPTER 1

INTRODUCTION

The flow of water under hydraulic structures and the accompanying hydraulic gradients and pressures that exist at different points in the foundations have long been recognised as important factors in the design of structures on permeable foundations such as weirs.

The formidable nature of flow through the foundations of structures on pervious soil defied empirical, graphical, experimental or analytical solution prior to 1856. In that year a simple kinematic law governing the flow of water through permeable soil was enunciated by Henry d' Aray, a French hydraulic engineer. According to this law, the velocity of water through the pervious material is directly proportional to the hydraulic gradient. Darcy's law is subject to certain limitations such as (i) the Reynolds numbers should not exceed one \(2 \leq 3\). However, the range for the upper limit of Reynolds number varies from 0.1 to 75 according to different investigators, and (ii) the soil medium in which the flow takes place must be completely saturated with water.

It is recognised that a weir should be safe against the efflux of seepage for the following two criteria:

a) Uplift Pressures
b) Piping, that is washing away of the sand particles by the force of seepage water
with regard to the determination of uplift or residual pressures and the exit gradients, the following approaches have been made:

1. Empirical methods based on hydraulic gradient theory vis. Bligh's creep theory and Lane's weighted creep theory

2. Graphical methods based on potential theory:
   a) Forchheimer's flownet method
   b) The Circle method

3. Experimental techniques
   a) Soil models
   b) Viscous fluid models
   c) Electric analogy models

4. Analytical approach based on
   a) Finite element method
   b) Conformal transformation method

The 'hydraulic gradient theory' for weir design originated between John Otlay and Thomas Higham and was developed as a result of experiments by Glibborn. Bligh first believed that the stability of a weir depended upon its weight but later admitted the fallacy of his original belief and became converted to the hydraulic gradient theory. He assumed that the hydraulic gradient was constant and the rate of loss along the creep was linear. This theory enabled the
determination of uplift pressure distribution at various points of the foundation profile of a weir, besides the fact that the average hydraulic gradient, according to Bligh, was an index of the security against piping. While a large number of structures in India and Egypt were built on the basis of this theory, some of them stood the test of time and others failed. Bligh's approach was empirical.

Based on the analysis of data of about two hundred and seventy dams all over the world, Lane found that the horizontal creep path was less effective than the vertical. He modified Bligh's creep theory, giving different weightages to horizontal and vertical creep paths. Since his approach was based on the statistical analysis of existing structures, it found favour with the engineers in the design of structures on permeable foundations, in the absence of a more realistic and scientific analysis although the basic concept of average pressure gradients and creep paths remained the same as in Bligh's theory.

It was Schlichter who proceeded on the assumption that the flow of a fluid in a soil is not different from the flow of a hypothetical fluid which may be supposed to replace both the fluid in the soil and the soil itself. With this assumption, the equation of continuity

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad (1.1)$$
will hold good. This equation, when coupled with Darcy's Law, gives the well-known Laplace Equation:

\[ \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = 0 \]  \hspace{1cm} (1.2)

Equation (1.2) deals with three-dimensional flow. In the case of weirs on permeable soils, the flow is mainly two-dimensional as the width of the river is considerably large and as such the flow at any cross-section of the weir is not appreciably influenced by cross-flow from the sides. For two-dimensional flow, Laplace equation reduces to the form:

\[ \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0 \]  \hspace{1cm} (1.3)

The fact that the flow of water through porous media is governed by Laplace equation implies that the potential theory for the flow of inviscid and incompressible fluids in classical hydrodynamics applies to the flow of water through porous media. The above discovery gave impetus to a number of investigators to apply techniques well-known in classical hydrodynamics for the solution of problems of flow of water through homogeneous soils. By potential theory the flownet can be constructed and uplift pressures and gradients at various points can be determined. Forchheimer has developed a geometrical method of plotting streamlines and potential lines. Modification of Forchheimer's method
has been suggested by Leliavsky. These are methods of trial and error, considered to be sufficiently accurate for practical purposes.

A better approach to the problem can be made by resorting to experimental techniques. In a soil model, a scale model of the structure constructed in a flume with granular soil to represent the soil in the foundation of the weir, is employed to study the seepage through the porous foundations of the weir. The flow lines are observed by injecting a dyed solution at suitable places. The flow net is then completed. This method, though it appears to be simple and easy, has serious limitations, some of which are listed below:

1) The soil model is very difficult to prepare and handle.

ii) Due to non-homogeneous character of the soil, the flow lines are not well-defined. Hence the exact flow-lines are difficult to demarcate. Thus the flow net and hence the pressure gradients may not be accurate.

More accurate and simpler method of solving problems of flow through porous media is by resorting to analogy with such other systems where the following three conditions namely:

1) kinetic law governing the flow
ii) equation of continuity and
iii) boundary conditions
are similar to those of the flow through porous media. In other words, Laplace's differential equation with specific boundary conditions is satisfied. Some such systems are:

1) Flow of viscous fluids through close parallel plates
2) Flow of electricity through a conducting medium

In the viscous fluid model, the actual flow of water through the soil is replaced by the flow of a more viscous fluid such as oil, glycerine or sugar solution through two parallel plates. The method was first suggested by Hele-Shaw in 1897. It has definite advantages over the soil model but does not give as accurate results as the electric analogy method.

The electric analogy model was first applied by Pavlovsky. The method consists essentially of producing and studying an analogous conformation in which the actual flow of water in the soil is replaced by the flow of electricity through an electrolyte in a tank or tray that has the same relative dimensions as the foundation of the weir. The cost of an experiment of this description is generally high and such expenditure is justified in special cases only.
A numerical procedure which has been used with some success to obtain approximate solution of complex flow problem is the relaxation process based on the calculus of finite differences. Essentially the procedure consists of reducing a partial differential equation in the vicinity of a point into an algebraic difference equation. Generally the procedure involves an evaluation of the residual at the point and a systematic refinement of this residual throughout the entire net. The application of relaxation method to find out the pressures beneath the profile of weirs on permeable foundations was suggested by Rao. The method is cumbersome and lengthy as a fine and large grid is required to get accurate results.

The solution of Laplace equation for any boundary conditions has been made possible by the method of conformal transformation devised by Schwarz and Christoffel. Using conformal transformation, mathematical solution for the following simple cases of foundations have been given by Pavlovsky, Zamarin, Weaver, Harza, Haffman, Malhotra, Malhotra, Malinov, Koslov, Bazanov and Fileshakov:

a) A vertical sheet pile
b) A horizontal flush floor with a vertical sheet pile at one end
c) A horizontal flush floor with a vertical sheet pile not at end
d) A stepped horizontal floor, with or without a sheet pile at the step

e) A horizontal floor with a pair of symmetrical sheet piles at ends

f) A depressed floor without sheet piles or with a pair of equal sheet piles at ends

g) A sloping floor without sheet pile

h) A horizontal floor with a pair of equal sheet piles at the ends and a third pile in the centre of the floor

These are the basic or elementary forms for which complete mathematical solutions are available. The solution will become too involved or indeterminate if two or more of these elementary forms are grouped in a weir section. In general, the practical weir sections rarely conform to any one single elementary form. They consist of a combination of almost all the forms mentioned above. Any method, to be of universal application, must aim at a complete solution of all the complex combinations of the elementary forms.

Leliwsky has attempted to give the mathematical solution for two unsymmetrical piles under a horizontal floor. However, the solution given by him is fallacious. For the problem of two or more unsymmetrical piles under a horizontal floor, Pavlovsky\(^27\) introduced his well-known "Method of Fragments" in 1935. This method is an approximate method applicable to weirs on a finite depth of permeable
stratum. Further improvements in Pavlovsky's method of fragments were obtained by Chertousov\textsuperscript{28} and independently by Chugaev\textsuperscript{29,30} and Christoulas.\textsuperscript{31} Another analytical approach to the problem of multiple sheet piles under a horizontal floor on an infinite stratum was made by Malechenko\textsuperscript{32} in his method of "Eliminated cut-offs". Bronzin\textsuperscript{33} devised a similar method called the method of "Successive transformations". However, none of the methods described above is applicable to composite weirs with inclined components.

The first attempt to give a complete solution for a practical weir profile was made by Khosla. He introduced a hypothesis known as "Method of Independent Variables". While accepting the potential theory of flow, complex practical profile is split up into elementary profiles (such as a floor with a pile at one end, a floor with an intermediate pile, a depressed floor without a pile, etc.) for which analytical solutions are available.

In the case of a horizontal floor with a number of vertical sheet piles, the floor is first split up into as many elementary forms as there are number of piles, each form containing one sheet pile under the whole horizontal floor. The pressures at the key points (viz. the points of contact of the pile with the floor and the bottom point
of the pile) are calculated in each case assuming the
other piles to be absent. Then the correction for the
interference of other (neighbouring) piles is applied to
the pressures so obtained. The empirical relation for the
correction for interference by other piles was derived from
the results of electric analogy experiments conducted by
Vaidynathan and Gurudas Ram34,35 at the Irrigation and Power
Research Institute, Amritsar (India). However, the formula
gives very erratic results in some of the cases. Khoala
himself states that "this formula does not apply to the
effect of an outer pile on an intermediate pile if the latter
is equal to or smaller than the former and is at a distance
less than twice the length of the outer pile". Another
shortcoming of the method is that whereas logically it is
evident that correction for interference by other piles
should be applied to pressures at all points, the formula
suggested by Khoala gives correction for the pressure at
only one point. The pressures at other points are not
corrected for the interference by other piles.

It may thus be observed that the problem of
determination of pressures at different points under a
horizontal floor with a number of sheet piles is not
amenable to simple analytical treatment.

Therefore, one aspect of the present study is to
derive an approximation for calculating pressures at various
points under a horizontal floor with a number of vertical
sheet piles.
In the case of a floor with a sloping component, in Khosla's method of "Independent Variables", the pressures at various points are first determined assuming the floor to be horizontal. Then the correction for the sloping floor is applied to the pressures at the upstream and downstream end points of the sloping component of the floor. It is claimed that the empirical correction suggested is based on the analytical solution for a simple sloping floor given by Malhotra. However, it is not logically deducible from Malhotra's solution. Moreover, the effect of a sloping component is assumed to reduce/increase pressures only at the upstream and downstream end points of the sloping component and no correction is applied to the pressures at other (key) points, however close these may be to the end points of the sloping component. Therefore, another aspect taken up for study in the present work is to evolve an analytical approximation to determine pressures at various points under a floor with sloping components. This will also include the case of a depressed floor by treating the vertical side of the depressed floor as a floor component inclined at right angle to the horizontal.

Seepage failure from seepage flow can also take place by undermining of the sub-soil. A knowledge of its causes and of the measures to prevent it is of utmost importance. The undermining of the sub-soil starts from
the tail-end of the work. It begins at the surface due to the residual force of seepage water at this end being in excess of the restraining forces of the sub-soil which tend to hold the latter in position. Once the surface is disturbed the dislocation of sub-soil particles works further upstream and leads to piping.

The conception of undermining by "Floatation" was put forward by Terszaghi\textsuperscript{35} in 1925 and independently arrived at by Haigh\textsuperscript{37} in 1930. According to this theory, the undermining starts when the exit gradient is greater than the critical or floatation gradient which is equal to the submerged unit weight of the soil (neglecting the side friction and cohesion). The critical gradient is a function of the type of soil and is approximately equal to unity in the case of the granular soils commonly encountered in practice. The exit gradient should be less than the critical gradient divided by a factor of safety to avoid failure of the weir by piping. Thus the determination of exit gradient is a very important feature of weir design.

In Khoala's method of "Independent Variables", the exit gradient is calculated by assuming that the floor is horizontal and all other piles except the downstream one are absent. This is evidently not a correct approach as the other piles and the sloping components of floor would definitely have an effect on the exit gradient. There is no solution available to calculate the exit gradient more rationally. Therefore, the third aspect of the present
study is to evolve an analytical approximation for calculating exit gradient for a practical profile of a weir.

SCOPE OF THE PRESENT WORK

From the above, it is evident that no universally applicable analytical solution exists for the practical profile of a weir. The right way to develop the theory, and to find an analytical method for the calculation of uplift pressures and exit gradient for a weir, should consist of using the method of successive conformal transformations of the flow region, until it becomes a simple case of a flat weir without cut-offs. An attempt is, therefore, made in this work to evolve an analytical approximation based on conformal transformations for the computation of pressures and exit gradients for the practical profile of a weir on homogeneous, infinite, permeable stratum. It is true that the model of homogeneous pervious stratum is very often far from reality. However, it is believed that the search for more accurate analytical methods on the basis of homogeneity is justified. The analytical methods are inexpensive, easily applicable and give a clear insight to the problem. Consequently they are preferable in case the model of homogeneous pervious system is considered as satisfactory or the lack of sufficient field measurements makes it necessarily applicable.

An anisotropic stratum can be easily transformed to isotropic one by multiplying the horizontal dimensions by
\[ \sqrt{k_y/k_x} \], where \( k_x \) and \( k_y \) are permeabilities in horizontal and vertical directions respectively.

In Chapter III, a simple approximation for determining pressures under a horizontal floor with two or more vertical sheet piles, where the depth of piles is not large as compared to the distance between them, is attempted. The results obtained by the method suggested are compared with the exact theoretical values in the case of a floor with two symmetrical piles and with the values obtained by Khoile's method of independent variables in the case of a floor with two unsymmetrical piles and a floor with three piles. An empirical formula giving the approximate expected maximum error is also suggested. Complete solutions of a few examples to illustrate the method are given.

In Chapter IV, a more accurate approximation for calculating pressures under a horizontal floor with two vertical piles is evolved. The results obtained are compared with the exact theoretical values in the case of a floor with two symmetrical piles and with the experimental values obtained in electric analogy experiments by Vaidyanathan and Gurdas Rai in the case of two unsymmetrical piles and three piles under a horizontal floor. Illustrative examples have been worked out.

In Chapter V, the second approximation method evolved in Chapter IV, is extended to the cases in which the whole or a part of the floor is inclined. The case of
depressed floor is also covered. The values of pressures obtained by the method are compared with exact theoretical values in the case of a stepped horizontal floor with a vertical sheet pile at the step. The method suggested is applied to a hypothetical problem by assuming the step in the floor as a sloping component of the floor inclined at right angle to the horizontal.

In Chapter VI a method for determining the exit gradient for a practical composite profile, is evolved. The formulae for correction for sloping components of floor and for sheet piles are also given.

In Chapter VII, the pressures at various points under the floor of a practical weir profile are calculated by the method outlined in Chapters IV and V. The pressures so obtained are compared with the experimental values taken from Khoala's publication "Design of weirs on permeable foundations". The exit gradient is also calculated by the method outlined in Chapter VI. The value of exit gradient so obtained is compared with that obtained by the Khoala's method.

In Chapter VIII, broad conclusions, based on the above studies, are outlined.